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Gears — Cylindrical involute gears and gear pairs — Concepts and geometry

Engrenages — Roues et engrenages cylindriques à développante — Concepts et géométrie

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ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

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ISO 21771 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This first edition of ISO 21771 cancels and replaces ISO/TR 4467:1982, of which it constitutes a technical revision.

Gears — Cylindrical involute gears and gear pairs — Concepts and geometry

1 Scope

This International Standard specifies the geometric concepts and parameters for cylindrical gears with involute helicoid tooth flanks. Flank modifications are included.

It also covers the concepts and parameters for cylindrical gear pairs with parallel axes and a constant gear ratio, which consist of cylindrical gears according to it. Gear and mating gear in these gear pairs have the same basic rack tooth profile.

The equations given are not restricted to the pressure angle, $\alpha_P = 20°$.

The standard is structured as follows.

— Listing of symbols and nomenclature for a unique description of gears and gear pairs (see Clause 3).

— Equations and explanations of the relevant values for defining a cylindrical gear and its tooth system. The equations for determination of the nominal values for zero-deviation gear description parameters are stated for radial tooth dimensions (gear tooth heights), the distance between flanks of the same hand, the distance between flanks of opposite hand, as well as the tooth flank characterizing parameters (see Clause 4).

— Equations and explanations of the relevant values for defining cylindrical gear pairs. The equations for the essential parameters characterizing the engagement conditions of the unloaded gear pair are listed (see Clause 5).

— Equations and suggestions for desired flank modifications (see Clause 6).

— Concepts and recommendations needed for a unique geometrical definition of the intended results from manufacture (Clause 7).

— Equations for determination of the nominal values or the limiting values for the most used inspection methods for tooth thickness (see Annex A).

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile*

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth*

ISO 1328-2:1997, *Cylindrical gears — ISO system of accuracy — Part 2: Definitions and allowable values of deviations relevant to radial composite deviations and runout information*

3 Symbols, subscripts and units

3.1 Symbols

Symbol	Description	Used in
a_{w}	centre distance of a cylindrical gear pair	5.2.3
a_0	centre distance in the generating gear unit	7.5
a_{L}	centre distance for tooth flank engagement	A.8
b	facewidth	4.2.8
b_{F}	usable facewidth	4.2.8
b_{M}	contact line overlap (for measuring base tangent length)	A.2.1
b_{w}	active facewidth (the facewidth used)	5.4.7.2
c	tip clearance	5.2.7
c_{F}	form over dimension	5.4.4
d	reference diameter	4.2.4
d_{a}	tip diameter	4.5.3
d_{a0}	tip diameter of tool	7.5
d_{aM}	tip diameter of overcut cylindrical gears	A.9
d_{b}	base diameter	4.3.10
d_{b0}	base diameter of the pinion-type cutter	7.6
d_{f}	root diameter (nominal dimension)	4.5.4
d_{fE}	root diameter produced	7.5
d_{f0}	root diameter of the pinion-type cutter	A.9
d_{v}	V-circle diameter	4.5.1
d_{w}	working pitch diameter	5.2.5
d_{y}	Y-circle diameter	4.3.3
d_{Fa}	tip form diameter	7.6
d_{Fa0}	tip form diameter of the pinion-type cutter	7.6
d_{Ff}	root form diameter	7.6
d_{K}	diameter of circle through centre of ball	A.5
d_{M}	diameter of a measuring circle	A.2.1
d_{Na}	active tip diameter	5.4.1
d_{Nf}	start of active profile diameter (SAP diameter, active root diameter)	5.4.1
e_{t}	space width on the reference cylinder	4.7.3
e_{yt}	space width on the Y-cylinder	4.7.3

Symbol	Description	Used in
e_P	space width of the standard basic rack tooth profile	4.2.3
g_a	length of addendum path of contact	5.4.5.2
g_f	length of dedendum path of contact	5.4.5.2
g_α	length of path of contact	5.4.5.2
$g_{\alpha y}$	distance of a point Y from pitch point C	5.6.1
g_β	arc of contact	5.4.7.4
h	tooth depth (between tip line and root line)	4.6.1
h_a	addendum	4.6.2
h_{aP}	addendum of the standard basic rack tooth profile	Figure 4
h_{aP0}	addendum of the tool standard basic rack tooth profile	7.5
h_f	dedendum	4.6.2
h_{fP}	dedendum of the standard basic rack tooth profile	Figure 4
h_{fP0}	dedendum of the tool standard basic rack tooth profile	A.9
h_w	working depth of teeth in a gear pair	5.2.6
h_{FfP}	depth of dedendum form of the standard basic rack tooth profile	Figure 4
h_K	radius of the tip corner chamfering or tip corner rounding	6.1.2
h_P	tooth depth of standard basic rack tooth profile	Figure 4
inv	involute function	4.3.9
j_{bn}	contact backlash	5.5
j_r	radial backlash	5.5
j_t	circumferential backlash at the reference circle	5.5.2
j_{wt}	circumferential backlash at the pitch circle	5.5
k	number of teeth, spaces or pitches in a span (e.g. number of teeth spanned)	A.2.1
k	addendum modification coefficient	4.5.2
l_{max}	path of engagement	5.4.8
$\sum l$	sum of path of contact	5.4.8
m_n	normal module	4.2.7
m_t	transverse module	4.2.7
m_x	axial module	4.2.7
n_a	number of revolutions of driving gear (rpm)	5.2.2
n_b	number of revolutions of driven gear (rpm)	5.2.2
p	pitch, pitch on the reference cylinder	Figure 4

Symbol	Description	Used in
p_{bn}	normal pitch on the base cylinder	4.4.5
p_{bt}	transverse pitch on the base cylinder	4.4.5.1
p_{en}	normal base pitch on the path of contact	4.4.5.2
p_{et}	transverse base pitch on the path of contact	4.4.5.1
p_n	normal pitch	4.4.2.2
p_t	transverse pitch	4.4.2.1
p_x	axial pitch	4.4.4
p_{yn}	normal pitch on the Y-cylinder	4.4.3
p_{yt}	transverse pitch on the Y-cylinder	4.4.3
p_z	lead	4.3.2
q	machining allowance on tooth flank	7.2
q_{Fs}	undercut	Figure 24
s_{aK}	residual tooth thickness at tip with tip corner chamfering or tip corner rounding	6.1.2
s_{bn}	normal tooth thickness on the base circle	A.2.2
s_n	normal tooth thickness on the reference circle	4.7.5
s_{ni}	minimum normal tooth thickness on the reference circle	7.3
s_{ns}	maximum normal tooth thickness on the reference circle	7.3
s_t	transverse tooth thickness on the reference circle	4.7.1
s_{yn}	normal tooth thickness on the Y-cylinder	4.7.5
s_{yt}	transverse tooth thickness on the Y-cylinder	4.7.1
s_P	tooth thickness of the standard basic rack tooth profile	4.2.3
u	gear ratio	5.2.1
v_g	sliding speed	5.6.1
v_{ga}	sliding speed at the addendum	5.6.1
v_{gf}	sliding speed at the dedendum	5.6.1
v_n	normal speed	5.6.1
x	profile shift coefficient	4.2.9
x_E	generating profile shift coefficient	7.4
x_{Emin}	generating profile shift coefficient at undercut limit	7.7
x_L	profile shift coefficient of master gear	A.8
z	number of teeth	4.1.5
z_a	number of teeth of driving gear	5.2.2

Symbol	Description	Used in
z_b	number of teeth of driven gear	5.2.2
z_L	number of teeth of master gear	A.8
z_0	number of teeth of pinion-type cutter	7.6
A	starting point of meshing	5.4.3
B	starting point of single tooth contact on driving gear	5.4.5.1
C	pitch point, depth of relief for modifications	5.4.3
C_{ay}	modification of the profile	6.5
$C_{\beta y}$	modification of the flank line	6.5
$C_{\Sigma y}$	modification of the flank surface	6.5
$C_{\alpha a}$	amount of tip relief	6.2.1
$C_{\alpha f}$	amount of root relief	6.2.1
C_{Ea}	tip amount of triangular end relief modification	6.4.2
C_{Ef}	root amount of triangular end relief modification	6.4.2
$C_{i,j}$	amount of modification at point (i,j)	6.4.1
$C_{H\alpha}$	amount of transverse profile slope modification	6.2.2
C_{α}	amount of profile crowning (barrelling)	6.2.3
$C_{\beta I}$, $C_{\beta II}$	amount of end relief	6.3.1
C_{β}	amount of flank line crowning	6.3.3
$C_{H\beta}$	amount of flank line slope modification	6.3.2
D_M	measuring ball or measuring cylinder diameter	A.5
D	end point of single tooth contact point on driving gear	5.4.5.1
E	end point of meshing	5.4.3
E_{sn}	normal tooth thickness deviation limit (or allowance)	A.9
E_{sni}	lower deviation limit for tooth thickness	7.3
E_{sns}	upper deviation limit for tooth thickness	7.3
K_g	sliding factor	5.6.2
K_{ga}	sliding factor at tooth tip	5.6.2
K_{gf}	sliding factor at tooth root	5.6.2
L_{AE}	roll length	6.2
L_{Ca}	tip relief roll length	6.2.1
L_{Cf}	root relief roll length	6.2.1
L_{CI}, L_{CII}	length of end relief	6.3.1
L_{Ea}	tip roll length of triangular end relief modification	6.4.2

Symbol	Description	Used in
L_{Ef}	root roll length of triangular end relief modification	6.4.2
M_{dK}	dimension over balls	A.7
M_{dZ}	dimension over cylinders	A.7.1
M_{rK}	radial single-ball dimension	A.5
M_{rZ}	radial single-cylinder dimension	A.6
N	number of tooth or pitch	4.1.6
O	centre of a circle	Figure 10
S_{α}	twist of the transverse profile	6.4.3
S_{β}	twist of the flank line	6.4.3
T_{sn}	tooth thickness tolerance	Figure 37
T	contact point of tangent (lines of engagement) at base circle	Figure 10
U	involute point of origin	4.3.7
W_{k}	base tangent length over k measured teeth or measured spaces	A.2.1
Y	any point on a tooth flank or involute	4.3.5
α_{n}	normal pressure angle	4.3.6
α_{t}	transverse pressure angle	4.3.5
α_{wt}	working transverse pressure angle of gear pair	5.2.4
α_{wt0}	working transverse pressure angle in the generating gear unit	7.6
α_{yn}	normal pressure angle at the Y-cylinder	4.3.6
α_{yt}	transverse pressure angle at the Y-cylinder	4.3.5
α_{Ff}	pressure angle at root form circle	7.6
α_{K}	pressure angle at circle through centre of ball	A.5
α_{Kt}	transverse pressure angle at a point at circle through centre of ball	A.5
α_{Mt}	transverse pressure angle at a point at measuring circle	A.5
α_{P}	pressure angle of the standard basic rack tooth profile	4.3.6
α_{P0}	pressure angle of the tool basic rack tooth profile	7
α_{L}	working transverse pressure angle for double-flank engagement	A.8
α_{vt}	transverse pressure angle at the V-cylinder	A.5
β	helix angle	4.3.3
β_{b}	base helix angle	4.3.3
β_{y}	helix angle at Y-cylinder	4.3.3
δ_{w}	angle of rocking for span measurement	A.2.1

Symbol	Description	Used in
γ	lead angle at reference cylinder	4.3.3
γ_y	lead angle at Y-cylinder	4.3.3
ε_α	transverse contact ratio	5.4.7.1
ε_β	overlap ratio	5.4.7.3
ε_γ	total contact ratio	5.4.7.5
ζ	specific sliding	5.6.3
ζ_f	specific sliding at end points of path of contact	5.6.3
η	space width half angle at reference circle	4.7.4
η_b	base space width half angle	4.7.4
η_y	space width half angle at Y-circle	4.7.4
ξ_y	rolling angle of the involute at point Y	4.3.7
ξ_{Fa0}	rolling angle at tip form circle of pinion-type cutter	7.6
ξ_{Ff}	rolling angle at root form circle	7.6
ξ_{Na}	rolling angle at active tip circle	5.4.1
ξ_{Nf}	rolling angle at active root circle	5.4.1
ρ_{fP}	root radius on the standard basic rack tooth profile	Figure 4
ρ_y	radius of curvature of the involute at point Y	4.3.8
τ	angular pitch	4.4.2
φ_j	backlash angle	5.5.2
φ_α	transverse angle of transmission	5.4.7.1
φ_β	overlap angle	5.4.7.3
φ_γ	total angle of transmission	5.4.7.5
ψ	tooth thickness half angle at reference circle	4.7.2
ψ_b	base tooth thickness half angle	4.7.2
ψ_y	tooth thickness half angle at Y-circle	4.7.2
ω_a	angular velocity of driving gear	5.2.2
ω_b	angular velocity of driven gear	5.2.2
$\sum x$	sum of profile shift coefficients	5.3
$\sum x_E$	sum of profile shift coefficient, non-zero backlash	5.3

3.2 Subscripts

Subscript	Description	Used in [b]
—	[a]	
a	for quantities associated with the tip of a tooth or for the driving gear	5.2.2
b	for quantities associated with the base cylinder	4.3.10
b	for quantities associated with the driven gear	5.2.2
e	for quantities associated with the plane of action	
f	for quantities associated with the root	
g	for “sliding”	
i	for the lower limit in the case of deviations	
k	for a number of teeth, spaces, pitches or spans	
l	for “left-hand”	
m	for a mean value	
max	for a maximum value	
min	for a minimum value	
n	for quantities in a normal section	4.2.6.2
r	for “right-hand”	
s	relating to “tooth thickness”, for the upper limit in the case of deviations	
t	for quantities in a transverse section	4.2.6.1
v	for quantities associated with the V-cylinder	4.5.1
w	for quantities associated with the pitch cylinder and working values of a gear pair	
x	for quantities in an axial section	4.2.6.3
y	for values at a point Y (on the Y-cylinder)	
E	relating to “generating” (e.g. quantities generated on the cylindrical gear) or “generator”	
F	for quantities determining form circles and maximum usable flank area	
K	for quantities resulting from corner chamfering or for ball dimensions	
L	for designating a master gear	
L	for designating left flanks	4.1.8.2
M	for designating a measured value	
N	for active circles	
P	for quantities of the standard basic rack tooth profile	
P0	for quantities of the tool standard basic rack tooth profile	
R	for designating right flanks	4.1.8.2
V	for working side, for rough gear cutting	
W	for measuring base tangent length	

Subscript	Description	Used in [b]
Z	for quantities associated with cylinder dimensions	
α	for quantities associated with contact	
β	for quantities associated with a tooth trace	
γ	for total contact ratio	
$\sum$	for “sum”	
0	for quantities associated with the generating tool or the generating gear unit	7
1	for quantities associated with the pinion (smaller gear) of a gear pair	5.1.3
2	for quantities associated with the wheel (larger gear) or internal gear, used for designating a coefficient relating to the module	5.1.3
I	for locating face	4.2.1
II	for the face opposite the locating face	4.2.1

[a] No subscript designates quantities associated with the reference cylinder.

[b] Used with the symbols listed in 3.1 or as additions.

3.3 Units

The quantities dealt with in this International Standard are to be stated in the following units:

— modules, lengths and linear dimensions in millimetres (mm);

— angles which are to be used in equations in radians (rad);

— angles which can be used for entries or to display results in degrees (°);

— angular velocity in radians per second (rad/s).

NOTE The notation $|z|$, denotes the absolute value, which is always positive, e.g. |–50| = +50. The expression $\frac{z}{|z|}$ is used to extract the sign of the tooth number and is convenient for programming. In particular, it is used often to determine the appropriate sign for an element of an expression; the result is 1 for external gears and –1 for internal gears.

4 Individual cylindrical gears

In this clause, the geometry of gear teeth is described using a generation process based on zero backlash engagement with a basic rack. The relationships are valid for any basic rack, but the standard basic rack (see ISO 53) is used for illustration. The standard basic rack tooth profile of the tooth system has straight flanks. Its datum line is the straight line on which the nominal dimensions of tooth thickness and space width are defined as equal to half the pitch. The standard basic rack tooth profile has the same pressure angles for the left and right flanks and the addendum plus bottom clearance equal to the dedendum. The helix angles for all the tooth flanks of a gear have the same nominal value.

4.1 Concepts for an individual gear

4.1.1 Gear, cylindrical gear, external gear, internal gear

A gear is a rotationally symmetrical object (gear blank) with a tooth system worked into the rim. A cylindrical gear is a gear with a cylindrical reference surface. A distinction is made between external and internal gears according to the radial arrangement of the teeth in each case. The tips of the teeth point outwards in an external gear and inwards in an internal gear.

4.1.2 Tooth system, external teeth, and internal teeth

The tooth system refers to all the teeth and space widths around the rim of a gear. As in 4.1.1, a distinction is made between internal and external gear teeth.

4.1.3 Tooth and space

A tooth is a geometrical element on the gearwheel body that enables the transmission of force and motion. The form and dimensions of the teeth and the distance between consecutive teeth are defined by the tooth system parameters. The space is the gap between two consecutive teeth.

4.1.4 Tooth system parameters

The nominal dimensions of involute cylindrical gear teeth are uniquely determined by the diameter of the reference cylinder, the associated basic rack and its position in relation to the reference circle. The nominal dimensions are defined by the following parameters, which are independent of each other:

— number of teeth, z;

— standard basic rack tooth profile;

— normal module, m_n;

— helix angle, β, and flank direction;

— profile shift coefficient, x;

— tip diameter, d_a;

— facewidth, b.

4.1.5 Number of teeth and sign of number of teeth

The number of teeth around the rim of the gearwheel is denoted by z.

The number of teeth, z, of an external cylindrical gear must be taken as a positive value in the following equations while the number of teeth, z, in an internal cylindrical gear is to be taken as a negative value.

In the case of segments, the number of teeth, z, used in calculations is the number that there would be on the whole circumference.

4.1.6 Tooth number

When numbering teeth, the designations tooth 1, tooth 2, etc. are to be defined on a transverse surface (datum face) viewed in an agreed direction so that the teeth are numbered in ascending order (moving in a clockwise direction). If the letter N is used to denote a reference tooth, the next tooth in the direction of counting is denoted by $N + 1$ and the previous tooth going in the opposite direction by $N - 1$. Tooth No. z is followed by tooth 1 in the direction of counting, see Figure 1.

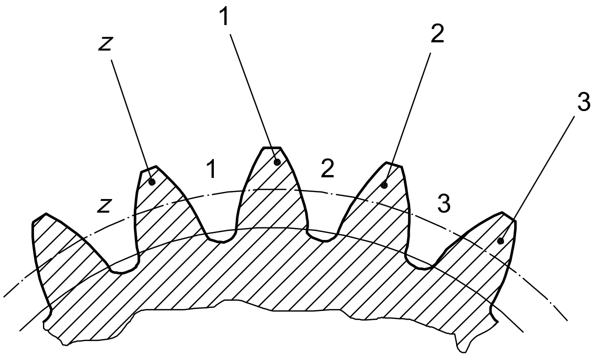

Figure 1 — Numbering of teeth and spaces on datum face

4.1.7 Top land and bottom land

4.1.7.1 Top land

The top land of a tooth is the outermost (innermost in the case of internal gears) periphery of the tooth concentric to the reference cylinder, see Figure 2.

4.1.7.2 Bottom land

The bottom land is the innermost (outermost in the case of internal gears) periphery of the space width concentric to the reference cylinder, see Figure 2.

4.1.8 Tooth flanks and flank sections

4.1.8.1 Tooth flank

Tooth flanks are those parts of the surface of a tooth that are located between the top land and the bottom land, see Figure 2.

4.1.8.2 Right flank, left flank

The right flank (or left flank) is the tooth flank that an observer sees on the right-hand (or left-hand) side when viewing the datum face of a tooth when it is pointing upwards. This definition applies to both external and internal gears, see Figure 2.

Right flank parameters are indicated by the subscript R and left flank parameters by the subscript L.