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Representation of results of particle size analysis —

Part 2:
Calculation of average particle sizes/diameters and moments from particle size distributions

Représentation de données obtenues par analyse granulométrique —

Partie 2: Calcul des tailles/diamètres moyens des particules et des moments à partir de distributions granulométriques

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this part of ISO 9276 may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 9276-2 was prepared by Technical Committee ISO/TC 24, *Sieves, sieving and other sizing methods*, Subcommittee SC 4, *Sizing by methods other than sieving*.

ISO 9276 consists of the following parts, under the general title *Representation of results of particle size analysis*:

— *Part 1: Graphical representation*

— *Part 2: Calculation of average particle sizes/diameters and moments from particle size distributions*

— *Part 3: Calculation of means and moments of particle size distributions*

— *Part 4: Characterization of a classification process.*

— *Part 5: Validation of calculations relating to particle size analyses using the logarithmic normal probability distribution*

Introduction

In particle size analysis, particulate matter is often characterized based on representative samples of the population with the final aim of linking the size information with some other important physical property such as strength, flowability, solubility, etc. In general, a correlation between the physical property and the size of the particles, the so-called property function, can be obtained if an average particle size has been derived or calculated from the measured distribution of sizes.

A unique definition of the average size, $\bar{x}_{k,r}$, is given in this part of ISO 9276, using the so-called moments, $M_{k,r}$, of a size distribution. Apart from average sizes, moments are also used in the calculation of volume related surface area, the spread and other statistical measures of a particle size distribution.

Representation of results of particle size analysis —

Part 2:
Calculation of average particle sizes/diameters and moments from particle size distributions

1 Scope

The object of this part of ISO 9276 is to provide the relevant equations for the calculation of average particle sizes or average particle diameters and moments from a given particle size distribution. It is assumed that the size distribution is available as a histogram. It is nevertheless also possible to apply the same mathematical treatment if the particle size distribution is represented by an analytical function.

It is furthermore assumed in this part of ISO 9276 that the particle size x of a particle of any other shape may also be represented by the diameter of an equivalent sphere, e.g. a sphere having the same volume as the particle concerned.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 9276. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 9276 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 565:1990, *Test sieves — Metal wire cloth, perforated metal plate and electroformed sheet — Nominal sizes of openings*.

ISO 9276-1, *Representation of results of particle size analysis — Part 1: Graphical representation*.

3 Symbols and abbreviated terms

For the purposes of this part of ISO 9276, the following symbols and abbreviated terms apply.

i number of the size class with upper particle size, x_i

k power of x

n total number of size classes

r type of quantity of a distribution (general description)

- $r = 0$ type of quantity, number
- $r = 1$ type of quantity, length

	$r = 2$	type of quantity, surface or projected area
	$r = 3$	type of quantity, volume or mass

$M_{k,r}$ complete k-th moment of a $q_r(x)$-distribution

$m_{k,r}$ complete k-th central moment of a $q_r(x)$-distribution

$q_r(x)$ density distribution

$\bar{q}_{r,i}$ average height of a density distribution in the i-th particle size interval, Δx_i

$\bar{q}_{r,i}(x_{i-1}, x_i)$ histogram

$Q_r(x)$ cumulative distribution

$\Delta Q_{r,i}$ difference of two values of the cumulative distribution, i.e. relative amount in the i-th particle size interval, Δx_i

s_r standard deviation of a $Q_r(x)$ distribution

s_g geometric standard deviation of a normal distribution

S surface area

S_V volume specific surface area

V particle volume

$\bar{V}$ average particle volume

x particle diameter, diameter of a sphere

x_i upper particle size of the i-th particle size interval

x_{i-1} lower particle size of the i-th particle size interval

x_{min} particle size below which there are no particles in a given size distribution

x_{max} particle size above which there are no particles in a given size distribution

$\bar{x}_{k,r}$ average particle diameter (general description)

$\bar{x}_{k,0}$ arithmetic average particle diameter (general description)

$\bar{x}_{1,0}$ arithmetic average length diameter

$\bar{x}_{2,0}$ arithmetic average surface diameter

$\bar{x}_{3,0}$ arithmetic average volume diameter

$\bar{x}_{1,r}$ weighted average particle diameter (general description)

$\bar{x}_{1,1}$ weighted average length diameter

$\bar{x}_{1,2}$ weighted average surface diameter, Sauter-Diameter

$\bar{x}_{1,3}$ weighted average volume diameter

$\bar{x}_{geo,r}$ geometric mean diameter (informative only)

$\bar{x}_{har,r}$ harmonic mean diameter (informative only)

$x_{50,3}$ median particle size of a cumulative volume distribution

$\Delta x_i = x_i - x_{i-1}$ width of the i-th particle size interval

z dimensionless variable of a logarithmic normal probability distribution

4 Basic definition of a moment

The complete k-th moment of a $q_r(x)$ density distribution (see [1] in the bibliography) is represented by integrals as defined in equation (1):

$$M_{k,r} = \int_{x_{min}}^{x_{max}} x^k q_r(x) \mathrm{d}x \tag{1}$$

where

M stands for moment;

k indicates the power of x;

r describes the type of quantity of the density distribution.

If $r = 0$, $q_0(x)$ represents a number density distribution, if $r = 3$, $q_3(x)$ represents a volume or mass density distribution.

Equation (1) describes a "complete moment" if the integral boundaries are represented by the minimum particle size x_{min} and the maximum particle size x_{max}.

A special complete moment is represented by $M_{0,r}$:

$$M_{0,r} = \int_{x_{min}}^{x_{max}} x^0 q_r(x) \mathrm{d}x = \int_{x_{min}}^{x_{max}} q_r(x) \mathrm{d}x = Q_r(x_{max}) - Q_r(x_{min}) = 1 \tag{2}$$

A moment is incomplete, if the integration is performed between two arbitrary particle diameters x_{i-1} and x_i within the given size range of a distribution $x_{min} < x_{i-1} < x < x_i < x_{max}$.

$$M_{k,r}(x_{i-1}, x_i) = \int_{x_{i-1}}^{x_i} x^k q_r(x) \mathrm{d}x \tag{3}$$

Apart from the moments related to the origin of the particle size axis and shown in equations (1) and (3), the so-called k-th central moment of a $q_r(x)$, density distribution, $m_{k,r}$, can be derived from a given density distribution. It is related to the weighted average particle diameter $\bar{x}_{1,r}$ [see equation (11)].

The complete k-th central moment is defined as:

$$m_{k,r} = \int_{x_{\min}}^{x_{\max}} \left(x - \bar{x}_{1,r}\right)^k q_r(x)\,\mathrm{d}x \qquad (4)$$

The incomplete k-th central moment is represented by:

$$m_{k,r}\left(x_{i-1}, x_i\right) = \int_{x_{i-1}}^{x_i} \left(x - \bar{x}_{1,r}\right)^k q_r(x)\,\mathrm{d}x \qquad (5)$$

5 Average particle diameters

All average particle diameters are defined by equation (6):

$$\bar{x}_{k,r} = \sqrt[k]{M_{k,r}} \qquad (6)$$

Depending on the numbers chosen for the subscripts, k and r, different average particle diameters may be defined. Since the average particle diameters calculated from equation (6) may differ considerably, the subscripts k and r should always be quoted.

The two groups of average particle diameters given in 5.1 and 5.2, should preferably be used.

5.1 Arithmetic average particle diameters

Arithmetic mean particle diameters, calculated from a number density distribution, $q_0(x)$, are:

$$\bar{x}_{k,0} = \sqrt[k]{M_{k,0}} \qquad (7)$$

Counting single particles in a microscope image is a typical example of obtaining number ($r = 0$) percentages as a basis for averaging.

The recommended average particle diameters (see [2] in the bibliography) are:

arithmetic average length diameter*:*

$$\bar{x}_{1,0} = M_{1,0} \qquad (8)$$

arithmetic average surface diameter*:*

$$\bar{x}_{2,0} = \sqrt[2]{M_{2,0}} \qquad (9)$$

arithmetic average volume diameter:

$$\bar{x}_{3,0} = \sqrt[3]{M_{3,0}} \qquad (10)$$

5.2 Weighted average particle diameters

Weighted average particle diameters are defined by:

$$\bar{x}_{1,r} = M_{1,r} \tag{11}$$

Weighing sieves before and after sieving is a typical example of obtaining mass (r = 3) percentages as a basis for averaging.

Weighted average particle diameters represent the abscissa of the centre of gravity of a $q_r(x)$ distribution. The recommended weighted average particle diameters are represented by equations (12) to (15).

The weighted average particle diameter of a number density distribution, $q_0(x)$, is equivalent to the arithmetic average length diameter [see equation (8)]. It is represented by the arithmetic average length diameter:

$$\bar{x}_{1,0} = M_{1,0} \tag{12}$$

The weighted average particle diameter of a length density distribution, $q_1(x)$, is given by the weighted average length diameter:

$$\bar{x}_{1,1} = M_{1,1} \tag{13}$$

The weighted average particle diameter of a surface density distribution, $q_2(x)$, is represented by the weighted average surface diameter:

$$\bar{x}_{1,2} = M_{1,2} \tag{14}$$

The weighted average particle diameter of a volume density distribution, $q_3(x)$, is given by the weighted average volume diameter:

$$\bar{x}_{1,3} = M_{1,3} \tag{15}$$

5.3 The calculation of $M_{k,r}$ and average particle diameters from a number or a volume density distribution, $q_0(x)$ or $q_3(x)$

In many cases of practical application, the measured data are either represented by a number density distribution, $q_0(x)$, or a volume density distribution, $q_3(x)$. The calculation of the average particle diameters described above can then be performed according to equation 16 (see [1] in the bibliography):

$$\bar{x}_{k,r} = \sqrt[k]{M_{k,r}} = \sqrt[k]{\frac{M_{k+r,0}}{M_{r,0}}} = \sqrt[k]{\frac{M_{k+r-3,3}}{M_{r-3,3}}} \tag{16}$$

This leads to:

$$\bar{x}_{2,0} = \sqrt{M_{2,0}} = \sqrt{\frac{M_{-1,3}}{M_{-3,3}}} \tag{17}$$

$$\bar{x}_{3,0} = \sqrt[3]{M_{3,0}} = \sqrt[3]{\frac{1}{M_{-3,3}}} \tag{18}$$

$$\bar{x}_{1,1} = M_{1,1} = \frac{M_{2,0}}{M_{1,0}} = \frac{M_{-1,3}}{M_{-2,3}} \tag{19}$$