INTERNATIONAL STANDARD

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Sampling procedures for inspection by variables —

Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)

Règles d'échantillonnage pour les contrôles par mesures —

Partie 5: Plans d'échantillonnage séquentiels indexés d'après la limite d'acceptation de qualité (LAQ) pour l'inspection par variables (écart-type connu)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 3951-5 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

This edition cancels and replaces Annex A of ISO 8423:1991, which has been technically revised to greatly improve its compatibility with the sampling systems in ISO 3951-1.

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

— *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

— *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

The following parts are under preparation:

— *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

— *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

Introduction

In contemporary production processes, quality is often expected to reach such high levels that the number of nonconforming items is reported in parts per million. Under such circumstances, popular acceptance sampling plans by attributes, such as those presented in ISO 2859-1, require prohibitively large sample sizes. When it is possible to apply acceptance sampling plans by variables, such as those presented in ISO 3951-1, the sample sizes are much smaller. However, especially in the case of acceptance of a product of extremely high quality, those sample sizes are still too large. Therefore, there is a need to apply standardized statistical procedures that require the smallest possible sample sizes. Sequential sampling plans are the only statistical procedures that satisfy that need. It has been mathematically proved that among all possible sampling plans having similar statistical properties, the sequential sampling plan has the smallest average sample size. Therefore, there is a strong need to present sequential sampling plans which are statistically equivalent to the commonly used acceptance sampling plans from ISO 3951-1, but which require significantly smaller average sample sizes.

The principal advantage of sequential sampling plans is the reduction in the average sample size. The *average sample number* is the average of all the sample sizes that may occur under a sampling plan for a given lot or process quality level. The use of sequential sampling plans leads to a smaller average sample number than single sampling plans having the equivalent operating characteristic. For the sequential sampling plans in this part of ISO 3951, a curtailment rule has been introduced involving an upper limit of 1,5 n_0 on the actual number of items to be inspected, where n_0 is the sample size of the corresponding single sampling plan.

Other factors that should be taken into account are as follows:

a) Complexity

   The rules of a sequential sampling plan are more easily misunderstood by inspectors than the simple rules for a single sampling plan.

b) Variability in the amount of inspection

   As the actual number of items inspected for a particular lot is not known in advance, the use of sequential sampling plans brings about various organizational difficulties. For example, scheduling of inspection operations may be difficult.

c) Ease of drawing sample items

   If drawing sample items is at different times rather difficult, the reduction in the average sample size by sequential sampling plans may be cancelled out by the increased sampling cost.

d) Duration of test

   If the test of a single item is of long duration and a number of items can be tested simultaneously, sequential sampling plans are much more time-consuming than the corresponding single sampling plans.

e) Variability of quality within the lot

   If the lot consists of two or more sublots from different sources and if there is likely to be any substantial difference between the qualities of the sublots, drawing of a random sample under a sequential sampling plan is more awkward than under the corresponding single sampling plan.

The balance between the advantage of a smaller average sample number of the sequential sampling plan and the above disadvantages leads to the conclusion that sequential sampling plans are suitable only when inspection of individual items is costly in comparison with inspection overheads.

The choice between single and sequential sampling plans should be made before the inspection of a lot is started. During inspection of a lot, it is not permitted to switch from one type of plan to another, because the operating characteristics of the plan may be drastically changed if the actual inspection results influence the choice of acceptability criteria.

Although a sequential sampling plan is on average much more economical than the corresponding single sampling plan, it may occur, during inspection of a particular lot, that acceptance and non-acceptance comes at a very late stage because the cumulative leeway (the statistic used for the determination of lot acceptability) remains between the acceptance value and the rejection value for a long time. With the graphical method, this corresponds to the random progress of the step-wise linear curve remaining in the indecision zone.

In order to alleviate this disadvantage, the sample size curtailment value is set before the inspection of a lot is started, and inspection terminates if the cumulative sample size reaches the curtailment value, n_t, without determination of lot acceptability. The acceptance and non-acceptance of the lot is then determined using the curtailment acceptance and rejection values.

For sequential sampling plans in common use, curtailment usually represents a deviation from their intended usage, leading to a distortion of their operating characteristics. In this part of ISO 3951, however, the operating characteristics of the sequential sampling plans have been determined with curtailment taken into account, so curtailment is an integral component of the provided plans.

Sampling procedures for inspection by variables —

Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)

1 Scope

This part of ISO 3951 specifies a system of sequential sampling plans (schemes) for lot-by-lot inspection by variables. The schemes are indexed in terms of a preferred series of acceptance quality limit (AQL) values, ranging from 0,01 to 10, which are defined in terms of percent nonconforming items.

The schemes of ISO 3951 are intended to induce a supplier through the economic and psychological pressure of lot non-acceptance to maintain a process average at least as good as the specified AQL value, while at the same time providing an upper limit for the risk to the consumer of accepting the occasional poor lot.

The schemes are designed to be applied to a continuing series of lots, that is, a series long enough to allow the switching rules (Clause 6) to be applied. These switching rules provide:

— automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;

— an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to reduced inspection) should consistently good quality be achieved.

This part of ISO 3951 is designed for use under the following conditions:

a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, apply this part of ISO 3951 to each one separately;

b) where only a single quality characteristic, x, of these products is taken into consideration, which must be measurable on a continuous scale;

c) where the uncertainty of the measurement system is negligible with respect to the production process standard deviation;

d) where production is stable (under statistical control) and the quality characteristic, x, is distributed according to a normal distribution (or a close approximation to the normal distribution) or a distribution which may be mathematically transformed to a normal distribution;

e) where the standard deviation of the quality characteristic, x, is known;

CAUTION — The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened previously for nonconforming items.

f) where a contract or standard defines an upper specification limit, U, a lower specification limit, L, or both; an item is qualified as conforming if and only if its measured quality characteristic, x, satisfies the appropriate one of the following inequalities:

1) $x \leqslant U$ (i.e. the single upper specification limit is not violated);

2) $x \geqslant L$ (i.e. the single lower specification limit is not violated);

3) $L \leqslant x \leqslant U$ (i.e. the upper and lower double specification limits are not violated).

In this part of ISO 3951, it is assumed that, where double specification limits apply, conformance to both specification limits is either equally important to the integrity of the product or is considered separately for both specification limits. In the first case, it is appropriate to apply a single AQL to the combined percentage of product outside the two specification limits. This is referred to as a combined AQL requirement. In the second case, separate AQLs apply to nonconformity beyond each of the limits, and this is referred to as a separate AQL requirement.

In this part of ISO 3951, the acceptability of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot. As such, it is not applicable for judging the acceptability of isolated lots or short series of lots. Refer to ISO 2859-2 for applicable sampling plans in this case.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1:1999, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-1:2005, *Sampling procedures for inspection by variables — Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 3534-2, ISO 2859-1, and ISO 3951-1 and the following apply.

3.1
inspection by variables
inspection by measuring the magnitude(s) of the characteristic(s) of an item

[ISO 3534-2]

3.2
sampling inspection
inspection of selected items in the group under consideration

[ISO 3534-2]

3.3
acceptance sampling
sampling after which decisions are made to accept or not to accept a lot, or other grouping of product, material or service, based on sample results

[ISO 3534-2]

3.4
acceptance sampling inspection by variables
acceptance sampling (3.3) inspection in which the acceptability of a process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

[ISO 3534-2]

3.5
process average
rate at which nonconforming items are generated by a process

3.6
acceptance quality limit
AQL
⟨acceptance sampling⟩ worst tolerable quality level

[ISO 3534-2]

NOTE 1 This concept only applies when a sampling scheme with rules for switching and for discontinuation, such as in ISO 2859-1, ISO 3951-1 or this part of ISO 3951 is used.

NOTE 2 Although individual lots with quality as bad as the acceptance quality limit may be accepted with fairly high probability, the designation of an acceptance quality limit does not suggest that this is a desirable quality level. Sampling schemes found in International Standards such as ISO 2859-1, ISO 3951-1 or this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to have process averages consistently better than the AQL. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection pending such improvement will be invoked.

3.7
quality level
quality expressed as a rate of occurrence of nonconforming units

3.8
nonconformity
non-fulfilment of a requirement

[ISO 3534-2]

3.9
nonconforming unit
unit with one or more nonconformities

[ISO 3534-2]

3.10
"σ" method acceptance sampling plan
sigma method
acceptance sampling plan by variables using the presumed value of the process standard deviation

NOTE Adapted from ISO 3534-2.

3.11
specification limit
limiting value stated for a characteristic

[ISO 3534-2]

3.12
lower specification limit
L
specification limit (3.11) that defines the lower limiting value

[ISO 3534-2]

3.13
upper specification limit
U
specification limit (3.11) that defines the upper limiting value

[ISO 3534-2]

3.14
combined AQL requirement
requirement when both upper and lower limits are specified for the quality characteristic and an AQL is given which applies to the combined percent nonconforming beyond the two limits

NOTE The use of a combined AQL requirement implies that nonconformities beyond either specification limit are believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.15
separate AQL requirement
requirement when both upper and lower limits are specified for the quality characteristic and separate AQLs are given which apply to each limit

NOTE The use of separate AQL requirements implies that nonconformities beyond either specification limit are either believed to be of different importance to the lack of integrity of the product or it is desired to control them separately.

3.16
maximum process standard deviation
MPSD
σ_{max}
largest process standard deviation for a given sample-size code letter and **acceptance quality limit** (3.6) for which it is possible to satisfy the acceptance criterion for the combined control of double specification limits under all inspection severities (i.e. normal, tightened and reduced) when the process variability is known

NOTE 1 The MPSD depends on whether the double specification limits are under combined, separate or complex control, but does not depend on the inspection severity.

NOTE 2 Adapted from ISO 3534-2.

3.17
switching rule
instruction within an acceptance sampling scheme for changing from one acceptance sampling plan to another of greater or lesser severity of sampling based on demonstrated quality history

NOTE Normal, tightened or reduced inspection, or discontinuation of inspection, are examples of “severity of sampling”.

[ISO 3534-2]

3.18
measurement
set of operations having the object of determining a value of a quantity

[ISO 3534-2]

3.19
leeway
quantity derived from a measured value of an item

NOTE In the case of a single lower specification limit and in the case of double specification limits, the leeway is obtained by subtracting the numerical value of the lower specification limit from the measured value. In the case of an upper specification limit, the leeway is obtained by subtracting the measured value from the numerical value of the upper specification limit.

3.20
cumulative leeway
value calculated by summing the leeways obtained from the start of the inspection up to, and including, that of the item last inspected

3.21
cumulative sample size
total number of inspected items, counting from the start of the inspection up to, and including, the item last inspected

3.22
acceptance value for sequential sampling
value derived from the specified parameters of the sampling plan and the cumulative sample size

NOTE Whether the lot may yet be accepted is determined by comparing the cumulative leeway with the acceptance value.

3.23
rejection value for sequential sampling
value derived from the specified parameters of the sampling plan and the cumulative sample size

NOTE Whether the lot may yet be considered unacceptable is determined by comparing the cumulative leeway with the rejection value.

3.24
responsible authority
concept used to maintain the neutrality of this part of ISO 3951 (primarily for specification purposes), irrespective of whether it is being invoked or applied by the first, second or third party

NOTE The responsible authority may be:

a) the quality department within a supplier's organization (first party);

b) the purchaser or procurement organization (second party);

c) an independent verification or certification authority (third party).

4 Symbols

The symbols used are as follows:

A acceptance value for sequential sampling

A_t acceptance value corresponding to the curtailed value of the cumulative sample size

f_σ factor given in Tables B.1 and B.2, that relates the maximum process standard deviation to the difference between U and L

g multiplier of the cumulative sample size that is used to determine the acceptance values and the rejection values (slope of the acceptance and rejection lines)

h_A constant that is used to determine the acceptance values (intercept of the acceptance line)

h_R constant that is used to determine the rejection values (intercept of the rejection line)

L lower specification limit (as a suffix or a superscript to a variable, denotes its value at L)

N lot size (number of items in a lot)

n sample size (number of items in a sample)

n_{cum} cumulative sample size

n_0 sample size of the corresponding single sampling plan

n_t curtailment value of the cumulative sample size ($n_t = 1{,}5\ n_0$)

P_a probability of acceptance

R rejection value for sequential sampling

U upper specification limit (as a suffix or a superscript to a variable, denotes its value at U)

x measured value of the quality characteristic for the measured item of the sample

y leeway, defined as

$y = U - x$ for a single upper specification limit

$y = x - L$ for a single lower specification limit

$y = x - L$ for double specification limits

Y cumulative leeway obtained by adding the leeways up to, and including, the item last inspected

μ process mean

σ standard deviation of a process that is under statistical control

σ_{max} maximum process standard deviation

NOTE σ^2, the square of the process standard deviation, is known as the process variance.

5 Acceptance quality limit (AQL)

5.1 Principle

The AQL is the quality level that is the worst tolerable process average when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the acceptance quality limit may be accepted with fairly high probability, the designation of an acceptance quality limit does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to have process averages consistently better than the AQL. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample-size code letter, is used to index the sampling plans in this part of ISO 3951.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification, contract or by the responsible authority. Where both upper and lower specification limits are given, this part of ISO 3951 addresses two cases:

— combined AQL requirement (this is known as "combined control of double specification limits"); and

— separate AQL requirements (this is known as "separate control of double specification limits").

5.4 Preferred AQLs

The sixteen AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. If, for any product or service, an AQL is designated other than a preferred AQL, then this part of ISO 3951 is not applicable.

5.5 Caution

From the above definition of the AQL, it follows that the desired protection can only be assured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming product.

6 Switching rules for normal, tightened and reduced inspection

6.1 General

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving his production process.

Tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this part of ISO 3951, if the protection implied by the AQL is to be maintained.