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Control charts —

Part 4:
Cumulative sum charts

Cartes de contrôle —

Partie 4: Cartes de contrôle de l'ajustement de processus

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Foreword

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International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 7870-4 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This first edition of ISO 7870-4 cancels and replaces ISO/TR 7871:1997.

ISO 7870 consists of the following parts, under the general title *Control charts*:

- *Part 1: General guidelines*
- *Part 3: Acceptance control charts*
- *Part 4: Cumulative sum charts*

The following part is under preparation:

- *Part 2: Shewhart control charts*

Additional parts on specialized control charts and on the application of statistical process control (SPC) charts are planned.

Introduction

This part of ISO 7870 demonstrates the versatility and usefulness of a very simple, yet powerful, pictorial method of interpreting data arranged in any meaningful sequence. These data can range from overall business figures such as turnover, profit or overheads to detailed operational data such as stock outs and absenteeism to the control of individual process parameters and product characteristics. The data can either be expressed sequentially as individual values on a continuous scale (e.g. 24,60, 31,21, 18,97...), in “yes”/“no”, “good”/“bad”, “success”/“failure” format, or as summary measures (e.g. mean, range, counts of events).

The method has a rather unusual name, cumulative sum, or, in short, “cusum”. This name relates to the process of subtracting a predetermined value, e.g. a target, preferred or reference value from each observation in a sequence and progressively cumulating (i.e. adding) the differences. The graph of the series of cumulative differences is known as a cusum chart. Such a simple arithmetical process has a remarkable effect on the visual interpretation of the data as will be illustrated.

The cusum method is already used unwittingly by golfers throughout the world. By scoring a round as “plus” 4, or perhaps even “minus” 2, golfers are using the cusum method in a numerical sense. They subtract the “par” value from their actual score and add (cumulate) the resulting differences. This is the cusum method in action. However, it remains largely unknown and hence is a grossly underused tool throughout business, industry, commerce and public service. This is probably due to cusum methods generally being presented in statistical language rather than in the language of the workplace.

This part of ISO 7870 is a revision of ISO/TR 7871:1997. The intention of this part is, thus, to be readily comprehensible to the extensive range of prospective users and so facilitate widespread communication and understanding of the method. The method offers advantages over the more commonly found Shewhart charts in as much as the cusum method will detect a change of an important amount up to three times faster. Further, as in golf, when the target changes per hole, a cusum plot is unaffected, unlike a standard Shewhart chart where the control lines would require a constant adjustment.

In addition to Shewhart charts, an EWMA (exponentially weighted moving average) chart, can be used. Each plotted point on an EWMA chart incorporates information from all of the previous subgroups or observations, but gives less weight to process data as they get “older” according to an exponentially decaying weight. In a similar manner to a cusum chart, an EWMA chart can be sensitized to detect any size of shift in a process. This subject is discussed further in another part of this International Standard.

Control charts —

Part 4:
Cumulative sum charts

1 Scope

This part of ISO 7870 provides statistical procedures for setting up cumulative sum (cusum) schemes for process and quality control using variables (measured) and attribute data. It describes general-purpose methods of decision-making using cumulative sum (cusum) techniques for monitoring, control and retrospective analysis.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions, abbreviated terms and symbols

For the purposes of this document, the terms and definitions given in ISO 3534-1 and ISO 3534-2 and the following apply.

3.1 Terms and definitions

3.1.1
target value
T
value for which a departure from an average level is required to be detected

NOTE 1 With a charted cusum, the deviations from the target value are cumulated.

NOTE 2 Using a “V” mask, the target value is often referred to as the reference value or the nominal control value. If so, it should be acknowledged that it is not necessarily the most desirable or preferred value, as may appear in other standards. It is simply a convenient target value for constructing a cusum chart.

3.1.2
datum value
⟨tabulated cusum⟩ value from which differences are calculated

NOTE The upper datum value is $T + f\sigma_{\mathrm{e}}$, for monitoring an upward shift. The lower datum value is $T - f\sigma_{\mathrm{e}}$, for monitoring a downward shift.

3.1.3
reference shift
F, f
⟨tabulated cusum⟩ difference between the **target value** (3.1.1) and **datum value** (3.1.2)

NOTE It is necessary to distinguish between f that relates to a standardized reference shift, and F to an observed reference shift, $F = f\sigma_e$.

3.1.4
reference shift
F, f
⟨truncated V-mask⟩ slope of the arm of the mask (tangent of the mask angle)

NOTE It is necessary to distinguish between f that relates to a standardized reference shift, and F to an observed reference shift, $F = f\sigma_e$.

3.1.5
decision interval
H, h
⟨tabulated cusum⟩ cumulative sum of deviations from a **datum value** (3.1.2) required to yield a signal

NOTE It is necessary to distinguish between h that relates to a standardized decision interval, and H to an observed decision interval, $H = h\sigma_e$.

3.1.6
decision interval
H, h
⟨truncated V-mask⟩ half-height at the datum of the mask

NOTE It is necessary to distinguish between h that relates to a standardized decision interval, and H to an observed decision interval, $H = h\sigma_e$.

3.1.7
average run length
L
average number of samples taken up to the point at which a signal occurs

NOTE Average run length (L) is usually related to a particular process level in which case it carries an appropriate subscript, as, for example, L_0, meaning the average run length when the process is at target level, i.e. zero shift.

3.2 Abbreviated terms

ARL	average run length
CS1	cusum scheme with a long ARL at zero shift
CS2	cusum scheme with a shorter ARL at zero shift
DI	decision interval
EWMA	exponentially weighted moving average
FIR	fast initial response
LCL	lower control limit
RV	reference value
UCL	upper control limit

3.3 Symbols

a	scale coefficient
C	cusum value
C_r	difference in the cusum value between the lead point and the out-of-control point
c_4	factor for estimating the within-subgroup standard deviation
δ	amount of change to be detected
Δ	standardized amount of change to be detected
d	lead distance
d_2	factor for estimating the within-subgroup standard deviation from within-subgroup range
F	observed reference shift
f	standardized reference shift
H	observed decision interval
h	standardized decision interval
J	index number
φ	size of process adjustment
K	cusum datum value for discrete data
k	number of subgroups
L_0	average run length at zero shift
L_δ	average run length at δ shift
μ	population mean value
m	mean count number
n	subgroup size
p	probability of “success”
$\overline{R}$	mean subgroup range
r	number of plotted points between the lead point and the out-of-control point
σ	process standard deviation
σ_0	within-subgroup standard deviation
$\hat{\sigma}_0$	estimated within-subgroup standard deviation

σ_{e}	standard error
s	observed within-subgroup standard deviation
$\bar{s}$	average subgroup standard deviation
$s_{\bar{x}}$	realized standard error of the mean from k subgroups
T	target value
T_m	reference or target rate of occurrence
T_p	reference or target proportion
τ	true change point
t	observed change point
V_{avg}	average voltage
$\hat{V}_{\mathrm{avg}}$	estimated average voltage
w	difference between successive subgroup mean values
x	individual result
$\bar{x}$	arithmetic mean value (of a subgroup)
$\bar{\bar{x}}$	mean of subgroup means

4 Principal features of cumulative sum (cusum) charts

A cusum chart is essentially a running total of deviations from some preselected reference value. The mean of any group of consecutive values is represented visually by the current slope of the graph. The principal features of a cusum chart are the following.

a) It is sensitive in detecting changes in the mean.

b) Any change in the mean, and the extent of the change, is indicated visually by a change in the slope of the graph:

    1) a horizontal graph indicates an “on-target” or reference value;

    2) a downward slope indicates a mean less than the reference or target value: the steeper the slope, the bigger the difference;

    3) an upward slope indicates a mean more than the reference or target value: the steeper the slope, the bigger the difference.

c) It can be used retrospectively for investigative purposes, on a running basis for control, and for prediction of performance in the immediate future.

Referring to point b) above, a cusum chart has the capacity to clearly indicate points of change; they will be clearly indicated by the change in gradient of the cusum plot. This has enormous benefit for process management: to be able to quickly and accurately pinpoint the moment when a process altered so that the appropriate corrective action can be taken.

A further very useful feature of a cusum system is that it can be handled without plotting, i.e. in tabular form. This is very helpful if the system is to be used to monitor a highly technical process, e.g. plastic film manufacture, where the number of process parameters and product characteristics is large. Data from such a process might be captured automatically, downloaded into cusum software to produce an automated cusum analysis. A process manager can then be alerted to changes on many characteristics on a simultaneous basis. Annex B contains an example of the method.

5 Basic steps in the construction of cusum charts — Graphical representation

The following steps are used to set up a cusum chart for individual values.

Step 1: Choose a reference, target, control or preferred value. The average of past results will generally provide good discrimination.

Step 2: Tabulate the results in a meaningful (e.g. chronological) sequence. Subtract the reference value from each result.

Step 3: Progressively sum the values obtained in Step 2. These sums are then plotted as a cusum chart.

Step 4: To obtain the best visual effect set up a horizontal scale no wider than about 2,5 mm between plotting points.

Step 5: For reasonable discrimination, without undue sensitivity, the following options are recommended:

a) choose a convenient plotting interval for the horizontal axis and make the same interval on the vertical axis equal to 2σ (or $2\sigma_e$ if a cusum of means is to be charted), rounding off as appropriate; or

b) where it is required to detect a known change, say δ, choose a vertical scale such that the ratio of the scale unit on the vertical scale divided by the scale unit on the horizontal scale is between δ and 2δ, rounding off as appropriate.

NOTE The scale selection is visually very important since an inappropriate scale will give either the impression of impending disaster due to the volatile nature of the plot, or a view that nothing is changing. The schemes described in a) and b) above should give a scale that shows changes in a reasonable manner, neither too sensitive nor too suppressed.

6 Example of a cusum plot — Motor voltages

6.1 The process

Suppose a set of 40 values in chronological sequence is obtained of a particular characteristic. These happen to be voltages, taken in order of production, on fractional horsepower motors at an early stage of production. But they could be any individual values taken in a meaningful sequence and expressed on a continuous scale. These are now shown:

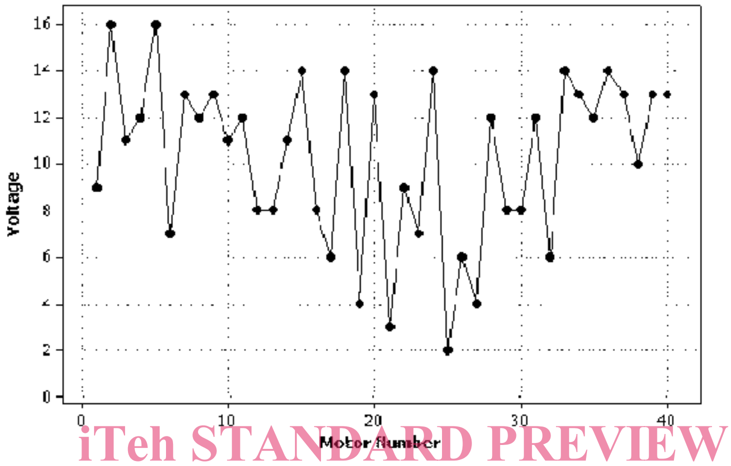
9, 16, 11, 12, 16, 7, 13, 12, 13, 11, 12, 8, 8, 11, 14, 8, 6, 14, 4, 13, 3, 9, 7, 14, 2, 6, 4, 12, 8, 8, 12, 6, 14, 13, 12, 14, 13, 10, 13, 13.

The reference or target voltage value is 10 V.

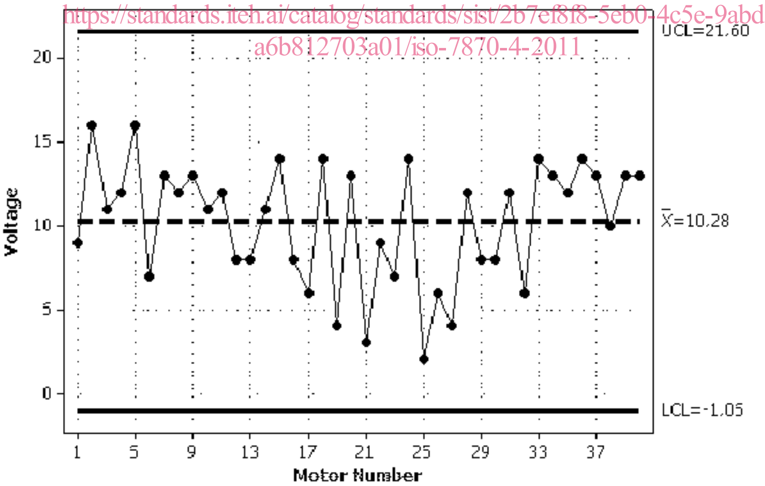
6.2 Simple plot of results

In order to gain a better understanding of the underlying behaviour of the process, by determining patterns and trends, a standard approach would be simply to plot these values in their natural order as shown in Figure 1 a).

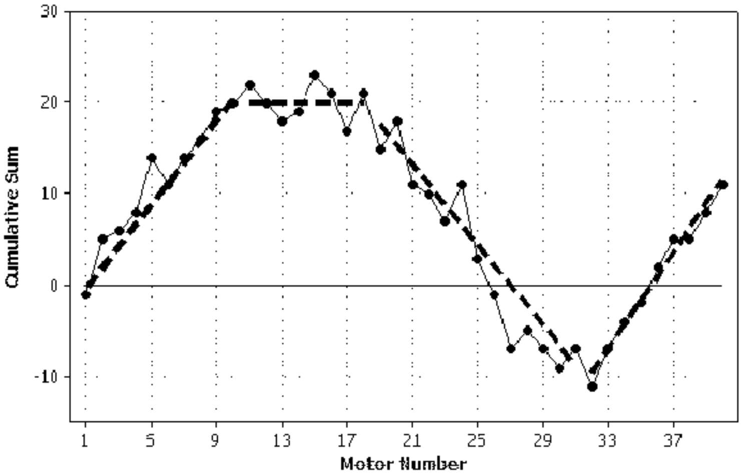
Apart from indicating a general drop away in the middle portion from a high start and with an equally high finish, Figure 1 a) is not very revealing because of the extremely noisy, or spiky, data throughout.



a) Simple plot of motor voltages



b) Standard control chart for individuals



c) Cusum chart

Figure 1 — Motor voltage example

6.3 Standard control chart for individual results

The next level of sophistication would be to establish a standard control chart for individuals as in Figure 1 b).

Figure 1 b) is even less revealing than the previous figure. It is, in fact, quite misleading. The standard statistical process control criteria to test for process stability and control are

a) no points lying above the upper control limit (UCL) or below the lower control limit (LCL),

b) no runs of seven or more intervals upwards or downwards,

c) no runs of seven points above or below the centreline.

The answer to all these criteria is “no”. Hence, one would be led to the conclusion that this is a stable process, one that is “in control” around its overall average value of about 10 V, which is the target value. Further standard analysis would reveal that although the process is stable, it is not capable of meeting specification requirements. However, this analysis would not in itself provide any further clues as to why it is incapable of meeting the requirements.

The reason for the inability of the standard control chart for individuals to be of value here is that the control limits are based on actual process performance and not on desired or specified requirements. Consequently, if the process naturally exhibits a large variation the control limits are correspondingly wide. What is required is a method that is better at indicating patterns and trends, or even pinpointing points of change, in order to help determine and remove primary sources of variation.

NOTE By using additional tools, such as an individual and moving range chart, the practitioner can study other process variation issues.

6.4 Cusum chart — Overall perspective

Another option here, the one recommended, would be to plot a cusum chart. Figure 1 c) illustrates the cusum plot of the same data.

It was not immediately apparent from the previous charts where, or whether, any significant changes in process level occurred, whereas the cusum chart indicates a well-defined pattern. The best fitting (by eye) indicates four changes in process level, changing after the 10th, 18th and 31st motors.

It is noted, from Clause 4, that an upward/downward slope indicates a value higher/lower than the preferred value and a horizontal line is indicative of a process at the preferred value. Hence, it is seen that this process appears to be on target only for a short period between around motor 11 and 18. Motors 1 to 10 were running higher similarly to motors 33 onwards, whereas the process between about motors 19 and 32 was delivering motors with low voltages.

These changes and their significance are further discussed and interpreted in detail in 6.6.

In a real life situation, the next step would be to seek out what happened operationally at these points of production to cause such changes in voltage performance. This poses certain questions directed specifically at improving the consistency of performance at the 10 V level. For instance, how did the build characteristics of motor 32 differ from those of 33? Or, what happened to the test gear calibration at this point? Did this correspond with a shift, manning or batch change? And so on. Used in this way, whatever the situation, the cusum chart can be a superb diagnostic tool. It pinpoints opportunities for improvement.

6.5 Cusum chart construction

The construction of a cusum chart using individual values, as in this example, is based on the very simple steps given in Clause 5.

Step 1: Choose a reference value, RV. Here the preferred or reference value is given as 10 V.

Step 2: Tabulate the results (voltages) in production sequence against motor number as in Table 1, column 2 (and 6). Subtract the reference value of 10 from each result as in Table 1, column 3 (and 7).

Step 3: Progressively sum the values of Table 1, column 3 (and 7) in column 4 (and 8). Plot column 4 (and 8) against the observation (motor) number as in Figure 1 c), taking note of the scale comments in Steps 4 and 5.

Table 1 — Tabular arrangement for calculating cusum values from a sequence of individual values

(1) Motor no.	(2) Voltage	(3) Voltage –10	(4) Cusum	(5) Motor no.	(6) Voltage	(7) Voltage –10	(8) Cusum
1	9	–1	–1	21	3	–7	+11
2	16	+6	+5	22	9	–1	+10
3	11	+1	+6	23	7	–3	+7
4	12	+2	+8	24	14	+4	+11
5	16	+6	+14	25	2	–8	+3
6	7	–3	+11	26	6	–4	–1
7	13	+3	+14	27	4	–6	–7
8	12	+2	+16	28	12	+2	–5
9	13	+3	+19	29	8	–2	–7
10	11	+1	+20	30	8	–2	–9
11	12	+2	+22	31	12	+2	–7
12	8	–2	+20	32	6	–4	–11
13	8	–2	+18	33	14	+4	–7
14	11	+1	+19	34	13	+3	–11
15	14	+4	+23	35	12	+2	–7
16	8	–2	+21	36	14	+4	–4
17	6	–4	+17	37	13	+3	–2
18	14	+4	+21	38	10	0	+2
19	4	–6	+15	39	13	+3	+5
20	13	+3	+18	40	13	+3	+5

6.6 Cusum chart interpretation

6.6.1 Introduction

When a cusum chart is used in retrospective diagnostic mode, as in this example, it is usually better not to focus on individual plotting points but to draw the minimum number of straight lines that are representative of lines of best fit by eye, through the data as in Figure 1 c).

One has to be very careful then not to interpret either the slope of these lines or their relative position related to the vertical axis, as with conventional data plots. It should be noted, too, that the vertical axis no longer represents actual voltages.

A straight line with an upward/downward slope does not indicate that the process level is increasing/decreasing, as is customary, but rather that it is constant at a value more/less than the reference value. The steeper the slope, the greater the difference. A horizontal line indicates that the process level is constant at the reference value. The interpretation of the cusum chart for the motor is now discussed in more detail.

6.6.2 The basics of interpretation of a cusum chart using “imaginary noiseless” data

Suppose that the sequence of the first 18 motor voltages had been 10, 10, 10, 13, 13, 13, 10, 10, 10, 9, 9, 9, 10, 10, 10, 8, 8, 8, as shown in Table 2, column 2, and that the reference value is still 10 V.

Table 2 — Imaginary motor data to illustrate the basic interpretation of a cusum chart

(1) Motor no.	(2) Voltage	(3) Voltage – 10	(4) Cusum
1	10	0	0
2	10	0	0
3	10	0	0
4	13	+3	+3
5	13	+3	+6
6	13	+3	+9
7	10	0	+9
8	10	0	+9
9	10	0	+9
10	9	–1	+8
11	9	–1	+7
12	9	–1	+6
13	10	0	+6
14	10	0	+6
15	10	0	+6
16	8	–2	+4
17	8	–2	+2
18	8	–2	0