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Sampling procedures for inspection by variables —

Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection

Règles d'échantillonnage pour les contrôles par mesures —
Partie 3: Plans d'échantillonnage doubles pour le contrôle lot par lot, indexés d'après le niveau de qualité acceptable (NQA)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 3951-3 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

ISO 3951 consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

— *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

— *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

— *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

— *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

The following part is under preparation:

— *Part 4: Procedures for assessment of declared quality levels*

Introduction

Inspection by variables for percentage nonconforming items, as described in this part of ISO 3951, includes several possible modes, the combination of which leads to a presentation which may appear quite complicated to the user:

a) procedures for unknown process standard deviation (the "s" method), or procedures for where the process standard deviation is originally unknown then estimated with fair precision, or known since the start of inspection (the "σ" method);

b) a single specification limit, or double specification limits with separate, combined or complex control;

c) normal inspection, tightened inspection or reduced inspection;

d) Form k plans and Form p^* plans;

e) a single quality characteristic (the univariate case) or a number of unrelated quality characteristics (the multivariate independent case).

The text has been arranged so that the simpler procedures may be implemented without necessarily understanding the more complicated procedures. The main text of this part of ISO 3951 is confined to the univariate case. The multivariate independent cases are treated separately in Annex A for the "s" method, in Annex B for the "σ" method and in Annex C for combined "s" method and "σ" method procedures. Annex D facilitates the use of the main text of the standard by directing the user to the clauses and tables concerning any univariate situation with which he might be confronted; it only deals with Clauses 16, 17, 21, 22 and 23 and, in every case, it is necessary to have read Clauses 1 to 15 and Clauses 18 to 20 first.

This part of ISO 3951 is complementary to the double sampling plans and procedures of ISO 2859-1. When specified by the responsible authority, it would be valid to reference both ISO 3951-3 and ISO 2859-1 in a product specification, a contract, inspection instructions, or other documents, and the provisions set forth therein shall govern. The "responsible authority" can then be designated in one of these documents.

In all parts of ISO 3951:

— the acronym AQL stands for "acceptance quality limit" rather than "acceptable quality level", in order to more accurately reflect its function;

— procedures are given for the case where the process standard is unknown (the "s" method) and for the case where it may be presumed to be known (the "σ" method);

— the sampling plans have been chosen so that their operating characteristic curves closely match those of the corresponding single sampling plans in ISO 2859-1;

— minimal statistical theory has been given (it being planned ultimately to provide this in a guidance document to sampling procedures for inspection by variables);

— text, charts and tables that are only informative have been consigned to annexes wherever practicable.

In none of the parts have methods been given based on the sample range, now that the availability of computers and calculators with a standard deviation function key is so widespread. Data for acceptance sampling by variables is often substantially more expensive to acquire than data for sampling by attributes, and the "s" method makes more efficient use of these data.

The coverage of ISO 3951-1 is constrained to the case of a single, normally distributed, quality characteristic with a single class of nonconformity, and includes the case of combined control of double specification limits.

ISO 3951-2 provides a more comprehensive treatment of single sampling plans by variables, including procedures for separate and complex control of double specification limits. Procedures are also given for multiple independent quality characteristics and/or multiple AQLs.

ISO 3951-3 provides plans for double sampling by variables, which on average provide substantial savings of inspection effort in comparison with plans for single sampling by variables. The savings are achieved by first selecting from the lot and inspecting a random sample that is typically nearly 40 % smaller than that of the corresponding single sampling plan. If these inspection results satisfy an acceptance criterion, an immediate decision is made to accept the lot without further inspection. Alternatively, if the inspection results satisfy a non-acceptance criterion, an immediate decision not to accept the lot is made without further inspection. Thus, when quality is very good or very poor, the saving in inspection effort can amount to nearly 40 %. Only when the inspection results from the first sample are equivocal is a second random sample, of the same size as the first, selected; the acceptability of the lot is then resolved by combining the results of the first and second samples and determining whether they satisfy a second acceptance criterion.

This publication does not purport to include all the necessary provisions of a contract. Users are responsible for its correct application.

Sampling procedures for inspection by variables —

Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection

1 Scope

This part of ISO 3951 specifies an acceptance sampling system of double sampling schemes for inspection by variables for percent nonconforming. It is indexed in terms of the acceptance quality limit (AQL).

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of acceptable quality have a high probability of acceptance and that the probability of non-accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide

— automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected, and

— an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this part of ISO 3951, the acceptability of a lot is implicitly or explicitly determined from an estimate of the percentage of nonconforming items in the process, based on either one or two random samples of items from the lot.

This part of ISO 3951 is primarily designed for use under the following conditions:

a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process; if there are different producers or production processes, apply this part of ISO 3951 to each one separately;

b) where the items of product have a single quality characteristic (for multiple quality characteristics, see informative Annexes A, B and C);

c) where the quality characteristic is measurable on a continuous scale;

d) where the measurement error is negligible (i.e. with a standard deviation of no more than 10 % of the corresponding process standard deviation);

e) where production is stable (under statistical control) and the quality characteristic is distributed, at least to a close approximation, according to a normal distribution;

CAUTION — The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened previously for nonconforming items.

f) where the possibility of having to select and inspect a second sample is administratively acceptable;

g) where a contract or standard defines an upper specification limit U, a lower specification limit L or both on the quality characteristic. An item is deemed to conform if its measured quality characteristic x satisfies the appropriate one of the following inequalities:

   1) $x \geqslant L$ (i.e. the lower specification limit is not violated);

   2) $x \leqslant U$ (i.e. the upper specification limit is not violated);

   3) $x \geqslant L$ and $x \leqslant U$ (i.e. neither the lower nor the upper specification limit is violated).

NOTE Inequalities 1) and 2) are called cases with a "single specification limit", and 3) is the case with "double specification limits". For double specification limits, a further distinction is made between combined control, separate control and complex control, as follows:

— combined control is where a single AQL applies to nonconformity beyond both limits;

— separate control is where separate AQLs apply to nonconformity beyond each of the limits;

— complex control is where one AQL applies to nonconformity beyond the limit that is of greater seriousness, and a larger AQL applies to the total nonconformity beyond both limits.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2:2006, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of this part of ISO 3951, the definitions given in ISO 3534-1, ISO 3534-2 and the following apply. References are given in square brackets for definitions that have been re-expressed in the vocabulary of this part of ISO 3951 for the user's convenience.

3.1
inspection by variables
inspection by measuring the magnitude(s) of the characteristic(s) of an item

[ISO 3534-2:2006, definition 4.1.4]

3.2
sampling inspection
inspection of selected items in the group under consideration

[ISO 3534-2:2006, definition 4.1.6]

3.3
acceptance sampling inspection
acceptance inspection where the acceptability is determined by means of sampling inspection

[ISO 3534-2:2006, definition 4.1.8]

3.4
double sampling inspection, double sampling
acceptance sampling inspection based initially on a first sample, of size n_1, which leads to a decision to accept, non-accept, or to inspect a second sample, of size n_2, before taking the decision whether or not to accept

NOTE 1 The decisions are made according to defined rules.

NOTE 2 In this part of ISO 3951, both sample sizes are equal and denoted by n, i.e. $n_1 = n_2 = n$.

3.5
acceptance sampling inspection by variables
acceptance sampling inspection in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

[ISO 3534-2:2006, definition 4.2.11]

NOTE An acceptable process is a process that generates nonconforming items at a rate no worse than the AQL.

3.6
process average
rate at which nonconforming items are generated by a process

3.7
acceptance quality limit
AQL
worst tolerable product quality level

NOTE See Clause 5.

3.8
quality level
quality expressed as a rate of nonconforming units

3.9
limiting quality
LQ
quality level, when a lot is considered in isolation, which, for the purposes of acceptance sampling inspection, is limited to a low probability of acceptance (in this part of ISO 3951: 10 %)

NOTE See Clause 8.

3.10
nonconformity
non-fulfilment of a requirement

[ISO 9000:2005, definition 3.6.2]

NOTE 1 Nonconformity will generally be classified by its degree of seriousness, such as:

Class A. Nonconformity of a type considered to be of the highest concern for the product or service. Such types of nonconformity will typically be assigned very small AQL values.

Class B. Nonconformity of a type considered to have the next lower degree of concern; this is typically assigned a larger AQL value than that in class A and smaller than that in class C if a third class exists, and so on.

The number of classes and the assignment into a class should be appropriate to the quality requirements of the specific situation.

NOTE 2 The main text of this part of ISO 3951 deals with the univariate case, for which there will be either one or two classes of nonconformity.

3.11
nonconforming unit
unit with one or more nonconformities

[ISO 3534-2:2006, definition 1.2.15]

3.12
"*s*" **method acceptance sampling plan**
acceptance sampling plan by variables using the sample mean(s) and sample standard deviation(s)

NOTE See Clause 16.

3.13
"σ" **method acceptance sampling plan**
acceptance sampling plan by variables using the sample mean(s) and the presumed value(s) of the process standard deviation(s)

NOTE See Clause 17.

3.14
specification limit
limiting value stated for a characteristic

[ISO 3534-2:2006, definition 3.1.3]

3.15
lower specification limit
L

specification limit that defines the lower limiting value

[ISO 3534-2:2006, definition 3.1.5]

3.16
upper specification limit
U
specification limit that defines the upper limiting value

[ISO 3534-2:2006, definition 3.1.4]

3.17
combined control
requirement when nonconformity beyond both the upper and the lower specification limits of a quality characteristic belongs to the same class to which a single AQL is applied

NOTE 1 See 5.3, 16.4 and 17.4.

NOTE 2 The use of a combined AQL requirement implies that nonconformities beyond either specification limit are believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.18
separate control
requirement when nonconformity beyond the upper and the lower specification limits of a quality characteristic belongs to different classes, to which separate AQLs are applied

NOTE See 5.3, 16.3 and 17.3.

3.19
complex control
requirement when nonconformity beyond both the upper and the lower specification limits of a quality characteristic belongs to one class and nonconformity beyond either the upper or the lower specification limit belongs to a different class, with separate AQLs being applied to the two classes

NOTE See 5.3, 16.5 and 17.5.

3.20
acceptability constants
k, p^*
constants depending on the specified value of the acceptance quality limit, sample size and inspection severity, used in the criteria for accepting the lot in an acceptance sampling plan by variables

NOTE 1 See Clauses 16 and 17.

NOTE 2 For double sampling, there will be three such pairs of acceptability constants, one for acceptance at the first sample, one for non-acceptance at the first sample and one for acceptance with the combined first and second samples.

3.21
quality statistic
Q
⟨acceptance sampling⟩ function of the specification limit, the sample mean, and the sample or process standard deviation, used in assessing the acceptability of a lot

[ISO 3534-2:2006, definition 4.4.9]

NOTE See also 3.22 and 3.23.

3.22
lower quality statistic

Q_L
function of the lower specification limit, the sample mean, and the sample or process standard deviation

NOTE See Clause 4 for further details.

3.23
upper quality statistic
Q_U
function of the upper specification limit, the sample mean, and the sample or process standard deviation

NOTE See Clause 4 for further details.

3.24
maximum sample standard deviation
MSSD
s_{max}
largest sample standard deviation for a given sample size code letter and acceptance quality limit for which it is possible to satisfy an acceptance criterion for double specification limits when the process variability is unknown

NOTE 1 The MSSD depends on whether the double specification limits are under combined, separate or complex control and on the inspection severity (i.e. normal, tightened or reduced).

NOTE 2 See 16.4.2 and Table 16, 17 or 18.

NOTE 3 For double sampling plans, there are two MSSDs under each combination of inspection severity and type of control, one for the first sample and one for the combined first and second samples.

3.25
maximum process standard deviation
MPSD
σ_{max}
largest process standard deviation for a given sample size code letter and acceptance quality limit for which it is possible to satisfy an acceptance criterion for double specification limits under all inspection severities (i.e. normal, tightened or reduced) when the process variability is known

NOTE 1 An MPSD depends on whether the double specification limits are under combined, separate or complex control, but does not depend on the inspection severity or on whether the sample is the first or second.

NOTE 2 See 17.3, 17.4 and 17.5 and Tables 19, 20 and 21.

3.26
switching rule
instruction within an acceptance sampling scheme for changing from one acceptance sampling plan to another of greater or lesser severity based on demonstrated quality history

NOTE 1 Normal, tightened or reduced inspection, or discontinuation of inspection, are examples of severity of sampling.

[ISO 3534-2:2006, definition 4.3.4]

NOTE 2 See Clause 21.

3.27
measurement

set of operations having the object of determining the value of a quantity

[ISO 3534-2:2006, definition 3.2.1]

3.28
average sample size
ASSI
average number of units in a sample inspected per lot in reaching decisions to accept or not to accept when using a given acceptance sampling scheme

NOTE ASSI is dependent on the actual quality level of the submitted lots.

[ISO 3534-2:2006, definition 4.7.3]

3.29
responsible authority
concept used to maintain the neutrality of this part of ISO 3951, irrespective of whether it is being invoked or applied by the first, second or third party

NOTE 1 The responsible authority may be:

a) the quality department within a supplier's organization (first party);

b) the purchaser or procurement organization (second party);

c) an independent verification or certification authority (third party);

d) any of a), b) or c), differing according to function (see Note 2) as described in a written agreement between two of the parties, for example a document between supplier and purchaser.

NOTE 2 The duties and functions of a responsible authority are outlined in this part of ISO 3951 (see 5.3, 6, 10, 11, 16.4.3.2.1, 17.1, 19.1, 20.2, 21.4, 23.1, 23.2 and 23.3).

4 Symbols and abbreviations

4.1 Symbols

For the purposes of this part of ISO 3951, the following symbols apply.

Ac	acceptance number
c_u	factor given in Table 29 for determining the upper control limit for sample standard deviations (see 23.2)
f_s	factor given in Tables 16, 17 and 18 for combined control of double specification limits, relating the maximum sample standard deviation (MSSD) to the difference between U and L, for normal, tightened and reduced inspection respectively (see 16.4.2 and 16.4.3.1)
	NOTE 1 $f_{s,1}$ and $f_{s,c}$ represent respectively the factors applicable to the standard deviation of the first sample and to the combined standard deviation of the first and second samples.
f_σ	factor given in Tables 19, 20 and 21 that relates the maximum process standard deviation (MPSD) to the difference between U and L, for combined, separate and complex control respectively (see 17.4 and 17.5)
k	Form k acceptability constant
	NOTE 2 k_a, k_r and k_c represent respectively the Form k acceptability and non-acceptability constants at the first sample and the acceptability constant for the combined first and second samples.
L	lower specification limit (as a suffix to a variable, denotes its value at L).
μ	process mean
N	lot size (number of items in a lot)
n	sample size (number of items in each sample)
p	process fraction nonconforming
$\hat{p}$	estimate of the process fraction nonconforming
$\hat{p}_L$	estimate of the process fraction nonconforming below the lower specification limit
$\hat{p}_U$	estimate of the process fraction nonconforming above the upper specification limit
p^*	Form p* acceptability constant, the largest acceptable value of the estimate of the process fraction nonconforming
	NOTE 3 p_a^*, p_r^* and p_c^* represent respectively the Form p* acceptability and non-acceptability constants at the first sample and the acceptability constant for the combined first and second samples.
P_a	probability of acceptance
Q	quality statistic
Q_L	lower quality statistic
	NOTE 4 Q_L is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and $(\bar{x} - L)/\sigma$ when it is presumed to be known.
	NOTE 5 $Q_{L,1}$ is defined as $(\bar{x}_1 - L)/s_1$ or $(\bar{x}_1 - L)/\sigma$; $Q_{L,c}$ is defined as $(\bar{x}_c - L)/s_c$ or $(\bar{x}_c - L)/\sigma$.
Q_U	upper quality statistic
	NOTE 6 Q_U is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and $(U - \bar{x})/\sigma$ when it is presumed to be known.
	NOTE 7 $Q_{U,1}$ is defined as $(U - \bar{x}_1)/s_1$ or $(U - \bar{x}_1)/\sigma$; $Q_{U,c}$ is defined as $(U - \bar{x}_c)/s_c$ or $(U - \bar{x}_c)/\sigma$.