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Quantities and units —

Part 0:
General principles

Grandeurs et unités —

Partie 0: Principes généraux

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 31-0 was prepared by Technical Committee ISO/TC 12, *Quantities, units, symbols, conversion factors.*

This third edition cancels and replaces the second edition (ISO 31-0:1981). The major technical changes from the second edition are the following:

— new tables of SI base units, SI derived units, SI prefixes and some other recognized units have been added;

— a new subclause (2.3.3) on the unit "one" has been added;

— a new annex C on international organizations in the field of quantities and units has been added.

The scope of Technical Committee ISO/TC 12 is standardization of units and symbols for quantities and units (and mathematical symbols) used within the different fields of science and technology, giving, where necessary, definitions of these quantities and units. Standard conversion factors for converting between the various units also come under the scope of the TC. In fulfilment of this responsibility, ISO/TC 12 has prepared ISO 31.

International Organization for Standardization
Case Postale 56 • CH-1211 Genève 20 • Switzerland

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ISO 31 consists of the following parts, under the general title *Quantities and units*:

- *Part 0: General principles*
- *Part 1: Space and time*
- *Part 2: Periodic and related phenomena*
- *Part 3: Mechanics*
- *Part 4: Heat*
- *Part 5: Electricity and magnetism*
- *Part 6: Light and related electromagnetic radiations*
- *Part 7: Acoustics*
- *Part 8: Physical chemistry and molecular physics*
- *Part 9: Atomic and nuclear physics*
- *Part 10: Nuclear reactions and ionizing radiations*
- *Part 11: Mathematical signs and symbols for use in the physical sciences and technology*
- *Part 12: Characteristic numbers*
- *Part 13: Solid state physics*

Annexes A, B and C of this part of ISO 31 are for information only.

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Quantities and units —

Part 0:
General principles

1 Scope

This part of ISO 31 gives general information about principles concerning physical quantities, equations, quantity and unit symbols, and coherent unit systems, especially the International System of Units, SI.

The principles laid down in this part of ISO 31 are intended for general use within the various fields of science and technology and as a general introduction to the other parts of ISO 31.

2 Quantities and units

2.1 Physical quantity, unit and numerical value

In ISO 31 only physical quantities used for the quantitative description of physical phenomena are treated. Conventional scales, such as the Beaufort scale, Richter scale and colour intensity scales, and quantities expressed as the results of conventional tests, e.g. corrosion resistance, are not treated here, neither are currencies nor information contents.

Physical quantities may be grouped together into categories of quantities which are mutually comparable. Lengths, diameters, distances, heights, wavelengths and so on would constitute such a category. Mutually comparable quantities are called "quantities of the same kind".

If a particular example of a quantity from such a category is chosen as a reference quantity called the *unit*, then any other quantity from this category can be expressed in terms of this unit, as a product of this unit and a number. This number is called the *numerical value* of the quantity expressed in this unit.

EXAMPLE

The wavelength of one of the sodium lines is

$$\lambda = 5{,}896 \times 10^{-7}\ \mathrm{m}$$

Here λ is the symbol for the physical quantity wavelength; m is the symbol for the unit of length, the metre; and $5{,}896 \times 10^{-7}$ is the numerical value of the wavelength expressed in metres.

In formal treatments of quantities and units, this relation may be expressed in the form

$$A = \{A\} \cdot [A]$$

where A is the symbol for the physical quantity, $[A]$ the symbol for the unit and $\{A\}$ symbolizes the numerical value of the quantity A expressed in the unit $[A]$. For vectors and tensors the components are quantities which may be expressed as described above.

If a quantity is expressed in another unit which is k times the first unit, then the new numerical value becomes $1/k$ times the first numerical value; the physical quantity, which is the product of the numerical value and the unit, is thus independent of the unit.

EXAMPLE

Changing the unit for the wavelength from the metre to the nanometre, which is 10^{-9} times the metre, leads to a numerical value which is 10^9 times the numerical value of the quantity expressed in metres.

Thus,

$\lambda = 5{,}896 \times 10^{-7}\ \mathrm{m} = 5{,}896 \times 10^{-7} \times 10^{9}\ \mathrm{nm} = 589{,}6\ \mathrm{nm}$

REMARK ON NOTATION FOR NUMERICAL VALUES

It is essential to distinguish between the quantity itself and the numerical value of the quantity expressed in a particular unit. The numerical value of a quantity expressed in a particular unit could be indicated by placing braces (curly brackets) around the quantity symbol and using the unit as a subscript. It is, however, preferable to indicate the numerical value explicitly as the ratio of the quantity to the unit.

EXAMPLE

$\lambda/\mathrm{nm} = 589{,}6$

NOTE 1 This notation is particularly useful in graphs and in the headings of columns in tables.

2.2 Quantities and equations

2.2.1 Mathematical operations with quantities

Two or more physical quantities cannot be added or subtracted unless they belong to the same category of mutually comparable quantities.

Physical quantities are multiplied or divided by one another according to the rules of algebra; the product or the quotient of two quantities, A and B, satisfies the relations

$$AB = \{A\}\{B\} \cdot [A][B]$$

$$\frac{A}{B} = \frac{\{A\}}{\{B\}} \cdot \frac{[A]}{[B]}$$

Thus, the product $\{A\}\{B\}$ is the numerical value $\{AB\}$ of the quantity AB, and the product $[A][B]$ is the unit $[AB]$ of the quantity AB. Similarly, the quotient $\{A\}/\{B\}$ is the numerical value $\{A/B\}$ of the quantity A/B, and the quotient $[A]/[B]$ is the unit $[A/B]$ of the quantity A/B.

EXAMPLE

The speed v of a particle in uniform motion is given by

$v = l/t$

where l is the distance travelled in the time-interval t.

Thus, if the particle travels a distance $l = 6\ \mathrm{m}$ in the time-interval $t = 2\ \mathrm{s}$, the speed v is equal to

$$v = \frac{l}{t} = \frac{6\ \mathrm{m}}{2\ \mathrm{s}} = 3\ \frac{\mathrm{m}}{\mathrm{s}}$$

The arguments of exponential, logarithmic and trigonometric functions, etc., are numbers, numerical values or combinations of dimension one of quantities (see 2.2.6).

EXAMPLES

$\exp(W/kT)$, $\ln(p/\mathrm{kPa})$, $\sin\alpha$, $\sin(\omega t)$

NOTE 2 The ratio of two quantities of the same kind and any function of that ratio, such as the logarithm of the ratio, are different quantities.

2.2.2 Equations between quantities and equations between numerical values

Two types of equation are used in science and technology: *equations between quantities*, in which a letter symbol denotes the physical quantity (i.e. numerical value × unit), and *equations between numerical values*. Equations between numerical values depend on the choice of units, whereas equations between quantities have the advantage of being independent of this choice. Therefore the use of equations between quantities should normally be preferred.

EXAMPLE

A simple equation between quantities is

$v = l/t$

as given in 2.2.1.

Using, for example, kilometres per hour, metres and seconds as the units for velocity, length and time, respectively, we may derive the following equation between numerical values:

$\{v\}_{\mathrm{km/h}} = 3{,}6\{l\}_{\mathrm{m}}/\{t\}_{\mathrm{s}}$

The number 3,6 which occurs in this equation results from the particular units chosen; with other choices it would generally be different.

If in this equation the subscripts indicating the unit symbols are omitted, one obtains

$\{v\} = 3{,}6\{l\}/\{t\}$

an equation between numerical values which is no longer independent of the choice of units and is therefore not recommended for use. If, nevertheless, equations between numerical values are used, the units shall be clearly stated in the same context.

2.2.3 Empirical constants

An empirical relation is often expressed in the form of an equation between the numerical values of certain physical quantities. Such a relation depends on the units in which the various physical quantities are expressed.

An empirical relation between numerical values can be transformed into an equation between physical quantities, containing one or more empirical constants. Such an equation between physical quantities has the advantage that the form of the equation is independent of the choice of the units. The numerical values of the empirical constants occurring in such an equation depend, however, on the units in which they are expressed, as is the case with other physical quantities.

EXAMPLE

The results of measuring the length l and the periodic time T at a certain station, for each of several pendulums, can be represented by one quantity equation

$$T = C \cdot l^{1/2}$$

where the empirical constant C is found to be

$$C = 2{,}006\ \mathrm{s/m}^{1/2}$$

(Theory shows that $C = 2\pi g^{-1/2}$, where g is the local acceleration of free fall.)

2.2.4 Numerical factors in quantity equations

Equations between quantities sometimes contain *numerical factors*. These numerical factors depend on the definitions chosen for the quantities occurring in the equations.

EXAMPLES

1 The kinetic energy E_k of a particle of mass m and speed v is

$$E_k = \frac{1}{2} m v^2$$

2 The capacitance C of a sphere of radius r in a medium of permittivity ε is

$$C = 4\pi\varepsilon r$$

2.2.5 Systems of quantities and equations between quantities; base quantities and derived quantities

Physical quantities are related to one another through equations that express laws of nature or define new quantities.

For the purpose of defining unit systems and introducing the concept of dimensions, it is convenient to consider some quantities as mutually independent, i.e. to regard these as *base quantities*, in terms of which the other quantities can be defined or expressed by means of equations; the latter quantities are called *derived quantities*.

It is a matter of choice how many and which quantities are considered to be base quantities.

The whole set of physical quantities included in ISO 31 is considered as being founded on seven base quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity.

In the field of mechanics a system of quantities and equations founded on three base quantities is generally used. In ISO 31-3, the base quantities used are length, mass and time.

In the field of electricity and magnetism a system of quantities and equations founded on four base quantities is generally used. In ISO 31-5, the base quantities used are length, mass, time and electric current.

In the same field, however, systems founded on only three base quantities, length, mass and time, in particular the "Gaussian" or symmetric system, have been widely used. (See ISO 31-5:1992, annex A.)

2.2.6 Dimension of a quantity

Any quantity Q can be expressed in terms of other quantities by means of an equation. The expression may consist of a sum of terms. Each of these terms can be expressed as a product of powers of base quantities $A, B, C, \ldots$ from a chosen set, sometimes multiplied by a numerical factor ξ, i.e. $\xi A^\alpha B^\beta C^\gamma \ldots$, where the set of exponents $(\alpha, \beta, \gamma, \ldots)$ is the same for each term.

The *dimension* of the quantity Q is then expressed by the dimensional product

$$\dim Q = \mathsf{A}^\alpha \mathsf{B}^\beta \mathsf{C}^\gamma \ldots$$

where A, B, C, ... denote the dimensions of the base quantities $A, B, C, \ldots$, and where $\alpha, \beta, \gamma, \ldots$ are called the *dimensional exponents*.

A quantity all of whose dimensional exponents are equal to zero is often called a *dimensionless* quantity. Its dimensional product or dimension is $A^0 B^0 C^0 \ldots = 1$. Such a quantity *of dimension one* is expressed as a number.

EXAMPLE

> If the dimensions of the three base quantities length, mass and time are denoted by L, M and T respectively, the dimension of the quantity work is expressed by $\dim W = \mathrm{L}^2\mathrm{M}\mathrm{T}^{-2}$, and the dimensional exponents are 2, 1 and −2.

In the system founded on the seven base quantities length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity, the base dimensions may be denoted by L, M, T, I, Θ, N and J respectively and the dimension of a quantity Q becomes in general

$$\dim Q = \mathrm{L}^{\alpha}\mathrm{M}^{\beta}\mathrm{T}^{\gamma}\mathrm{I}^{\delta}\Theta^{\varepsilon}\mathrm{N}^{\zeta}\mathrm{J}^{\eta}$$

EXAMPLES

Quantity	Dimension
velocity	LT^{-1}
angular velocity	T^{-1}
force	LMT^{-2}
energy	$\mathrm{L}^2\mathrm{MT}^{-2}$
entropy	$\mathrm{L}^2\mathrm{MT}^{-2}\Theta^{-1}$
electric potential	$\mathrm{L}^2\mathrm{MT}^{-3}\mathrm{I}^{-1}$
permittivity	$\mathrm{L}^{-3}\mathrm{M}^{-1}\mathrm{T}^4\mathrm{I}^2$
magnetic flux	$\mathrm{L}^2\mathrm{MT}^{-2}\mathrm{I}^{-1}$
illuminance	$\mathrm{L}^{-2}\mathrm{J}$
molar entropy	$\mathrm{L}^2\mathrm{MT}^{-2}\Theta^{-1}\mathrm{N}^{-1}$
Faraday constant	TIN^{-1}
relative density	1

In ISO 31, the dimensions of the quantities are not explicitly stated.

2.3 Units

2.3.1 Coherent unit systems

Units might be chosen arbitrarily, but making an independent choice of a unit for each quantity would lead to the appearance of additional numerical factors in the equations between the numerical values.

It is possible, however, and in practice more convenient, to choose a system of units in such a way that the equations between numerical values have exactly the same form (including the numerical factors) as the corresponding equations between the quantities. A unit system defined in this way is called *coherent* with respect to the system of quantities and equations in question. The SI is such a system. The corresponding system of quantities is given in ISO 31-1 to ISO 31-10 and in ISO 31-12 and ISO 31-13.

For a particular system of quantities and equations, a coherent system of units is obtained by first defining units for the base quantities, the *base units*. Then for each derived quantity, the definition of the corresponding *derived unit* in terms of the base units is given by an algebraic expression obtained from the dimensional product (see 2.2.6) by replacing the symbols for the base dimensions by those of the base units. In particular, a quantity of dimension one acquires the unit 1. In such a coherent unit system no numerical factor other than the number 1 ever occurs in the expressions for the derived units in terms of the base units.

EXAMPLES

Quantity	Equation	Dimension	Symbol for derived unit
speed	$v = \dfrac{\mathrm{d}l}{\mathrm{d}t}$	LT^{-1}	m/s
force	$F = m\dfrac{\mathrm{d}^2 l}{\mathrm{d}t^2}$	MLT^{-2}	$\mathrm{kg \cdot m/s^2}$
kinetic energy	$E_{\mathrm{k}} = \dfrac{1}{2}mv^2$	$\mathrm{ML}^2\mathrm{T}^{-2}$	$\mathrm{kg \cdot m^2/s^2}$
potential energy	$E_{\mathrm{p}} = mgh$	$\mathrm{ML}^2\mathrm{T}^{-2}$	$\mathrm{kg \cdot m^2/s^2}$
energy	$E = \dfrac{1}{2}mv^2 + mgh$	$\mathrm{ML}^2\mathrm{T}^{-2}$	$\mathrm{kg \cdot m^2/s^2}$
relative density	$d = \dfrac{\varrho}{\varrho_0}$	1	1

2.3.2 SI units and their decimal multiples and sub-multiples

The name *International System of Units* (Système International d'Unités), with the international abbreviation *SI*, was adopted by the 11th *General Conference on Weights and Measures* (Conférence Générale des Poids et Mesures, CGPM) in 1960.

This system includes

— base units
— derived units including supplementary units

which together form the coherent system of *SI units*.

2.3.2.1 Base units

The seven base units are listed in table 1.

Table 1 — SI base units

Base quantity	SI base unit	
	Name	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

2.3.2.2 Derived units including supplementary units

The expressions for the coherent derived units in terms of the base units can be obtained from the dimensional products by using the following formal substitutions:

$L \rightarrow m$ $\quad I \rightarrow A$
$M \rightarrow kg$ $\quad \Theta \rightarrow K$
$T \rightarrow s$ $\quad N \rightarrow mol$
$\quad J \rightarrow cd$

In 1960, the CGPM classified the SI units radian, rad, and steradian, sr, for plane angle and solid angle respectively as "supplementary units".

In 1980, the *International Committee for Weights and Measures* (Comité International des Poids et Mesures, CIPM) decided to interpret the class of supplementary units in the SI as a class of dimensionless derived units for which the CGPM allows the freedom of using or not using them in expressions for SI derived units.

Although, as a consequence of this interpretation, the coherent unit for plane angle and for solid angle is the number 1, it is convenient to use the special names radian, rad, and steradian, sr, instead of the number 1 in many practical cases.

EXAMPLES

Quantity	Symbol for SI unit expressed in terms of the seven base units (and the supplementary units in some cases)
velocity	m/s
angular velocity	rad/s or s^{-1}
force	$kg \cdot m/s^2$
energy	$kg \cdot m^2/s^2$
entropy	$kg \cdot m^2/(s^2 \cdot K)$
electric potential	$kg \cdot m^2/(s^3 \cdot A)$
permittivity	$A^2 \cdot s^4/(kg \cdot m^3)$
magnetic flux	$kg \cdot m^2/(s^2 \cdot A)$
illuminance	$cd \cdot sr/m^2$
molar entropy	$kg \cdot m^2/(s^2 \cdot K \cdot mol)$
Faraday constant	$A \cdot s/mol$
relative density	1

For some of the SI derived units, special names and symbols exist; those approved by the CGPM are listed in tables 2 and 3.

It is often of advantage to use special names and symbols in compound expressions for units.

EXAMPLES

1 Using the derived unit joule $(1\ J = 1\ m^2 \cdot kg \cdot s^{-2})$, one may write

Quantity	Symbol for SI unit
molar entropy	$J \cdot K^{-1} \cdot mol^{-1}$

2 Using the derived unit volt $(1\ V = 1\ m^2 \cdot kg \cdot s^{-3} \cdot A^{-1})$, one may write

Quantity	Symbol for SI unit
permittivity	$s \cdot A \cdot m^{-1} \cdot V^{-1}$

Table 2 — SI derived units with special names, including SI supplementary units

Derived quantity	SI derived unit		
	Special name	**Symbol**	**Expressed in terms of SI base units and SI derived units**
plane angle	radian	rad	1 rad = 1 m/m = 1
solid angle	steradian	sr	1 sr = 1 m^2/m^2 = 1
frequency	hertz	Hz	1 Hz = 1 s^{-1}
force	newton	N	1 N = 1 kg · m/s^2
pressure, stress	pascal	Pa	1 Pa = 1 N/m^2
energy, work, quantity of heat	joule	J	1 J = 1 N · m
power, radiant flux	watt	W	1 W = 1 J/s
electric charge, quantity of electricity	coulomb	C	1 C = 1 A · s
electric potential, potential difference, tension, electromotive force	volt	V	1 V = 1 W/A
capacitance	farad	F	1 F = 1 C/V
electric resistance	ohm	Ω	1 Ω = 1 V/A
electric conductance	siemens	S	1 S = 1 Ω^{-1}
magnetic flux	weber	Wb	1 Wb = 1 V · s
magnetic flux density	tesla	T	1 T = 1 Wb/m^2
inductance	henry	H	1 H = 1 Wb/A
Celsius temperature	degree Celsius[1]	°C	1 °C = 1 K
luminous flux	lumen	lm	1 lm = 1 cd · sr
illuminance	lux	lx	1 lx = 1 lm/m^2

1) Degree Celsius is a special name for the unit kelvin for use in stating values of Celsius temperature. (See also ISO 31-4:1992, items 4-1.a and 4-2.a.)

Table 3 — SI derived units with special names admitted for reasons of safeguarding human health

Derived quantity	SI derived unit		
	Special name	Symbol	Expressed in terms of SI base units and SI derived units
activity (of a radionuclide)	becquerel	Bq	1 Bq = 1 s^{-1}
absorbed dose, specific energy imparted, kerma, absorbed dose index	gray	Gy	1 Gy = 1 J/kg
dose equivalent, dose equivalent index	sievert	Sv	1 Sv = 1 J/kg

2.3.2.3 SI prefixes

In order to avoid large or small numerical values, decimal multiples and sub-multiples of the SI units are added to the coherent system within the framework of the SI. They are formed by means of the prefixes listed in table 4.

Table 4 — SI prefixes

Factor	Prefix	
	Name	Symbol
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

For information about the use of the prefixes, see 3.2.4.

The SI units and their decimal multiples and sub-multiples formed by use of the prefixes are specially recommended.

2.3.3 The unit one

The coherent SI unit for any quantity of dimension one is the unit one, symbol 1. It is generally not written out explicitly when such a quantity is expressed numerically.

EXAMPLE

Refractive index $n = 1{,}53 \times 1 = 1{,}53$

In the case of certain such quantities, however, the unit 1 has special names that could be used or not, depending on the context.

EXAMPLES

Plane angle	$\alpha = 0{,}5$ rad $= 0{,}5$
Solid angle	$\Omega = 2{,}3$ sr $= 2{,}3$
Level of a field quantity	$L_F = 12$ Np $= 12$

Decimal multiples and sub-multiples of the unit one are expressed by powers of 10. They shall not be expressed by combining the symbol 1 with a prefix.

In some cases the symbol % (per cent) is used for the number 0,01.

EXAMPLE

Reflection factor $r = 0{,}8 = 80$ %