INTERNATIONAL STANDARD

ISO 7870-6

First edition
2016-02-15

Control charts —

Part 6: EWMA control charts

Cartes de contrôle —

Partie 6: Cartes de contrôle de EWMA

Reference number
ISO 7870-6:2016(E)

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT), see the following URL: Foreword — Supplementary information.

The committee responsible for this document is ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

ISO 7870 consists of the following parts, under the general title *Control charts*:

— *Part 1: General guidelines*

— *Part 2: Shewhart control charts*

— *Part 3: Acceptance control charts*

— *Part 4: Cumulative sum charts*

— *Part 5: Specialized control charts*

— *Part 6: EWMA control charts*

A future part on charting techniques for short runs and small mixed batches is planned.

Introduction

Shewhart control charts are the most widespread statistical control methods used for controlling a process, but they are slow in signalling shifts of small magnitude in the process parameters. The exponentially weighted moving average[10] (EWMA) control chart makes possible faster detection of small to moderate shifts.

The Shewhart control chart is simple to implement and it rapidly detects shifts of major magnitude. However, it is fairly ineffective for detecting shifts of small or moderate magnitude. It happens quite often that the shift of the process is slow and progressive (in case of continuous processes in particular); this shift has to be detected very early in order to react before the process deviates seriously from its target value. There are two possibilities for improving the effectiveness of the Shewhart control charts with respect to small and moderate shifts.

— The simplest, but not the most economical possibility is to increase the subgroup size. This may not always be possible due to low production rate; time consuming or too costly testing. As a result, it may not be possible to draw samples of size more than 1 or 2.

— The second possibility is to take into account the results preceding the control under way in order to try to detect the existence of a shift in the production process. The Shewhart control chart takes into account only the information contained in the last sample observation and it ignores any information given by the entire sequence of points. This feature makes the Shewhart control chart relatively insensitive to small process shifts. Its effectiveness may be improved by taking into account the former results.

Where it is desired to detect slow, progressive shifts, it is preferable to use specific charts which take into account the past data and which are effective with a moderate control cost. Two very effective alternatives to the Shewhart control chart in such situations are

a) Cumulative Sum (CUSUM) control chart. This chart is described in ISO 7870-4. The CUSUM control chart reacts more sensitively than the X-bar chart to a shift of the mean value in the range of half or two sigma. If one plots the cumulative sum of deviations of successive sample means from a specified target, even minor, permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, this chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the X-bar chart.

b) Exponentially Weighted Moving Average (EWMA) control chart which is covered by this document. This chart is presented like the Shewhart control chart; however, instead of placing on the chart the successive averages of the samples, one monitors a weighted average of the current average and of the previous averages.

EWMA control charts are generally used for detecting small shifts in the process mean. They will detect shifts of half sigma to two sigma much faster. They are, however, slower in detecting large shifts in the process mean. EWMA control charts may also be preferred when the subgroups are of size $n = 1$.

The joint use of an EWMA control chart with a small value of lambda and a Shewhart control chart has been recommended as a means of guaranteeing fast detection of both small and large shifts. The EWMA control chart monitors only the process mean; monitoring the process variability requires the use of some other technique.

Control charts —

Part 6:
EWMA control charts

1 Scope

This International Standard covers EWMA control charts as a statistical process control technique to detect small shifts in the process mean. It makes possible the faster detection of small to moderate shifts in the process average. In this chart, the process average is evaluated in terms of exponentially weighted moving average of all prior sample means. EWMA weights samples in geometrically decreasing order so that the most recent samples are weighted most highly while the most distant samples contribute very little depending upon the smoothing parameter (λ).

NOTE 1 The basic objective is the same as that of the Shewhart control chart described in ISO 7870-2.

The Shewhart control chart's application is worthwhile in the rare situations when

— production rate is slow,

— sampling and inspection procedure is complex and time consuming,

— testing is expensive, and

— it involves safety risks.

NOTE 2 Variables control charts can be constructed for individual observations taken from the production line, rather than samples of observations. This is sometimes necessary when testing samples of multiple observations would be too expensive, inconvenient, or impossible. For example, the number of customer complaints or product returns may only be available on a monthly basis; yet, one would like to chart those numbers to detect quality problems. Another common application of these charts occurs in cases when automated testing devices inspect every single unit that is produced. In that case, one is often primarily interested in detecting small shifts in the product quality (for example, gradual deterioration of quality due to machine wear).

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 7870-1, *Control charts — Part 1: General guidelines*

ISO 7870-2, *Control charts — Part 2: Shewhart control charts*

ISO 7870-4, *Control charts — Part 4: Cumulative sum charts*

3 Symbols and abbreviated terms

μ_0	Target value for the average of the process
U_μ, L_μ	Upper rejectable value of the average, lower rejectable value of the average
$\overline{x}_i$	Mean of the sample *i*
N	Number of units in a sample (sample size)

z_i	EWMA value placed on the control chart
z_0	Initial value of z_i
λ	Value of the smoothing parameter
L_z	Parameter used to establish the control limit for z_i (expressed in number of standard deviations of z)
s	Estimator of the standard deviation σ
σ	True standard deviation of the distribution of x
σ_0	True standard deviation of binomial distribution for $P = p_0$
$\sigma_{\overline{x}}$	Standard deviation of the averages of n individual observations; $\sigma_{\overline{x}} = \sigma / \sqrt{n}$
σ_z	Standard deviation of z_i when i tends towards infinity
δ	Drift related to the average expressed in number of standard deviations
δ_1	Maximum acceptable drift of the average, expressed in number of standard deviations
p	Proportion of nonconforming units of the process
p_0	Target value for the proportion of nonconforming units of the process
p_1	Upper refusable value of the proportion of nonconforming units
p_i	Proportion of nonconforming units in the ith sample
c	Average number of nonconformities
c_0	Target value for the average number of nonconformities
c_1	Refusable average of nonconformities
c_i	Number of nonconforming units in the ith sample
U_{CL}	Upper control limit value for the EWMA control chart
L_{CL}	Lower control limit value for the EWMA control chart. If L_{CL} is negative, then it is taken as zero
ARL	Average Run Length
ARL_0	Average Run Length of the process in control
ARL_1	Average Run Length of the process with setting drift
CL	Centre line of the control limit
MAXRL	Maximum Run Length (5 % overrun probability), expressed as an integer

4 EWMA for inspection by variables

4.1 General

An EWMA control chart plots geometric moving averages of past and current data in which the values being averaged are assigned weights that decrease exponentially from the present into the past. Consequently, the average values are influenced more by recent process performance. The exponentially weighted moving average is defined as Formula (1):

$$z_i = \lambda x_i + (1 - \lambda)\, z_{i-1} \tag{1}$$

NOTE 1 When the EWMA control chart is used with rational subgroups of size $n > 1$ then x_i is simply replaced with $\overline{x}_i$.

Where $0 < \lambda < 1$ is a constant and the starting value (required with the first sample at $i = 1$) is the process target, so that $z_0 = \mu_0$.

NOTE 2 μ_0 can be estimated by the average of preliminary data.

The EWMA control chart becomes an $\overline{X}$ chart for $\lambda = 1$.

4.2 Weighted average explained

To demonstrate that the EWMA is a weighted average of all previous sample means, the right-hand side of Formula (1) in 4.1 can be substituted with z_{i-1} to obtain Formula (2):

$$\begin{aligned} z_i &= \lambda x_i + (1-\lambda)\left[\lambda x_{i-1} + (1-\lambda) z_{i-2}\right] \\ &= \lambda x_i + \lambda(1-\lambda) x_{i-1} + (1-\lambda)^2 z_{i-2} \end{aligned} \quad (2)$$

Continuing to substitute recursively for z_{i-j}, where j = 2, 3, ..., we obtain Formula (3):

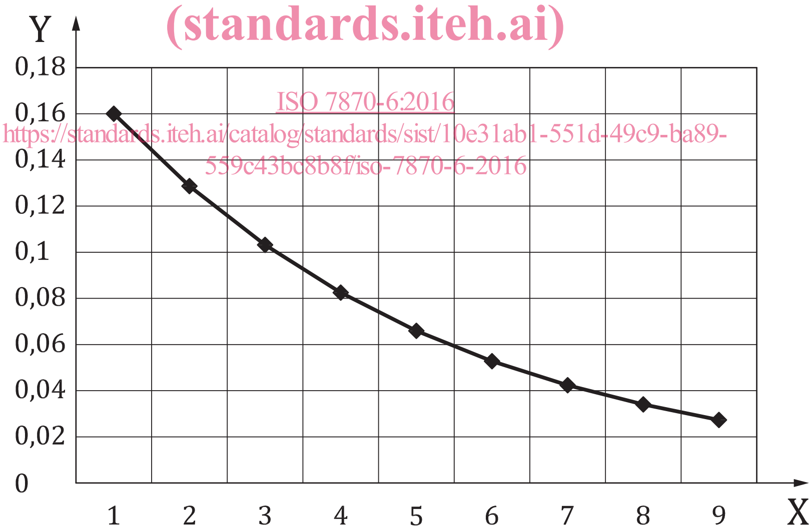
$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \quad (3)$$

For $i = 1$, $z_1 = \lambda x_1 + (1 - \lambda)\mu_0$.

The weights, $\lambda(1 - \lambda)^j$, decrease geometrically with the age of the sample mean. Furthermore, the weights sum to unity, since

$$\lambda \sum_{j=0}^{i-1} (1-\lambda)^j = \lambda \left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)} \right] = 1-(1-\lambda)^i \quad (4)$$

If λ = 0,2, then the weight assigned to the current sample mean is 0,2 and the weights given to the preceding means are 0,16; 0,128; 0,102 4 and so forth. These weights are shown in Figure 1. Because these weights decline geometrically, the EWMA is sometimes called a geometric moving average (GMA).



Key

X age of sample mean (EWMA λ = 0,2)

Y weights $\lambda(1-\lambda)^j$

Figure 1 — Weights of past sample means

Since the EWMA value can be viewed as a weighted average of all past and current observations, it is very insensitive to the normality assumption. It is, therefore an ideal control chart to use with individual observations.

4.3 Control limits for EWMA control chart

If the observations x_i are independent random variables with variance σ^2, then the variance of z_i is represented by Formula (5):

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-\left(1-\lambda\right)^{2i}\right] \tag{5}$$

Therefore, the EWMA control chart would be constructed by plotting z_i versus the sample number i (or time). The centre line and control limits for the EWMA control chart are as follows:

Centre line = μ_0

$$U_{\mathrm{CL}} = \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{\left(2-\lambda\right)}\left[1-\left(1-\lambda\right)^{2i}\right]} \tag{6}$$

$$L_{\mathrm{CL}} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{\left(2-\lambda\right)}\left[1-\left(1-\lambda\right)^{2i}\right]} \tag{7}$$

The factor L_z is the width of the control limits and its value depends upon the confidence level. In the case of $\overline{X}$ - R charts, 3σ limits are plotted for 99,73 % ($\pm 3\sigma$) confidence. Similarly, on EWMA control chart, this confidence level can vary depending on the requirements (e.g. L_z = 2,7 gives the confidence of 99,307 %).

No action is taken as long as z_i falls between these limits, and the process is considered to be out of control as soon as z_i overshoots the control limits. In this case, reset the process and resume the EWMA control chart after reinitializing it, i.e. by not taking into account the results obtained prior to this resetting, but by taking z_0 as the initial value.

The term $[1 - (1 - \lambda)^{2i}]$ approaches unity as i gets larger. This means that after the EWMA control chart has been running for several time periods, the control limits will approach steady state values obtained using Formulae (8) and (9):

Centre line = μ_0

$$U_{\mathrm{CL}} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{\left(2-\lambda\right)}} \tag{8}$$

$$L_{\mathrm{CL}} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{\left(2-\lambda\right)}} \tag{9}$$

However, it is strongly recommended to use the exact control limits. This will greatly improve the performance of the control chart in detecting an off-target process immediately after the EWMA control chart is initiated.

NOTE For practical purposes, use the estimate of σ, denoted by s, estimated from the data.

4.4 Construction of EWMA control chart

To illustrate the construction of an EWMA control chart, consider a process with the following parameters calculated from historical data:

μ_0 = 50

s = 2,053 9

with λ chosen to be 0,3; so that

$$\sqrt{\frac{\lambda}{(2-\lambda)}} = \sqrt{\frac{0,3}{1,7}} = 0,420\,1 \quad (10)$$

The control limits at steady-state are given, obtained using Formulae (11) and (12):

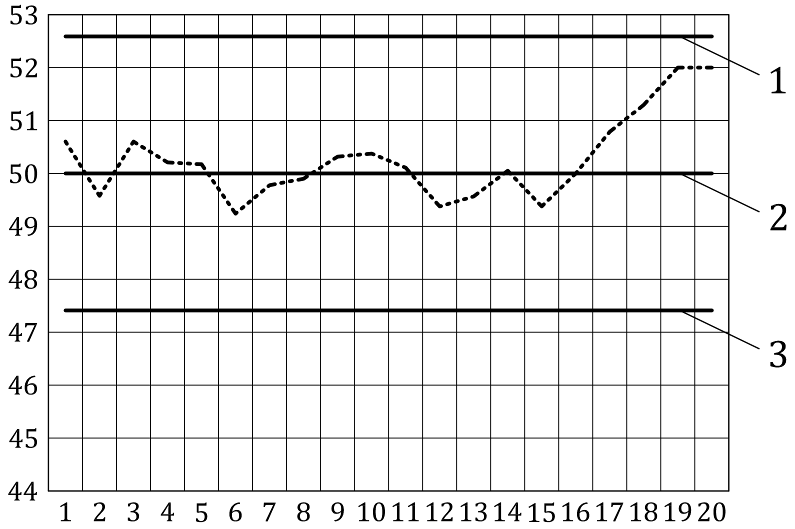
$$U_{CL} = 50 + 3\,(0{,}420\,1)(2{,}053\,9) = 52{,}588\,5 \quad (11)$$

$$L_{CL} = 50 - 3\,(0{,}420\,1)(2{,}053\,9) = 47{,}411\,5 \quad (12)$$

Consider the data consisting of 20 points as given in Table 1.

Table 1 — Calculation of EWMA values

Sample	X_i	EWMA values
1	52,0	50,600 0
2	47,0	49,520 0
3	53,0	50,564 0
4	49,3	50,184 8
5	50,1	50,159 4
6	47,0	49,211 6
7	51,0	49,748 1
8	50,1	49,853 7
9	51,2	50,257 6
10	50,5	50,330 3
11	49,6	50,111 2
12	47,6	49,357 8
13	49,9	49,520 5
14	51,3	50,054 3
15	47,8	49,378 0
16	51,2	49,924 6
17	52,6	50,727 2
18	52,4	51,229 1
19	53,6	51,940 3
20	52,1	51,988 2



Key

1 U_{CL} = 52,588 5

2 CL = 50

3 L_{CL} = 47,411 5

Figure 2 — EWMA plot

The EWMA control chart in Figure 2 shows that the process is in control because all EWMA points lie between the control limits.

4.5 Example

Consider the data in Table 2 (observations x_1). The first 20 observations were drawn at random from a normal distribution with mean μ = 10 and standard deviation σ = 1. The last 10 observations were drawn from a normal distribution with mean μ = 11 and standard deviation σ = 1, i.e. after the process has experienced a shift in the mean of one sigma.

Set up an EWMA control chart with λ = 0,10 and L_z = 2,7 to the data in Table 2.

The target value of the mean is μ = 10 and the standard deviation is σ = 1.

The calculations for EWMA control chart are summarized in Table 2 and the control chart is shown in Figure 3.

To illustrate the calculations, consider the first observations, x_i = 9,45.

The first value of the EWMA statistic is shown in Formula (13):

$$\begin{aligned} z_1 = \lambda x_1 + \left(1-\lambda\right) z_0 &= 0,1 \times 9,45 + 0,9 \times 10 \\ &= 9,945\ 00 \end{aligned} \tag{13}$$

Therefore, z_1 = 9,945 00 is the first value plotted on the control chart in Figure 3.

The second value of the EWMA is shown in Formula (14):

$$\begin{aligned} z_2 = \lambda x_2 + \left(1-\lambda\right) z_1 &= 0,1 \times 7,99 + 0,9 \times 9,945 \\ &= 9,749\ 50 \end{aligned} \tag{14}$$

The other values of the EWMA statistic are computed similarly.

The control limits are calculated following Formulae (15) and (16):

