# INTERNATIONAL STANDARD

# ISO 13786

Second edition
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# Thermal performance of building components — Dynamic thermal characteristics — Calculation methods

*Performance thermique des composants de bâtiment — Caractéristiques thermiques dynamiques — Méthodes de calcul*



Reference number
ISO 13786:2007(E)

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# Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 13786 was prepared by Technical Committee ISO/TC 163, *Thermal performance and energy use in the built environment*, Subcommittee SC 2, *Calculation methods*.



This second edition cancels and replaces the first edition (ISO 13786:1999), which has been technically revised.

The following principal changes have been made to the first edition:



— all equations in Clause 3 have been reviewed and corrected as appropriate; the definition of heat capacity (3.1.1.5) has been modified;

— all equations in 7.2.1 and 7.2.2 have been reviewed and corrected as appropriate;

— 7.2.4 contains a new equation for periodic thermal transmittance, and a new note;

— Equation (A.4) has been corrected;

— B.2 has undergone minor revisions;

— Table C.1 has been added;

— Annex D contains amended examples to align with changes to the formulae in the main body of the text.

 

## Introduction

This International Standard provides the means (in part) to assess the contribution that building products and services make to energy conservation and to the overall energy performance of buildings.

The dynamic thermal characteristics of a building component describe the thermal behaviour of the component when it is subject to variable boundary conditions, i.e. variable heat flow rate or variable temperature on one or both of its boundaries. In this International Standard, only sinusoidal boundary conditions are considered: boundaries are submitted to sinusoidal variations of temperature or heat flow rate.

The properties considered are thermal admittances and thermal dynamic transfer properties, relating cyclic heat flow rate to cyclic temperature variations. Thermal admittance relates heat flow rate to temperature variations on the same side of the component. Thermal dynamic transfer properties relate physical quantities on one side of the component to those on the other side. From the aforementioned properties, it is possible to define the heat capacity of a given component which quantifies the heat storage property of that component.

The dynamic thermal characteristics defined in this International Standard can be used in product specifications of complete building components.

The dynamic thermal characteristics can also be used in the calculation of:

— the internal temperature in a room;

— the daily peak power and energy needs for heating or cooling;

— the effects of intermittent heating or cooling, etc.

# Thermal performance of building components — Dynamic thermal characteristics — Calculation methods

## 1 Scope

This International Standard specifies the characteristics related to the dynamic thermal behaviour of a complete building component and provides methods for their calculation. It also specifies the information on building materials required for the use of the building component. Since the characteristics depend on the way materials are combined to form building components, this International Standard is not applicable to building materials or to unfinished building components.

The definitions given in this International Standard are applicable to any building component. A simplified calculation method is provided for plane components consisting of plane layers of substantially homogeneous building materials.

Annex A specifies simpler methods for the estimation of the heat capacities in some limited cases. These methods are suitable for the determination of dynamic thermal properties required for the estimation of energy use. These approximations are not appropriate, however, for product characterization.



Annex B gives the basic principle and examples of applications of the dynamic thermal characteristics defined in this International Standard.



Annex C provides information for programming the calculation method.

Annex D gives examples of calculation for a building component.

## 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 6946, *Building components and building elements — Thermal resistance and thermal transmittance — Calculation method*

ISO 7345, *Thermal insulation — Physical quantities and definitions*

ISO 10211, *Thermal bridges in building construction — Heat flows and surface temperatures — Detailed calculations*

## 3 Terms, definitions, symbols and units

### 3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 7345 and the following apply.

#### 3.1.1 Definitions valid for any component

**3.1.1.1**
**component**
part of a building, such as a wall, floor or roof, or a part of such an element

**3.1.1.2**
**thermal zone of a building**
part of a building throughout which the internal temperature is assumed to have negligible spatial variations

NOTE 1 A component separates two zones, designated in this International Standard by $m$ and $n$.

NOTE 2 The external environment can also be considered a zone.

**3.1.1.3**
**sinusoidal conditions**
conditions in which the variations of the temperature and heat flows around their long term average values are described by a sine function of time

NOTE Using complex numbers, the temperature in zone $n$ can be described by Equation (1) and the heat flow by Equation (2):



$$\theta_n(t) = \overline{\theta}_n + \left|\hat{\theta}_n\right| \cos(\omega t + \psi) = \overline{\theta}_n + \frac{1}{2}\left[\hat{\theta}_{+n} e^{\mathrm{j}\omega t} + \hat{\theta}_{-n} e^{-\mathrm{j}\omega t}\right] \quad (1)$$



$$\Phi_n(t) = \overline{\Phi}_n + \left|\hat{\Phi}_n\right| \cos(\omega t + \varphi) = \overline{\Phi}_n + \frac{1}{2}\left[\hat{\Phi}_{+n} e^{\mathrm{j}\omega t} + \hat{\Phi}_{-n} e^{-\mathrm{j}\omega t}\right] \quad (2)$$

where

$\overline{\theta}_n$ and $\overline{\Phi}_n$ are average values of temperature and heat flow;

$\left|\hat{\theta}_n\right|$ and $\left|\hat{\Phi}_n\right|$ are amplitudes of temperature and heat flow variations;

$\hat{\theta}_{\pm n}$ and $\hat{\Phi}_{\pm n}$ are complex amplitudes defined by:

$$\hat{\theta}_{\pm n} = \left|\hat{\theta}_n\right| e^{\pm \mathrm{j}\psi} \text{ and } \hat{\Phi}_{\pm n} = \left|\hat{\Phi}_n\right| e^{\pm \mathrm{j}\varphi} \quad (3)$$

$\omega$ is the angular frequency of the variations.

**3.1.1.4**
**periodic thermal conductance**
$L_{mn}$
complex number relating the periodic heat flow into a component to the periodic temperatures on either side of it under sinusoidal conditions

Another representation of the concept:

$$\hat{\Phi}_m = L_{mm}\hat{\theta}_m - L_{mn}\hat{\theta}_n \quad (4)$$

 

NOTE 1 $L_{mm}$ relates the periodic heat flow on side $m$ to the periodic temperature on side $m$ when the temperature amplitude on side $n$ is zero. $L_{mn}$ relates the periodic heat flow on side $m$ to the periodic temperature on side $n$ when the temperature amplitude on side $m$ is zero.

NOTE 2 As a convention within this International Standard, the heat flow rate is defined as positive when it enters the surface of the component.

**3.1.1.5**
**heat capacity**
modulus of the net periodic thermal conductance divided by the angular frequency

Another representation of the concept:

$$C_m = \frac{1}{\omega}\left|L_{mm} - L_{mn}\right| \quad (5)$$

**3.1.1.6**
**time shift**
$\Delta t$
period of time between the maximum amplitude of a cause and the maximum amplitude of its effect

### 3.1.2 Definitions valid only for one dimensional heat flow

**3.1.2.1**
**plane component**
component for which the smallest curvature radius is at least five times its thickness



**3.1.2.2**
**homogeneous material layer**
layer of material in which the largest size of inhomogeneities does not exceed one fifth of the thickness of the layer



**3.1.2.3**
**thermal admittance**
complex quantity defined as the complex amplitude of the density of heat flow rate through the surface of the component adjacent to zone $m$, divided by the complex amplitude of the temperature in the same zone when the temperature on the other side is held constant

Another representation of the concept:

$$Y_{mm} = \frac{\hat{q}_m}{\hat{\theta}_m} \quad (6)$$

**3.1.2.4**
**periodic thermal transmittance**
complex quantity defined as the complex amplitude of the density of heat flow rate through the surface of the component adjacent to zone $m$, divided by the complex amplitude of the temperature in zone $n$ when the temperature in zone $m$ is held constant

Another representation of the concept:

$$Y_{mn} = -\frac{\hat{q}_m}{\hat{\theta}_n} \quad (7)$$

**3.1.2.5**
**areal heat capacity**
heat capacity divided by area of element

Another representation of the concept:

$$\kappa_m = \frac{C_m}{A} = \frac{1}{\omega}\left|Y_{mm} - Y_{mn}\right| \qquad (8)$$

NOTE 1 Using Equation (8), the heat capacities are then:

$$C_m = A\kappa_m \qquad (9)$$

NOTE 2 There are two thermal admittances and two heat capacities for a component separating two zones, all of which depend on the period of the thermal variations.

**3.1.2.6**
**decrement factor**
ratio of the modulus of the periodic thermal transmittance to the steady-state thermal transmittance $U$

Another representation of the concept:

$$f = \frac{\left|\hat{q}_m\right|}{\left|\hat{\theta}_n\right|U} = \frac{\left|Y_{mn}\right|}{U} \qquad (10)$$



where $m \neq n$

**3.1.2.7**
**periodic penetration depth**
$\delta$



depth at which the amplitude of the temperature variations are reduced by the factor "e" in a homogeneous material of infinite thickness subjected to sinusoidal temperature variations on its surface

Another representation of the concept:

$$\delta = \sqrt{\frac{\lambda\, T}{\pi \rho\, c}} \qquad (11)$$

NOTE e is the base of natural logarithms; e = 2,718...

**3.1.2.8**
**heat transfer matrix**
$\boldsymbol{Z}$
matrix relating the complex amplitudes of temperature and heat flow rate on one side of a component to the complex amplitudes of temperature and heat flow rate on the other side

Another representation of the concept:

$$\boldsymbol{Z} = \begin{pmatrix} \hat{\theta}_2 \\ \hat{q}_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \cdot \begin{pmatrix} \hat{\theta}_1 \\ \hat{q}_1 \end{pmatrix} \qquad (12)$$

## 3.2 Symbols and units

| Symbol | Quantity | Unit |
|---|---|---|
| $A$ | area | $m^2$ |
| $C$ | heat capacity | J/K |
| $L_{mn}$ | periodic thermal conductance | W/K |
| $R$ | thermal resistance | $m^2 \cdot K/W$ |
| $T$ | period of the variations | s |
| $U$ | thermal transmittance under steady state boundary conditions | $W/(m^2 \cdot K)$ |
| $Y_{mm}$ | thermal admittance | $W/(m^2 \cdot K)$ |
| $Y_{mn}$ | periodic thermal transmittance | $W/(m^2 \cdot K)$ |
| $\boldsymbol{Z}$ | heat transfer matrix environment to environment | — |
| $Z_{mn}$ | element of the heat transfer matrix | — |
| $a$ | thermal diffusivity | $m^2/s$ |
| $c$ | specific heat capacity | $J/(kg \cdot K)$ |
| $d$ | thickness of a layer | m |
| $f$ | decrement factor | — |
| j | unit on the imaginary axis for a complex number; $\mathrm{j} = \sqrt{-1}$ | — |
| $q$ | density of heat flow rate | $W/m^2$ |
| $t$ | time | s or h |
| $x$ | distance through the component | m |
| $\Delta t$ | time shift: time lead (if positive), or time lag (if negative) | s or h |
| $\delta$ | periodic penetration depth of a heat wave in a material | m |
| $\Phi$ | heat flow rate | W |
| $\xi$ | ratio of the thickness of the layer to the penetration depth | — |
| $\kappa$ | areal heat capacity | $J/(m^2 \cdot K)$ |
| $\lambda$ | design thermal conductivity | $W/(m \cdot K)$ |
| $\rho$ | density | $kg/m^3$ |
| $\theta$ | temperature | °C |
| $\omega$ | angular frequency; $\omega = \frac{2\pi}{T}$ | rad/s |
| $\varphi$, $\psi$ | phase differences | rad |

### 3.3 Subscripts

| Subscript | Definition |
|---|---|
| a | air layer |
| e | external |
| i | internal |
| $m$, $n$ | for the thermal zones |
| s | related to surface |
| ee | from environment to environment |

### 3.4 Other symbols

| Symbol | Definition |
|---|---|
| ^ | complex amplitude |
| – | mean value |
| \| \| | modulus of a complex number |
| arg | argument of a complex number |



## 4 Period of the thermal variations

The definition of dynamic thermal characteristics and the formulae for their calculation are valid for any period of thermal variations.

The values of dynamic thermal characteristics depend on the periods. If more than one period is considered, an additional suffix shall be added to all quantities affected so as to distinguish between the values for different periods.

Practical time periods are:

— one hour (3 600 s), which corresponds to very short time variations, such as those resulting from temperature control systems;

— one day (86 400 s), corresponding to daily meteorological variations and temperature setback;

— one week (604 800 s), corresponding to longer term averaging of the building;

— one year (31 536 000 s), useful for treatment of heat transfer through the ground.

## 5 Data required

The data required to compute the dynamic thermal characteristics are:

a) the detailed drawings of the product, with dimensions;

b) for each material used in the product:

  — the thermal conductivity, $\lambda$;

 