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Statistical Methods in Process Management — Control Charts —

Part 6: EWMA Control Charts

Cartes de contrôle de EWMA —

Partie 6: Cartes de contrôle de EWMA

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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

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ISO 7870-6 was prepared by Technical Committee ISO/TC 69, *TC Applications of statistical methods*, Subcommittee SC 4, *SC Applications of statistical methods in process management*.

ISO 7870 consists of the following parts, under the general title *Control charts*:

— *Part 1: General guidelines*

— *Part 2: Shewhart control charts*

— *Part 3: Acceptance control charts*

— *Part 4: Cumulative sum charts*

The following part is planned:

— *Part 5: Specialized control charts*

Introduction

Shewhart charts are the most widespread statistical control methods used for controlling a process, but they are slow in signaling shifts of small magnitude in the process parameters. The exponentially weighted moving average (EWMA) chart makes possible faster detection of small to moderate shifts.

The Shewhart chart is simple to implement and it rapidly detects shifts of major magnitude. However, it is fairly ineffective for detecting shifts of small or moderate magnitude. It happens quite often that the shift of the process is slow and progressive (in case of continuous processes in particular); this shift has to be detected very early in order to react before the process deviates seriously from its target value. There are two possibilities for improving the effectiveness of the Shewhart chart with respect to small and moderate shifts:

- The simplest, but not the most economical possibility, is to increase the sample size taken from each rational subgroup. This may not always be possible due to low production rate; testing may be time consuming or too costly. As a result it may not be possible to draw samples sizes of more than 1 or 2.

- The second possibility is to take into account the results preceding the control under way in order to try to detect the existence of a shift in the production process. The Shewhart control chart takes into account only the information contained in the last sample observation and it ignores any information given by the entire sequence of points. This feature makes the Shewhart control chart relatively insensitive to small process shifts. Its effectiveness may be improved by taking into account the former results.

Where it is desired to detect slow, progressive shifts, it is preferable to use specific charts which take into account the past data and which are effective with a moderate control cost. Two very effective alternatives to the Shewhart control chart in such situations are:

a) Cumulative Sum (CUSUM) charts. These charts are described in ISO 7870-4. The CUSUM chart reacts more sensitively than the X-bar chart to a shifting of the mean value in the range of 0.5-2 sigma. If one plots the cumulative sum of deviations of successive sample means from a target specification, even minor, permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, this chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the X-bar chart.

b) Exponentially Weighted Moving Average (EWMA) chart, which is covered by this document. This chart is presented like the Shewhart chart; however, instead of placing on the chart the successive averages of the samples, one monitors a weighted average of the local average and of the previous averages.

The performance of the EWMA control chart is approximately equivalent to that of the CUSUM chart and in some ways it is easier to setup and operate.

EWMA Charts are generally used for detecting small shifts in the process mean. They will detect shifts of 0,5 sigma to 2 sigma much faster. They are, however, slower in detecting large shifts in the process mean. EWMA Charts may also be preferred when the subgroups are of size n=1.

The joint use of an EWMA chart with a small value and a Shewhart chart has been recommended as a means of guaranteeing fast detection of both small and large shifts. This was first proposed by Lucas and Saccucci [13].

Statistical methods in process management — Control charts — Part 6:

1 Scope

This International standard covers EWMA charts as a statistical process control technique to detect small shifts in the process mean. It makes possible faster detection of small to moderate shifts in the process average. In this chart, the process average is evaluated in terms of exponentially-weighted moving average of current and all prior sample means. EWMA weights samples in geometrically decreasing order so that the most recent samples are weighted most highly while the most distant samples contribute very little.

The basic objective is therefore the same as that of the Shewhart control chart described in ISO 7870-2.

Its application is worthwhile in the rare situations when:

- production rate is slow
- sampling and inspection procedure is complex and time consuming
- testing is expensive
- it involves safety risks
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The EWMA is used extensively in time series modeling and in forecasting.

Variable control charts can by constructed for individual observations taken from the production line, rather than samples of observations. This is sometimes necessary when testing samples of multiple observations would be too expensive, inconvenient, or impossible. For example, the number of customer complaints or product returns may only be available on a monthly basis; yet, one would like to chart those numbers to detect quality problems. Another common application of these charts occurs in cases when automated testing devices inspect every single unit that is produced. In that case, one is often primarily interested in detecting small shifts in the product quality (for example, gradual deterioration of quality due to machine wear).

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics -- Vocabulary and symbols -- Part 2: Applied statistics*

ISO 7870-1, *Control charts -- Part 1: General guidelines*

ISO 7870-2, *Control charts -- Part 2: Shewhart control charts*

ISO 7870-4, *Control charts -- Part 4: Cumulative sum charts*

3 Symbols (and abbreviated terms)

μ_0 Target value for the average of the process

μ_1, μ_1' Upper refusable value of the average, lower refusable value of the average

$\overline{x}_i$ Average of the sample i

N	Number of samples
n	Number of units in a samples (sample size)
z_i	EWMA value placed on the control chart
z_0	Initial value of z_i
λ	Value of the smoothing parameter
L	Value of the control limit for z_i (expressed in number of standard deviations of z, σ_z)
σ	True standard deviation of the distribution of x
s	Estimation of the standard deviation σ
$\sigma_{\bar{x}}$	Standard deviation of the averages of n individual observations; $\sigma_{\bar{x}} = \sigma / \sqrt{n}$
σ_z	Standard deviation of z_i when i tends towards infinity
δ	Drift related to the average expressed in number of standard deviations
δ_1	Maximum acceptable drift of the average, expressed in number of standard deviations
p	Proportion of nonconforming units of the process
p_0	Target value for the proportion of nonconforming units of the process
p_1	Upper refusable value of the proportion of nonconforming units
σ_0	True standard deviation of binomial distribution for $p = p_0$
p_i	Proportion of nonconforming units in the i^{th} sample
c	Average number of nonconformities
c_0	Target value for the average number of nonconformities
c_1	Refusable average of nonconformities
c_i	Number of nonconformities in the i^{th} sample
U_{CL}	Upper control limit value for the EWMA chart
L_{CL}	Lower control limit value for the EWMA chart. If *LCL* is negative, then it is taken as zero
ARL	Average Run Length
ARL_0	Average Run Length of the controlled process
ARL_1	Average Run Length of the process with setting drift
MAXRL	Maximum Run Length (5 % overrun probability), expressed as an integer

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4 EWMA for variables

4.1 General

An EWMA control chart plots geometric moving averages of past and current data in which the values being averaged are assigned weights that decrease exponentially from the present into the past. Consequently, the average values are influenced more by recent process performance. The exponentially weighted moving average is defined as

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}$$

where 0 <λ< 1 is a constant and the starting value (required with the first sample at i=1), is the process target, so that

$$z_0 = \mu_0$$

Sometimes the average of preliminary data is used as the starting value of the EWMA, so that $z_0 = \bar{x}$.

The EWMA chart becomes an $\bar{X}$ chart for λ =1.

4.2 Weighted average explained

To demonstrate that the EWMA z_i is a weighted average of all previous sample means, we may substitute for z_{i-1} on the right hand side of the equation in Clause4.1to obtain

$$z_i = \lambda x_i + (1-\lambda)\left[\lambda x_{i-1} + (1-\lambda) z_{i-2}\right]$$

$$= \lambda x_i + \lambda(1-\lambda) x_{i-1} + (1-\lambda)^2 z_{i-2}$$

Continuing to substitute recursively for z_{i-j}, j =2,3,…t, we obtain

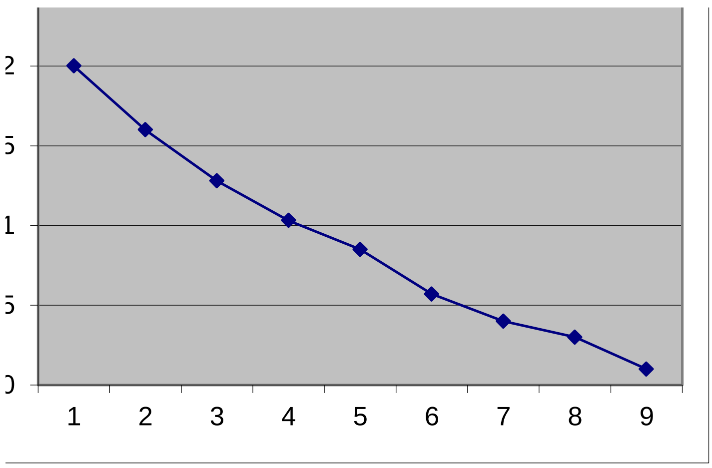
$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0$$

For i= 1, $z_1 = \lambda x_1 + (1-\lambda)\mu_0$

The weights $\lambda(1-\lambda)^j$ decrease geometrically with the age of the sample mean. Furthermore, the weights sum to unity, since

$$\lambda \sum_{j=0}^{i-1} (1-\lambda)^j = \lambda \left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)}\right] = 1-(1-\lambda)^i$$

If λ = 0,2, then the weight assigned to the current sample mean is 0,2 and the weights given to the preceding means are 0,16; 0,128; 0,1024 and so forth. A comparison of these weights with those of a five-period moving average is shown in Figure 1. Because these weights decline geometrically when connected by a smooth curve, the EWMA is sometimes called a geometric moving average (GMA). The common use of the EWMA chart is in the chemical industry where large day-to-day fluctuations are common but may not be indicative of the lack of process predictability.



Age of sample mean (EWMA, λ =0.2)

Figure 1 - Weights of past sample means

Since the EWMA can be viewed as a weighted average of all past and current observations, it is very sensitive to the normality assumption. It is, therefore an ideal control chart to use with individual observations.

4.3 Control limits for the EWMA chart

If the observations x_i are independent random variables with variance σ^2, then the variance of z_i is

$$\sigma_{z_i}^2 = \sigma^2\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]$$

Therefore, the EWMA control chart would be constructed by plotting z_i versus the sample number *i* (or time). The centre line and control limits for the EWMA control chart are as follows:

Centre Line = μ_0

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

The factor L is the width of the control limits and its value depends upon the confidence level. Like in case of $\bar{X}$-R charts, 3σ limits are plotted for 99,73 % (3σ) confidence. Similarly, on EWMA chart, we can vary this confidence level as per our requirements (e.g. L = 2,7 gives the confidence of 99,65 %).

No action is taken as long as z_i falls between these limits, and the process is considered to be out of control as soon as z_i overshoots the control limits. in this case, reset the process and one resume the EWMA chart after re-initialising it, i.e. by not taking into account the results obtained prior to this resetting, but by taking $z_0$as the initial value.

The term $\left[1-(1-\lambda)^{2i}\right]$ approaches unity as *i* gets larger. This means that after the EWMA control chart has been running for several time periods, the control limits will approach steady state values given by

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$

$CL = \mu_0$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}}$$

However, it is strongly recommended to use the exact control limits. This will greatly improve the performance of the control chart in detecting an off-target process immediately after the EWMA is initiated.

NOTE For practical purposes the estimate of *σ*, denoted by *s*, estimated from the data.

4.4 EWMA control chart for rational subgroups of size, *n*>1

The EWMA control chart when used with rational subgroups of size n > 1, then x_i is simply replaced with $\bar{x}_i$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ with *σith.*

4.5 Illustration

Consider the data in Table 1 (observations x_i). The first 20 observations were drawn at random from a normal distribution with mean μ=10 and standard deviation σ=1. The last 10 observations were drawn from a normal distribution with mean μ=11 and standard deviation σ=1, i.e, after the process has experienced a shift in the mean of 1σ.

It is desired to set up an EWMA chart with λ =0,10 and L=2,7 to the data in Table 1.

The target value of the mean is $\mu_0 = 10$ and the standard deviation is $\sigma = 1$.

The calculations for EWMA control chart are summarized in Table1 and the control chart is shown in Figure 2.

To illustrate the calculations, consider the first observations, $x_1 = 9.45$. The first value of the EWMA is

$$z_1 = \lambda x_1 + (1-\lambda) z_0$$

=0,1(9,45)+0,9(10)

=9,945

Therefore, $z_1 = 9.945$ is the first value plotted on the control chart in Figure 2. The second value of the EWMA is

$$z_2 = \lambda x_2 + (1-\lambda) z_1$$

=0,1(7,99) + 0,9(9,945)

= 9,7495

The other values of the EWMA statistic are computed similarly.

Table 1 EWMA Calculations

S.No	x_i	EWMA z_i
1	9,45	9,945
2	7,99	9,7495
3	9,29	9,70355
4	11,66	9,8992
5	12,16	10,1253
6	10,18	10,1307
7	8,04	9,92167
8	11,46	10,0755
9	9,2	9,98796
10	10,34	10,0232
11	9,03	9,92384
12	11,47	10,0785
13	10,51	10,1216
14	9,4	10,0495
15	10,08	10,0525