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**Ball screws —**

Part 4:  
**Static axial rigidity**

*Vis à billes —*

*Partie 4: Rigidité axiale statique*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 3408-4 was prepared by Technical Committee ISO/TC 39, *Machine tools*.

ISO 3408 consists of the following parts, under the general title *Ball screws*.

- *Part 1: Vocabulary and designation* (standards.iteh.ai)
- *Part 2: Nominal diameters and nominal leads — Metric series*
- *Part 3: Acceptance conditions and acceptance tests*
- *Part 4: Static axial rigidity*
- *Part 5: Static and dynamic axial load ratings and operational life*

# Ball screws —

## Part 4: Static axial rigidity

### 1 Scope

This part of ISO 3408 sets forth terms and mathematical relations relevant to the determination of the static axial rigidity of the ball screw.

### 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3408-1:2006, *Ball screws — Part 1: Vocabulary and designation*

### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3408-1 apply.

### 4 Symbols and subscripts

#### 4.1 Symbols

Symbol	Description	Unit
$\alpha$	Contact angle	degrees, °
$\rho$	Reciprocal curvature radius	mm <sup>-1</sup>
$\tau$	Ratio of the semi-major to the semi-minor axes of the contact ellipse	—
$\varphi$	Lead angle	degrees, °
$\Delta l$	Elastic deflection	µm
$c_E$	Material constant	—
$c_K$	Geometry factor	N <sup>-2/3</sup> µm
$d_{bo}$	Diameter of the deep hole bore	mm
$d_C$	Diameter of load application on the ball screw shaft	mm
$D_C$	Diameter of load application on the ball nut	mm
$D_{pw}$	Ball pitch circle diameter	mm
$D_w$	Ball diameter	mm
$D_1$	Outer diameter of ball nut	mm

Symbol	Description	Unit
$E$	Modulus of elasticity	N/mm <sup>2</sup>
$f_{ar}$	Correction factor for accuracy classes (rigidity)	—
$f_{al}$	Correction factor for load application	—
$f_{rs}, f_{rn}$	Conformity (ratio of ball/balltrack radius to ball diameter) of ball screw shaft and ball nut	—
$F$	Axial force, load	N
$i$	Number of loaded turns	—
$k$	Rigidity characteristic	N/μm <sup>3/2</sup>
$l$	Length	mm
$l_s$	Unsupported length of ball screw shaft	mm
$m$	Poisson's constant (e.g. for steel $m = 10/3$ )	—
$n$	Rotational speed	min <sup>-1</sup>
$P_h$	Lead	mm
$q$	Time percentage	%
$R$	Rigidity	N/μm
$s_a$	Backlash (axial play)	μm
$Y$	Auxiliary value according to Hertz for the description of the elliptic integrals of the first and second kinds	N <sup>-2/3</sup> ·μm <sup>4/3</sup>
$z_1$	Number of effectively loaded balls per turn	—
$z_2$	Number of unloaded balls in the recirculation system, only for systems where balls will be recirculated after one turn	—

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## 4.2 Subscripts

Symbol	Description
ar	refers to accuracy
b	refers to ball
bs	refers to ball screw
c	refers to nut body/ball screw shaft
e	refers to external load or the resulting deformation respectively
lim	refers to limit load (at this value the contact between balls and balltracks of ball screw shaft and ball nut is eliminated)
m	refers to equivalent
N	refers to normal load which acts upon balls and balltracks of the ball screw shaft and ball nut in the direction of the contact angle
n	refers to ball nut
pr	refers to preload
s	refers to ball screw shaft
b/t	refers to ball/balltrack area
nu	refers to ball screw within the loaded ball nut area
1	refers to ball nut 1
2	refers to ball nut 2

## 5 Determination of static axial rigidity, $R$

### 5.1 General

The static axial rigidity of a ball screw exerts a major influence on its positioning accuracy. It is a function of the design of the ball screw, its support and bearing arrangement. For the purpose of the calculation given below support and bearing arrangement have been disregarded.

The static axial rigidity of ball screws is not linear. For the purpose of the study of rigidity, a ball screw can be conceived as a combination of several linear and non-linear spring elements. For this reason the rigidity value indicated is correct only for one load application.

The deflection to be determined is caused by

- axial deflections of the screw shaft and the ball nut body,
- radial deflections of the screw shaft and the ball nut body,
- deflections of the balls and the thread land.

The calculation of the deflections attributable to the ball contact is based on the theory related to Hertz stress. The following preconditions should be met as closely as possible:

- the material of the contacting partners shall be homogenous and isotropic,
- in addition, Hooke's law applies, i.e., no plastic deformation, and
- in the contact area only normal stress shall be acting, i.e., a level pressure surface is generated.

Moreover, the applied simplified theory of Hertz specifies identical elasticity modulus and transversal contraction parameter for the material of ball screw shaft, ball nuts and balls.

When calculating axial rigidity it is important to differentiate between ball nuts that have backlash and those that have none, i.e. preloaded ball nuts.

It is possible to generate preload by different methods:

- a) **Single ball nut with continuous thread.** Preloading by oversize balls, resulting in four-point-ball-contact.

See Figure 1.

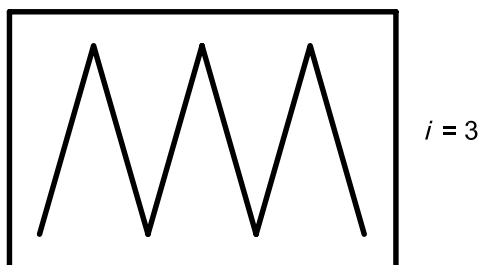


Figure 1

b) **Single ball nut with shifted thread between the preloaded areas**, achieving two-point-ball-contact.

See Figure 2.

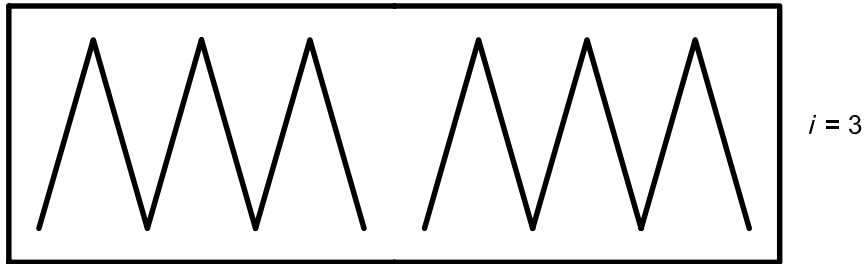


Figure 2

c) **Single ball nut with double start thread and shifted pitch** (two-point-ball-contact).

See Figure 3.



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Figure 3

d) **Double ball nut consisting of two single ball nuts, each with continuous thread.** Axial displacement of the two single ball nuts against each other.

See Figure 4.

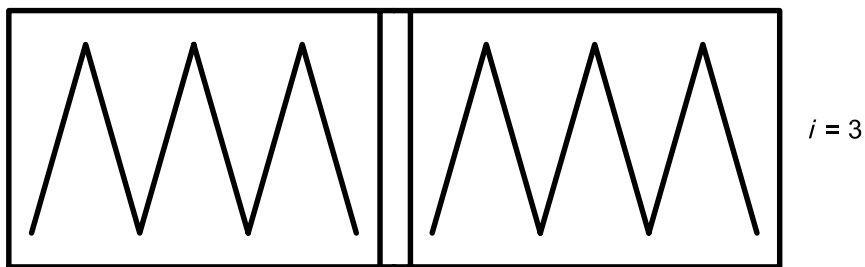


Figure 4

The rigidity calculation set forth in this standard can be applied to all preloading methods described.

As it is very time-consuming — and hence unsuitable for practical purposes — to determine the precise axial deflection on the basis of the corresponding formulae, a reasonably simplified calculation method is outlined below so that the calculation may be effected with a pocket calculator.



## 5.2 Static axial rigidity, $R$

The static axial rigidity,  $R$ , constitutes the resistance to deformation and denotes the force  $\Delta F$ , in newtons, which is required to effect a component deflection  $\Delta l$  by 1  $\mu\text{m}$  in the axial direction of load application:

$$R = \frac{\Delta F}{\Delta l} \quad (1)$$

## 5.3 Static axial rigidity of ball screw, $R_{\text{bs}}$

The overall rigidity,  $R_{\text{bs}}$ , is arrived at by adding the pertinent rigidity values of the components:

$$\frac{1}{R_{\text{bs}}} = \frac{1}{R_{\text{s}}} + \frac{1}{R_{\text{nu,ar}}} \quad (2)$$

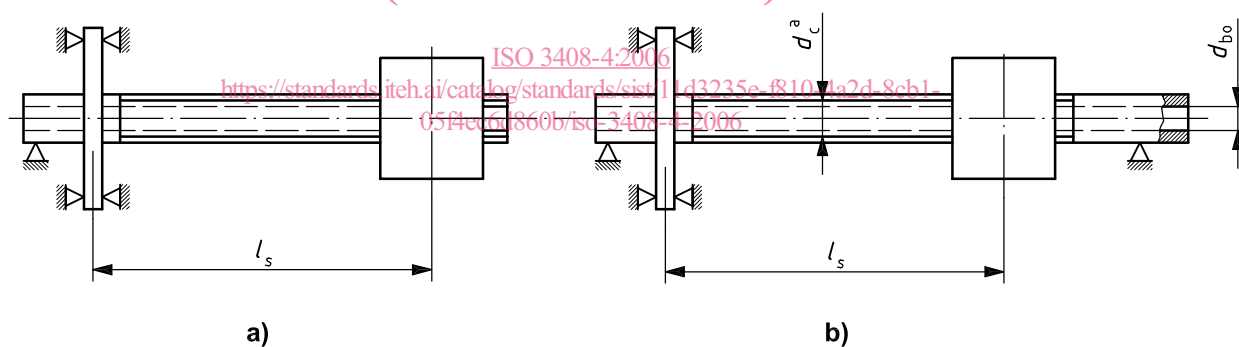
## 5.4 Static axial rigidity of ball screw shaft, $R_{\text{s}}$

### 5.4.1 General

The rigidity of the ball screw shaft follows from the elastic deflection of the ball screw shaft  $\Delta l_{\text{s}}$  caused by an axial force  $\Delta F$  and depends on the bearing arrangement.

### 5.4.2 Rigid mounting of ball screw shaft at one end

See Figure 5.



<sup>a</sup> See Equation (4).

Figure 5

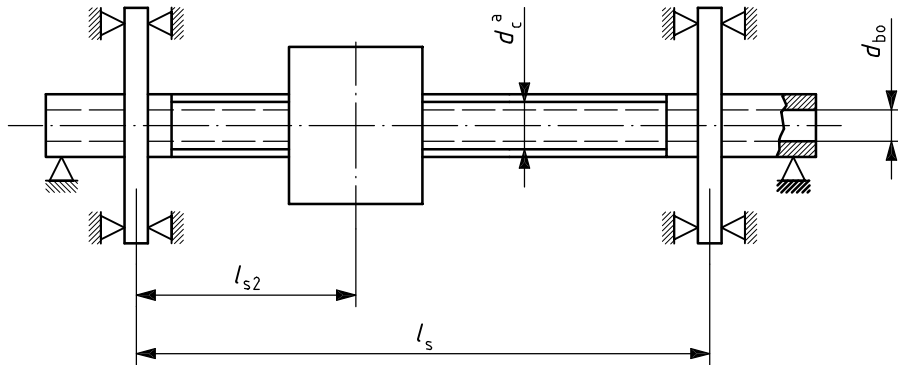
Where the rigidity is

$$R_{s1} = \frac{\pi \cdot (d_c^2 - d_{bo}^2) \cdot E}{4 \cdot l_s \cdot 10^3} \quad \text{in case of a solid shaft } d_{bo} = 0 \quad (3)$$

$$d_c = D_{\text{pw}} - D_w \cdot \cos \alpha \quad (4)$$

5.4.3 Rigid mounting of ball screw shaft at both ends

See Figure 6.



<sup>a</sup> See Equation (4).

Figure 6

Where the rigidity is

$$R_{s2} = \frac{\pi \cdot (d_c^2 - d_{bo}^2) \cdot E}{4 \cdot l_{s2} \cdot 10^3} \cdot \frac{l_s}{l_s - l_{s2}} \tag{5}$$

the minimum of rigidity is obtained at

$$l_{s2} = \frac{l_s}{2}$$

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and thus is

$$R_{s2,min} = \frac{\pi \cdot (d_c^2 - d_{bo}^2) \cdot E}{l_s \cdot 10^3} \tag{6}$$

5.5 Static axial rigidity of ball nut unit,  $R_{nu}$

5.5.1 Static axial rigidity of ball nut unit with backlash,  $R_{nu1}$

5.5.1.1 Static axial rigidity of nut body and screw shaft under resulting radial components of load

$R_{n/s}$

Determination of  $R_{n/s}$ :

$$R_{n/s} = \frac{\Delta F}{\Delta l_{n/s}} \tag{7}$$

$$\Delta l_{n/s} = \frac{\Delta F}{R_{n/s}} \tag{8}$$

Nut: thick-walled cylinder subjected to “internal pressure” (radial component of normal ball thrust).

Screw shaft: cylinder subjected to “external pressure” (radial component of normal ball thrust).

Premise:

- the ball screw shaft is either solid or deephole drilled;
- ball screw shaft and ball nut have the same Young’s modulus and Poisson’s ratio.

The axial rigidity of the nut body and screw shaft under this type of load is

$$R_{n/s} = \frac{2 \cdot \pi \cdot i \cdot P_h \cdot E \cdot \tan^2 \alpha}{\left( \frac{D_1^2 + D_c^2}{D_1^2 - D_c^2} + \frac{d_c^2 + d_{bo}^2}{d_c^2 - d_{bo}^2} \right) \cdot 10^3} \quad (9)$$

where

$$D_c = D_{pw} + D_w \cdot \cos \alpha \quad (10)$$

#### 5.5.1.2 Static axial rigidity in ball/balltrack area, $R_{b/t}$

In order to simplify, the ball nut body and the screw shaft deformations have been disregarded in this calculation. The same applies to

- uneven distribution of load on the balls and threads,
- machining inaccuracies, and
- change of contact angle.

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The relative displacement between ball nut and ball screw shaft due to the axial backlash has not been taken into account because it is not an elastic deflection [see Figure 7 a) and b)].