INTERNATIONAL STANDARD

ISO 31-11

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Quantities and units —

Part 11:
Mathematical signs and symbols for use in the physical sciences and technology

Grandeurs et unités —

Partie 11: Signes et symboles mathématiques à employer dans les sciences physiques et dans la technique

INTERNATIONAL ISO

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 31-11 was prepared by Technical Committee ISO/TC 12, *Quantities, units, symbols, conversion factors*.

This second edition cancels and replaces the first edition (ISO 31-11:1978). The major technical changes from the first edition are the following:

— a new clause on coordinate systems has been added;

— some new items have been added in the old clauses.

The scope of Technical Committee ISO/TC 12 is standardization of units and symbols for quantities and units (and mathematical symbols) used within the different fields of science and technology, giving, where necessary, definitions of the quantities and units. Standard conversion factors for converting between the various units also come under the scope of the TC. In fulfilment of this responsibility, ISO/TC 12 has prepared ISO 31.

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ISO 31 consists of the following parts, under the general title *Quantities and units*:

— *Part 0: General principles*

— *Part 1: Space and time*

— *Part 2: Periodic and related phenomena*

— *Part 3: Mechanics*

— *Part 4: Heat*

— *Part 5: Electricity and magnetism*

— *Part 6: Light and related electromagnetic radiations*

— *Part 7: Acoustics*

— *Part 8: Physical chemistry and molecular physics*

— *Part 9: Atomic and nuclear physics*

— *Part 10: Nuclear reactions and ionizing radiations*

— *Part 11: Mathematical signs and symbols for use in the physical sciences and technology*

— *Part 12: Characteristic numbers*

— *Part 13: Solid state physics*

Introduction

0.1 General

If more than one sign, symbol or expression is given for the same item, they are on an equal footing. Signs, symbols and expressions in the "Remarks" column are given for information.

Where the numbering of an item has been changed in the revision of a part of ISO 31, the number in the preceding edition is shown in parentheses below the new number for the item; a dash is used to indicate that the item in question did not appear in the preceding edition.

0.2 Variables, functions and operators

Variables, such as x, y, etc., and running numbers, such as i in $\Sigma_i x_i$, are printed in italic (sloping) type. Also parameters, such as a, b, etc., which may be considered as constant in a particular context, are printed in italic (sloping) type. The same applies to functions in general, e.g. f, g.

An explicitly defined function is, however, printed in Roman (upright) type, e.g. sin, exp, ln, Γ. Mathematical constants, the values of which never change, are printed in Roman (upright) type, e.g. e = 2,718 281 8...; π = 3,141 592 6...; $\mathrm{i}^2 = -1$. Well defined operators are also printed in upright style, e.g. div, δ in δx and each d in $\mathrm{d}f/\mathrm{d}x$.

Numbers expressed in the form of digits are always printed upright, e.g. 351 204; 1,32; 7/8.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\omega t + \varphi)$. If the symbol for the function consists of two or more letters and the argument contains no operation sign, such as +; −; ×; ·; or /, the parentheses around the argument may be omitted. In these cases, there should be a thin space between the symbol for the function and the argument, e.g. ent 2,4; sin $n\pi$; arcosh $2A$; Ei x.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$ or $(\cos x) + y$; do not write $\cos x + y$, which could be mistaken for $\cos(x + y)$.

If an expression or equation must be split into two or more lines, the line-breaks should preferably be immediately after one of the signs =; +; −; ±; or ∓; or, if necessary, immediately after one of the signs ×; ·; or /. In this case, the sign works like a hyphen at the end of the first line, informing the reader that the rest will follow on the next line or even on the next page. The sign should not be repeated at the beginning of the following line; two minus signs could for example give rise to sign errors.

0.3 Scalars, vectors and tensors

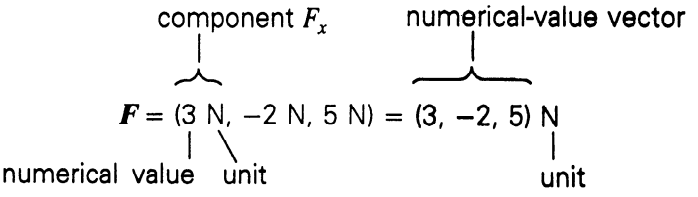
Scalars, vectors and tensors are used to denote certain physical quantities. They are as such independent of the particular choice of coordinate system, whereas each component of a vector or a tensor depends on that choice.

It is important to distinguish between the "components of a vector" $\boldsymbol{a}$, i.e. a_x, a_y and a_z, and the "component vectors", i.e. $a_x\boldsymbol{e}_x$, $a_y\boldsymbol{e}_y$ and $a_z\boldsymbol{e}_z$.

The cartesian components of the position vector are equal to the cartesian coordinates of the point given by the position vector.

Instead of treating each component as a physical quantity (i.e. numerical value × unit), the vector could be written as a numerical-value vector multiplied by the unit. All units are scalars.

EXAMPLE



The same considerations apply to tensors of second and higher orders.

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Quantities and units —

Part 11:
Mathematical signs and symbols for use in the physical sciences and technology

1 Scope

This part of ISO 31 gives general information about mathematical signs and symbols, their meanings, verbal equivalents and applications.

The recommendations in this part of ISO 31 are intended mainly for use in the physical sciences and technology.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this part of ISO 31. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this part of ISO 31 are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 31-0:1992, *Quantities and units — Part 0: General principles.*

3 MATHEMATICAL LOGIC

Item No.	Symbol, sign	Application	Name of symbol	Meaning, verbal equivalent and remarks
11-3.1 (*11-2.1*)	$\wedge$	$p \wedge q$	conjunction sign	p and q
11-3.2 (*11-2.2*)	$\vee$	$p \vee q$	disjunction sign	p or q (or both)
11-3.3 (*11-2.3*)	$\neg$	$\neg p$	negation sign	negation of p; not p; non p
11-3.4 (*11-2.4*)	$\Rightarrow$	$p \Rightarrow q$	implication sign	if p then q; p implies q Can also be written $q \Leftarrow p$. Sometimes $\rightarrow$ is used.
11-3.5 (*11-2.5*)	$\Leftrightarrow$	$p \Leftrightarrow q$	equivalence sign	$p \Rightarrow q$ and $q \Rightarrow p$; p is equivalent to q Sometimes $\leftrightarrow$ is used.
11-3.6 (*11-2-6*)	$\forall$	$\forall x \in A \quad p(x)$ $(\forall x \in A) \quad p(x)$	universal quantifier	for every x belonging to A, the proposition $p(x)$ is true If it is clear from the context which set A is being considered, the notation $\forall x\, p(x)$ can be used. For $x \in A$, see 11-4.1.
11-3.7 (*11-2.7*)	$\exists$	$\exists x \in A \quad p(x)$ $(\exists x \in A) \quad p(x)$	existential quantifier	there exists an x belonging to A for which $p(x)$ is true If it is clear from the context which set A is being considered, the notation $\exists x\, p(x)$ can be used. For $x \in A$, see 11-4.1. $\exists!$ or $\exists^{1}$ is used to indicate the existence of one and only one element for which $p(x)$ is true.

4 SETS

Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.1 (*11-1.1*)	$\in$	$x \in A$	x belongs to A; x is an element of the set A	
11-4.2 (*11-1.2*)	$\notin$	$y \notin A$	y does not belong to A; y is not an element of the set A	The symbol $\not\in$ is also used.
11-4.3 (*11-1.3*)	$\ni$	$A \ni x$	the set A contains x (as element)	$A \ni x$ has the same meaning as $x \in A$.
11-4.4 (*11-1.4*)	$\not\ni$	$A \not\ni y$	the set A does not contain y (as element)	$A \not\ni y$ has the same meaning as $y \notin A$. The symbol $\not\ni$ is also used.
11-4.5 (*11-1.5*)	{ }	$\{x_1, x_2, \ldots, x_n\}$	set with elements $x_1, x_2, \ldots, x_n$	Also $\{x_i : i \in I\}$, where I denotes a set of indices.
11-4.6 (*11-1.6*)	{ | }	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbb{R} \mid x \leqslant 5\}$ If it is clear from the context which set A is being considered, the notation $\{x \mid p(x)\}$ can be used. EXAMPLE $\{x \mid x \leqslant 5\}$
11-4.7 (—)	card	card (A)	number of elements in A; cardinal of A	
11-4.8 (*11-1.7*)	$\varnothing$		the empty set	
11-4.9 (*11-1.8*)	$\mathbb{N}$ **N**		the set of natural numbers; the set of positive integers and zero	$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ Exclusion of zero from the sets 11-4.9 to 11-4.13 is denoted by an asterisk, e.g. $\mathbb{N}^*$. $\mathbb{N}_k = \{0, 1, \ldots, k-1\}$
11-4.10 (*11-1.9*)	$\mathbb{Z}$ **Z**		the set of integers	$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ See remark to 11-4.9.
11-4.11 (*11-1.10*)	$\mathbb{Q}$ **Q**		the set of rational numbers	See remark to 11-4.9.
11-4.12 (*11-1.11*)	$\mathbb{R}$ **R**		the set of real numbers	See remark to 11-4.9.
11-4.13 (*11-1.12*)	$\mathbb{C}$ **C**		the set of complex numbers	See remark to 11-4.9.

4 SETS (*continued*)				
Item No.	**Symbol, sign**	**Application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-4.14 (—)	[,]	$[a, b]$	closed interval in $\mathbb{R}$ from a (included) to b (included)	$[a, b] = \{x \in \mathbb{R} \mid a \leqslant x \leqslant b\}$
11-4.15 (—)	] ,] (,]	$]a, b]$ $(a, b]$	left half-open interval in $\mathbb{R}$ from a (excluded) to b (included)	$]a, b] = \{x \in \mathbb{R} \mid a < x \leqslant b\}$
11-4.16 (—)	[, [[,)	$[a, b[$ $[a, b)$	right half-open interval in $\mathbb{R}$ from a (included) to b (excluded)	$[a, b[= \{x \in \mathbb{R} \mid a \leqslant x < b\}$
11-4.17 (—)	] , [(,)	$]a, b[$ (a, b)	open interval in $\mathbb{R}$ from a (excluded) to b (excluded)	$]a, b[= \{x \in \mathbb{R} \mid a < x < b\}$
11-4.18 (*11-1.13*)	$\subseteq$	$B \subseteq A$	B is included in A; B is a subset of A	Every element of B belongs to A. $\subset$ is also used, but see remark to 11-4.19.
11-4.19 (*11-1.14*)	$\subset$	$B \subset A$	B is properly included in A; B is a proper subset of A	Every element of B belongs to A, but B is not equal to A. If $\subset$ is used for 11-4.18, then $\subsetneq$ shall be used for 11-4.19.
11-4.20 (*11-1.15*)	$\not\subseteq$	$C \not\subseteq A$	C is not included in A; C is not a subset of A	$\not\subset$ is also used. The symbols $\nsubseteq$ and $\not\subset$ are also used.
11-4.21 (*11-1.16*)	$\supseteq$	$A \supseteq B$	A includes B (as subset)	A contains every element of B. $\supset$ is also used, but see remark to 11-4.22. $A \supseteq B$ has the same meaning as $B \subseteq A$.
11-4.22 (*11-1.17*)	$\supset$	$A \supset B$	A includes B properly	A contains every element of B, but A is not equal to B. If $\supset$ is used for 11-4.21, then $\supsetneq$ shall be used for 11-4.22. $A \supset B$ has the same meaning as $B \subset A$.
11-4.23 (*11-1.18*)	$\not\supseteq$	$A \not\supseteq C$	A does not include C (as subset)	$\not\supset$ is also used. The symbols $\nsupseteq$ and $\not\supset$ are also used. $A \not\supseteq C$ has the same meaning as $C \not\subseteq A$.
11-4.24 (*11-1.19*)	$\cup$	$A \cup B$	union of A and B	The set of elements which belong to A or to B or to both A and B. $A \cup B = \{x \mid x \in A \vee x \in B\}$

4 SETS (*continued*)				
Item No.	**Symbol, sign**	**Application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-4.25 (*11-1.20*)	$\cup$	$\bigcup_{i=1}^{n} A_i$	union of a collection of sets $A_1, \ldots, A_n$	$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$, the set of elements belonging to at least one of the sets $A_1, \ldots, A_n$. $\bigcup_{i=1}^{n}$ and $\bigcup_{i \in I}$, $\bigcup_{i \in I}$ are also used, where I denotes a set of indices.
11-4.26 (*11-1.21*)	$\cap$	$A \cap B$	intersection of A and B, read as A inter B	The set of elements which belong to both A and B. $A \cap B = \{x \mid x \in A \wedge x \in B\}$
11-4.27 (*11-1.22*)	$\bigcap$	$\bigcap_{i=1}^{n} A_i$	intersection of a collection of sets $A_1, \ldots, A_n$	$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$, the set of elements belonging to all sets $A_1, A_2, \ldots$ and A_n. $\bigcap_{i=1}^{n}$ and $\bigcap_{i \in I}$, $\bigcap_{i \in I}$ are also used, where I denotes a set of indices.
11-4.28 (*11-1.23*)	$\setminus$	$A \setminus B$	difference between A and B; A minus B	The set of elements which belong to A, but not to B. $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ $A - B$ should not be used.
11-4.29 (*11-1.24*)	$\complement$	$\complement_A B$	complement of subset B of A	The set of those elements of A which do not belong to the subset B. If it is clear from the context which set A is being considered, the symbol A is often omitted. Also $\complement_A B = A \setminus B$
11-4.30 (*11-1.25*)	(,)	(a, b)	ordered pair a, b; couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. $\langle a, b \rangle$ is also used.
11-4.31 (*11-1.26*)	(, ...,)	$(a_1, a_2, \ldots, a_n)$	ordered n-tuplet	$\langle a_1, a_2, \ldots, a_n \rangle$ is also used.

4 SETS (*concluded*)				
Item No.	**Symbol, sign**	**Application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-4.32 (*11-1.27*)	×	$A \times B$	cartesian product of A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$ $A \times A \times \ldots \times A$ is denoted by A^n, where n is the number of factors in the product.
11-4.33 (—)	Δ	Δ_A	set of pairs (x, x) of $A \times A$, where $x \in A$; diagonal of the set $A \times A$	$\Delta_A = \{(x, x) \mid x \in A\}$ id_A is also used.