INTERNATIONAL STANDARD

ISO 9276-5

First edition
2005-08-01

Representation of results of particle size analysis —

Part 5: Methods of calculation relating to particle size analyses using logarithmic normal probability distribution

Représentation de données obtenues par analyse granulométrique —

Partie 5: Méthodes de calcul relatif à l'analyse granulométrique à l'aide de la distribution de probabilité logarithmique normale

Reference number
ISO 9276-5:2005(E)

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Published in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 9276-5 was prepared by Technical Committee ISO/TC 24, *Sieves, sieving and other sizing methods*, Subcommittee SC 4, *Sizing by methods other than sieving*.

ISO 9276 consists of the following parts, under the general title *Representation of results of particle size analysis*:

— *Part 1: Graphical representation*

— *Part 2: Calculation of average particle sizes/diameters and moments from particle size distributions*

— *Part 4: Characterization of a classification process*

— *Part 5: Methods of calculation relating to particle size analyses using logarithmic normal probability distribution*

Further parts are under preparation:

— *Part 3: Fitting of an experimental cumulative curve to a reference model*

— *Part 6: Descriptive and quantitative representation of particle shape and morphology*

Introduction

Many cumulative particle size distributions, $Q_r(x)$, may be plotted on special graph paper which allow the cumulative size distribution to be represented as a straight line. Scales on the ordinate and the abscissa are generated from various mathematical formulae. In this part of ISO 9276, it is assumed that the cumulative particle size distribution follows a logarithmic normal probability distribution.

In this part of ISO 9276, the size, x, of a particle represents the diameter of a sphere. Depending on the situation, the particle size, x, may also represent the equivalent diameter of a particle of some other shape.

Representation of results of particle size analysis —

Part 5: Methods of calculation relating to particle size analyses using logarithmic normal probability distribution

1 Scope

The main objective of this part of ISO 9276 is to provide the background for the representation of a cumulative particle size distribution which follows a logarithmic normal probability distribution, as a means by which calculations performed using particle size distribution functions may be unequivocally checked. The design of logarithmic normal probability graph paper is explained, as well as the calculation of moments, median diameters, average diameters and volume-specific surface area. Logarithmic normal probability distributions are often suitable for the representation of cumulative particle size distributions of any dimensionality. Their particular advantage lies in the fact that cumulative distributions, such as number-, length-, area-, volume- or mass-distributions, are represented by parallel lines, all of whose locations may be determined from a knowledge of the location of any one.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 9276-1, *Representation of results of particle size analysis — Part 1: Graphical representation*

ISO 9276-2:2001, *Representation of results of particle size analysis — Part 2: Calculation of average particle sizes/diameters and moments from particle size distributions*

3 Symbols

For the purposes of this part of ISO 9276, the following symbols apply.

c	cumulative percentage
e = 2,718 28...	base of natural logarithms
k	power of x in a moment
$M_{k,r}$	complete kth moment of a density distribution of dimensionality r
p	dimensionality (type of quantity) of a distribution, $p = 0$: number, $p = 1$: length, $p = 2$: area, $p = 3$: volume or mass
$q_r(x)$	density distribution of dimensionality r
$Q_r(x)$	cumulative distribution of dimensionality r

r	dimensionality (type of quantity) of a distribution, $r = 0$: number, $r = 1$: length, $r = 2$: area, $r = 3$: volume or mass
s	standard deviation of the density distribution
s_g	geometric standard deviation, exponential function of the standard deviation
S_V	volume-specific surface area
x	particle size, diameter of a sphere
x_{min}	particle size below which there are no particles in a given size distribution
x_{max}	particle size above which there are no particles in a given size distribution
$x_{84,r}$	particle size at which $Q_r = 0{,}84$
$x_{50,r}$	median particle size of a cumulative distribution of dimensionality r
$x_{16,r}$	particle size at which $Q_r = 0{,}16$
$\bar{x}_{k,r}$	average particle size based on the kth moment of a distribution of dimensionality r
z	dimensionless variable proportional to the logarithm of x (see Equation 3)
ξ	integration variable based on x (see Equation 11)
ζ	integration variable based on z (see Equation 2)

Subscripts of different sense are separated by a comma in this and all other parts of ISO 9276.

4 Logarithmic normal probability function

Normal probability density distributions are described in terms of a dimensionless variable z:

$$q^*_r(z) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-0{,}5z^2} \tag{1}$$

The cumulative normal probability distribution is represented by:

$$Q^*_r(z) = \int_{-\infty}^{z} q^*_r(\zeta)\,\mathrm{d}\zeta = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \mathrm{e}^{-0{,}5\zeta^2}\,\mathrm{d}\zeta \tag{2}$$

A sample table of values for $Q^*_r(z)$ as a function of z is given in Table A.1.

The logarithmic normal probability distribution is a formulation in which z is defined as a logarithm of x scaled by two parameters, the mean size $x_{50,r}$ and either the dimensionless standard deviation, s, or the geometric standard deviation, s_g, that characterize the distribution:

$$z = \frac{1}{s}\ln\left[\frac{x}{x_{50,r}}\right] = \frac{1}{\ln s_g}\ln\left[\frac{x}{x_{50,r}}\right] = \frac{1}{\log s_g}\log\left[\frac{x}{x_{50,r}}\right] \tag{3}$$

which is equivalent to

$$x = x_{50,r}\ \mathrm{e}^{s\,z} \tag{4}$$

According to Equation 3, the standard deviation, s, is linked with the geometric standard deviation, s_g, by:

$$s = \ln s_g \text{ or } s_g = \mathrm{e}^{s} \tag{5}$$

Although Equation 1 has no explicit dependences on r, the dimensionality of the density distribution is involved through the relationship of z to $x_{50,r}$ in Equation 3. The value of $x_{50,r}$ for a specific size distribution may be determined from experimental data according to ISO 9276-1. The standard deviation of a logarithmic normal probability distribution may be calculated from the values of the cumulative distribution at certain characteristic values of z:

either at $z = 1$, for which

$$Q^*_r\ (z = 1) = 0{,}84 \text{ and } s = \ln\left[\frac{x_{84,r}}{x_{50,r}}\right] \tag{6}$$

or at $z = -1$, for which

$$Q^*_r\ (z = -1) = 0{,}16 \text{ and } s = \ln\left[\frac{x_{50,r}}{x_{16,r}}\right] \tag{7}$$

Throughout this part of ISO 9276, the values 0,84 and 0,16 (and their representation as percentages 84 and 16) are used in place of the more precise values 0,841 34 and 0,158 65.

Logarithmic probability graph presentation: Useful information about the nature of a particle size distribution may be obtained by plotting the cumulative distribution on special graph paper, on which the abscissa (representing particle size) is marked with an exponential scale and the ordinate (representing cumulative distribution) is marked with a scale of $Q^*_r(z)$ values (see Annex A). Preprinted paper marked with these scales is available. Graphical representation is now more often displayed as a specific graphical screen created by software in a computer. Experimental values of each cumulative fraction (expressed in terms of number, length, area or volume) of undersize particles, $Q_r(x)$, (that is, of particles smaller than x) are plotted at the size corresponding to the upper size limit of the particles in that cumulative fraction. A logarithmic normal probability distribution gives a straight line in Figure 1.

To fulfil the condition of normalization, the cumulative fraction smaller than or equal to the particle having the largest size in the sample must be unity, that is, $Q_r(x_{max})$ must be equal to 1. If this is so, then

$$q^*_r(z)\,\mathrm{d}z = q_r(x)\,\mathrm{d}x \tag{8}$$

NOTE The superscript* is used to distinguish the distributions defined in terms of the dimensionless integration variable z, such as $q^*_r(z)$, from those defined in terms of the size x, such as $q_r(x)$. This is because z, the integration variable, is related to the particle size x, as shown in Equation 3.

$$q_r(x) = q^*_r(z)\frac{\mathrm{d}z}{\mathrm{d}x} = q^*_r(z)\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{1}{s}\ln\left[\frac{x}{x_{50,r}}\right]\right\} = \frac{1}{x\,s}\,q^*_r(z) \tag{9}$$

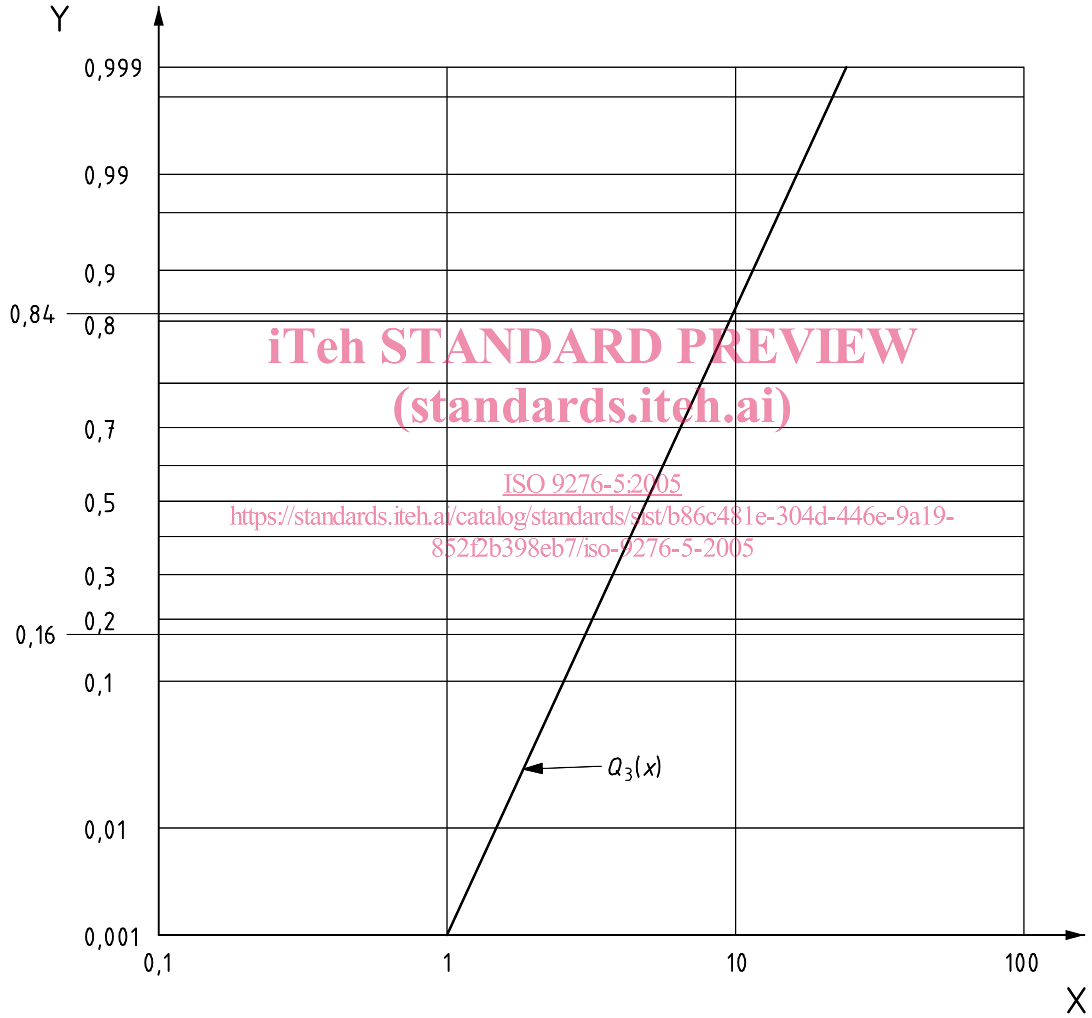
or, using Equation 1,

$$q_r(x) = \frac{1}{x\,s\sqrt{2\pi}}\,\mathrm{e}^{-0{,}5\,z^2} \tag{10}$$

and, parallel to Equation 2,

$$Q_r(x) = \int_{x_{\min}}^{x} q_r(\xi)\,\mathrm{d}\xi \qquad (11)$$

EXAMPLE A logarithmic normal probability distribution of volume ($r = 3$), with a median size of $x_{50,3} = 5$ µm and a standard deviation of $s = 0{,}5$, has $x_{16,3} = 3{,}0$ µm and $x_{84,3} = 8{,}2$ µm (see ISO 9276-2:2001, Annex A). Figure 1 shows a plot of the cumulative volume distribution, $Q_3(x)$, on logarithmic probability graph paper.



Key

X particle size, x, µm

Y cumulative distribution, Q

Figure 1 — Plot of a logarithmic normal probability distribution on logarithmic probability graph paper

5 Special values of a logarithmic normal probability distribution

5.1 Complete kth moments

The *complete* kth *moment* of a logarithmic normal probability distribution, $q_r(x)$, is

$$M_{k,r} = x_{50,r}{}^{k}\ \mathrm{e}^{0{,}5\,k^2 s^2} = \mathrm{e}^{k \ln x_{50,r} + 0{,}5\,k^2 s^2} \tag{12}$$

with $k = 2$ and $r = 3$:

$$M_{2,3} = x_{50,3}{}^{2}\ \mathrm{e}^{2\,s^2} = \mathrm{e}^{2 \ln x_{50,3} + 2\,s^2} \tag{13}$$

5.2 Average particle sizes

A series of average particle sizes, $\bar{x}$, of a logarithmic normal probability distribution, $q_r(x)$, can be calculated from the kth root of the kth moment (or from the $x_{50,r}$ and s) of that distribution using Equation 14:

$$\bar{x}_{k,r} = \sqrt[k]{M_{k,r}} = x_{50,r}\ \mathrm{e}^{0{,}5\,k\,s^2} \tag{14}$$

For a logarithmic normal probability distribution, the median is the same as the geometric mean and the average size in one dimension, r, may be calculated from the parameters describing the distribution in a different dimensionality, p, using:

$$\bar{x}_{k,r} = x_{50,p}\ \mathrm{e}^{(0{,}5\,k + r - p)\,s^2} \tag{15}$$

or

$$\ln \bar{x}_{k,r} = \ln x_{50,r} + 0{,}5\,k\,s^2 = \ln x_{50,p} + (0{,}5\,k + r - p)\,s^2 \tag{16}$$

EXAMPLE The first several moments ($k = 1$, 2 or 3) of the arithmetic average particle size ($r = 0$) for a logarithmic normal probability distribution may be computed from the parameters for any of the dimensionalities ($p = 0$, 1, 2 or 3) using:

$$\bar{x}_{1,0} = x_{50,0}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{-0{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{-1{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-2{,}5\,s^2} \tag{17}$$

$$\bar{x}_{2,0} = x_{50,0}\ \mathrm{e}^{\,s^2} = x_{50,1} = x_{50,2}\ \mathrm{e}^{-s^2} = x_{50,3}\ \mathrm{e}^{-2\,s^2} \tag{18}$$

$$\bar{x}_{3,0} = x_{50,0}\ \mathrm{e}^{1{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{-0{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-1{,}5\,s^2} \tag{19}$$

EXAMPLE The first moment ($k = 1$) weighted average particle size for the different dimensionalities ($r = 0$, 1, 2, or 3) of a logarithmic normal probability distribution may be computed from the parameters for any of the dimensionalities ($p = 0$, 1, 2 or 3) using:

$$\bar{x}_{1,0} = x_{50,0}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{-0{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{-1{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-2{,}5\,s^2} \tag{17}$$

$$\bar{x}_{1,1} = x_{50,0}\ \mathrm{e}^{1{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{-0{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-1{,}5\,s^2} \tag{20}$$

$$\bar{x}_{1,2} = x_{50,0}\ \mathrm{e}^{2{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{1{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{0{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{-0{,}5\,s^2} \tag{21}$$

$$\bar{x}_{1,3} = x_{50,0}\ \mathrm{e}^{3{,}5\,s^2} = x_{50,1}\ \mathrm{e}^{2{,}5\,s^2} = x_{50,2}\ \mathrm{e}^{1{,}5\,s^2} = x_{50,3}\ \mathrm{e}^{0{,}5\,s^2} \tag{22}$$