TECHNICAL REPORT

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Calculation of micropitting load capacity of cylindrical spur and helical gears —

Part 1:
Introduction and basic principles

Calcul de la capacité de charge aux micropiqûres des engrenages cylindriques à dentures droite et hélicoïdale —

Partie 1: Introduction et principes fondamentaux

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Foreword

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Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 15144-1 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

ISO/TR 15144 consists of the following parts, under the general title *Calculation of micropitting load capacity of cylindrical spur and helical gears*:

— *Part 1: Introduction and basic principles*

Introduction

This part of ISO/TR 15144 provides principles for the calculation of the micropitting load capacity of cylindrical involute spur and helical gears with external teeth.

The basis for the calculation of the micropitting load capacity of a gear set is the model of the minimum operating specific lubricant film thickness in the contact zone. There are many influence parameters, such as surface topology, contact stress level, and lubricant chemistry. Whilst these parameters are known to affect the performance of micropitting for a gear set, it must be stated that the subject area remains a topic of research and, as such, the science has not yet developed to allow these specific parameters to be included directly in the calculation methods. Furthermore, the correct application of tip and root relief (involute modification) has been found to greatly influence micropitting; the suitable values should therefore be applied. Surface finish is another crucial parameter. At present *R*a is used, but other aspects such as *R*z or skewness have been observed to have significant effects which could be reflected in the finishing process applied.

Although the calculation of specific lubricant film thickness does not provide a direct method for assessing micropitting load capacity, it can serve as an evaluation criterion when applied as part of a suitable comparative procedure based on known gear performance.

Calculation of micropitting load capacity of cylindrical spur and helical gears —

Part 1:
Introduction and basic principles

1 Scope

This part of ISO/TR 15144 describes a procedure for the calculation of the micropitting load capacity of cylindrical gears with external teeth. It has been developed on the basis of testing and observation of oil-lubricated gear transmissions with modules between 3 mm and 11 mm and pitch line velocities of 8 m/s to 60 m/s. However, the procedure is applicable to any gear pair where suitable reference data is available, providing the criteria specified below are satisfied.

The formulae specified are applicable for driving as well as for driven cylindrical gears with tooth profiles in accordance with the basic rack specified in ISO 53. They are also applicable for teeth conjugate to other basic racks where the virtual contact ratio is less than $\varepsilon_{\alpha n} = 2{,}5$. The results are in good agreement with other methods for normal working pressure angles up to 25°, reference helix angles up to 25° and in cases where pitch line velocity is higher than 2 m/s.

This part of ISO/TR 15144 is not applicable for the assessment of types of gear tooth surface damage other than micropitting.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 53:1998, *Cylindrical gears for general and heavy engineering — Standard basic rack tooth profile*

ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth*

ISO 6336-1:2006, *Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors*

ISO 6336-2:2006, *Calculation of load capacity of spur and helical gears — Part 2: Calculation of surface durability (pitting)*

ISO 21771:2007, *Gears — Cylindrical involute gears and gear pairs — Concepts and geometry*

ISO/TR 13989-1:2000, *Calculation of scuffing load capacity of cylindrical, bevel and hypoid gears — Part 1: Flash temperature method*

ISO/TR 13989-2:2000, *Calculation of scuffing load capacity of cylindrical, bevel and hypoid gears — Part 2: Integral temperature method*

3 Terms, definitions, symbols and units

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 1122-1, ISO 6336-1 and ISO 6336-2 apply.

3.2 Symbols and units

The symbols used in this document are given in Table 1. The units of length metre, millimetre and micrometre are chosen in accordance with common practice. The conversions of the units are already included in the given equations.

Table 1 — Symbols and units

Symbol	Description	Unit
a	centre distance	mm
B_{M1}	thermal contact coefficient of pinion	$N/(m \cdot s^{0,5} \cdot K)$
B_{M2}	thermal contact coefficient of wheel	$N/(m \cdot s^{0,5} \cdot K)$
b	face width	mm
C_{a1}	tip relief of pinion	µm
C_{a2}	tip relief of wheel	µm
C_{eff}	effective tip relief	µm
c_{M1}	specific heat per unit mass of pinion	J/(kg·K)
c_{M2}	specific heat per unit mass of wheel	J/(kg·K)
c'	maximum tooth stiffness per unit face width (single stiffness) of a tooth pair	N/(mm·µm)
$c_{\gamma\alpha}$	mean value of mesh stiffness per unit face width	N/(mm·µm)
d_{a1}	tip diameter of pinion	mm
d_{a2}	tip diameter of wheel	mm
d_{b1}	base diameter of pinion	mm
d_{b2}	base diameter of wheel	mm
d_{w1}	pitch diameter of pinion	mm
d_{w2}	pitch diameter of wheel	mm
d_{Y1}	Y-circle diameter of pinion	mm
d_{Y2}	Y-circle diameter of wheel	mm
E_r	reduced modulus of elasticity	N/mm^2
E_1	modulus of elasticity of pinion	N/mm^2
E_2	modulus of elasticity of wheel	N/mm^2
F_{bt}	nominal transverse load in plane of action (base tangent plane)	N
F_t	(nominal) transverse tangential load at reference cylinder per mesh	N
G_M	material parameter	–
g_Y	parameter on the path of contact (distance of point Y from point A)	mm
g_α	length of path of contact	mm
H_v	load losses factor	–

Table 1 (*continued*)

Symbol	Description	Unit
h_Y	local lubricant film thickness	µm
K_A	application factor	–
$K_{H\alpha}$	transverse load factor	–
$K_{H\beta}$	face load factor	–
K_v	dynamic factor	–
n_1	rotation speed of pinion	min^{-1}
P	transmitted power	kW
p_{et}	transverse base pitch on the path of contact	mm
$p_{dyn,Y}$	local Hertzian contact stress including the load factors K	N/mm^2
$p_{H,Y}$	local nominal Hertzian contact stress	N/mm^2
Ra	effective arithmetic mean roughness value	µm
Ra_1	arithmetic mean roughness value of pinion	µm
Ra_2	arithmetic mean roughness value of wheel	µm
$S_{GF,Y}$	local sliding parameter	–
S_λ	safety factor against micropitting	–
$S_{\lambda,min}$	minimum required safety factor against micropitting	–
T_1	nominal torque at the pinion	Nm
U_Y	local velocity parameter	–
u	gear ratio	–
$v_{g,Y}$	local sliding velocity	m/s
$v_{r1,Y}$	local tangential velocity on pinion	m/s
$v_{r2,Y}$	local tangential velocity on wheel	m/s
$v_{\Sigma,C}$	sum of tangential velocities at pitch point	m/s
$v_{\Sigma,Y}$	sum of tangential velocities at point Y	m/s
W_W	material factor	–
W_Y	local load parameter	–
$X_{but,Y}$	local buttressing factor	–
X_{Ca}	tip relief factor	–
X_L	lubricant factor	–
X_R	roughness factor	–
X_S	lubrication factor	–
X_Y	local load sharing factor	–
Z_E	elasticity factor	$(N/mm^2)^{0,5}$
z_1	number of teeth of pinion	–
z_2	number of teeth of wheel	–
α_t	transverse pressure angle	°
α_{wt}	pressure angle at the pitch cylinder	°
$\alpha_{\theta B,Y}$	pressure-viscosity coefficient at local contact temperature	m^2/N
$\alpha_{\theta M}$	pressure-viscosity coefficient at bulk temperature	m^2/N
α_{38}	pressure-viscosity coefficient at 38 °C	m^2/N
β_b	base helix angle	°

Table 1 (*continued*)

Symbol	Description	Unit
ε_{max}	maximum addendum contact ratio	–
ε_{α}	transverse contact ratio	–
$\varepsilon_{\alpha n}$	virtual contact ratio, transverse contact ratio of a virtual spur gear	–
ε_{β}	overlap ratio	–
ε_{γ}	total contact ratio	–
ε_1	addendum contact ratio of the pinion	–
ε_2	addendum contact ratio of the wheel	–
$\eta_{\theta B,Y}$	dynamic viscosity at local contact temperature	$N \cdot s/m^2$
$\eta_{\theta M}$	dynamic viscosity at bulk temperature	$N \cdot s/m^2$
$\eta_{\theta oil}$	dynamic viscosity at oil inlet/sump temperature	$N \cdot s/m^2$
η_{38}	dynamic viscosity at 38 °C	$N \cdot s/m^2$
$\theta_{B,Y}$	local contact temperature	°C
$\theta_{fl,Y}$	local flash temperature	°C
θ_M	bulk temperature	°C
θ_{oil}	oil inlet/sump temperature	°C
$\lambda_{GF,min}$	minimum specific lubricant film thickness in the contact area	–
$\lambda_{GF,Y}$	local specific lubricant film thickness	–
λ_{GFP}	permissible specific lubricant film thickness	–
λ_{GFT}	limiting specific lubricant film thickness of the test gears	–
λ_{M1}	specific heat conductivity of pinion	$W/(m \cdot K)$
λ_{M2}	specific heat conductivity of wheel	$W/(m \cdot K)$
μ_m	mean coefficient of friction	–
$\nu_{\theta B,Y}$	kinematic viscosity at local contact temperature	mm^2/s
$\nu_{\theta M}$	kinematic viscosity at bulk temperature	mm^2/s
ν_1	Poisson's ratio of pinion	–
ν_2	Poisson's ratio of wheel	–
ν_{100}	kinematic viscosity at 100 °C	mm^2/s
ν_{40}	kinematic viscosity at 40 °C	mm^2/s
ρ_{M1}	density of pinion	kg/m^3
ρ_{M2}	density of wheel	kg/m^3
$\rho_{n,C}$	normal radius of relative curvature at pitch diameter	mm
$\rho_{n,Y}$	normal radius of relative curvature at point Y	mm
$\rho_{t,Y}$	transverse radius of relative curvature at point Y	mm
$\rho_{t1,Y}$	transverse radius of curvature of pinion at point Y	mm
$\rho_{t2,Y}$	transverse radius of curvature of wheel at point Y	mm
$\rho_{\theta B,Y}$	density of lubricant at local contact temperature	kg/m^3
$\rho_{\theta M}$	density of lubricant at bulk temperature	kg/m^3
ρ_{15}	density of lubricant at 15 °C	kg/m^3
Subscripts to symbols		
parameter for any contact point Y in the contact area for Method A and on the path of contact for Method B; (all parameters subscript Y have to be calculated with local values)		

4 Definition of micropitting

Micropitting is a phenomenon that occurs in Hertzian type of rolling and sliding contact that operates in elastohydrodynamic or boundary lubrication regimes. Micropitting is influenced by operating conditions such as load, speed, sliding, temperature, surface topography, specific lubricant film thickness and chemical composition of the lubricant. Micropitting is more commonly observed on materials with a high surface hardness.

Micropitting is the generation of numerous surface cracks. The cracks grow at a shallow angle to the surface forming micropits. The micropits are small relative to the size of the contact zone, typically of the order 10 - 20 µm deep. The micropits can coalesce to produce a continuous fractured surface which appears as a dull, matte surface during unmagnified visual inspection.

Micropitting is the preferred name for this phenomenon, but it has also been referred to as grey staining, grey flecking, frosting and peeling. Illustrations of micropitting can be found in ISO 10825 [8].

Micropitting may arrest. However, if micropitting continues to progress, it may result in reduced gear tooth accuracy, increased dynamic loads and noise. If it does not arrest and continues to propagate it can develop into macropitting and other modes of gear failure.

5 Basic formulae

5.1 General

The calculation of micropitting load capacity is based on the local specific lubricant film thickness $\lambda_{GF,Y}$ in the contact area and the permissible specific lubricant film thickness λ_{GFP} [11]. It is assumed that micropitting can occur, when the minimum specific lubricant film thickness $\lambda_{GF,min}$ is lower than a corresponding critical value λ_{GFP}. Both values $\lambda_{GF,min}$ and λ_{GFP} shall be calculated separately for pinion and wheel in the contact area. It has to be recognized that the determination of the minimum specific lubricant film thickness and the permissible specific lubricant film thickness have to be based on the operating parameters.

The micropitting load capacity can be determined by comparing the minimum specific lubricant film thickness with the corresponding limiting value derived from gears in service or from specific gear testing. This comparison will be expressed by the safety factor S_λ which shall be equal or higher than a minimum safety factor against micropitting $S_{\lambda,min}$.

Micropitting mainly occurs in areas of negative specific sliding. Negative specific sliding is to be found along the path of contact (see Figure 1) between point A and C on the driving gear and between point C and E on the driven gear. Considering the influences of lubricant, surface roughness, geometry of the gears and operating conditions the specific lubricant film thickness $\lambda_{GF,Y}$ can be calculated for every point in the field of contact.

5.2 Safety factor against micropitting S_λ

To account for the micropitting load capacity the safety factor S_λ according to equation (1) is defined.

$$S_\lambda = \frac{\lambda_{GF,min}}{\lambda_{GFP}} \geq S_{\lambda,min} \tag{1}$$

where

$\lambda_{GF,min} = \min(\lambda_{GF,Y})$	is the minimum specific lubricant film thickness in the contact area;
$\lambda_{GF,Y}$	is the local specific lubricant film thickness (see 5.3);
λ_{GFP}	is the permissible specific lubricant film thickness (see 5.4);
$S_{\lambda,min}$	is the minimum required safety factor (see 5.5).

The minimum specific lubricant film thickness is determined from all calculated local values of the specific lubricant film thickness $\lambda_{GF,Y}$ obtained by equation (2).

5.3 Local specific lubricant film thickness $\lambda_{GF,Y}$

For the determination of the safety factor S_λ the local lubricant film thickness h_Y according to Dowson/Higginson [5] in the field of contact has to be known and compared with the effective surface roughness.

$$\lambda_{GF,Y} = \frac{h_Y}{Ra} \tag{2}$$

where

$$Ra = 0{,}5 \cdot (Ra_1 + Ra_2) \tag{3}$$

$$h_Y = 1600 \cdot \rho_{n,Y} \cdot G_M{}^{0,6} \cdot U_Y{}^{0,7} \cdot W_Y{}^{-0,13} \cdot S_{GF,Y}{}^{0,22} \tag{4}$$

Ra is the effective arithmetic mean roughness value;

Ra_1 is the arithmetic mean roughness value of pinion (compare ISO 6336-2);

Ra_2 is the arithmetic mean roughness value of wheel (compare ISO 6336-2);

h_Y is the local lubricant film thickness;

$\rho_{n,Y}$ is the normal radius of relative curvature at point Y (see clause 10);

G_M is the material parameter (see clause 6);

U_Y is the local velocity parameter (see clause 7);

W_Y is the local load parameter (see clause 8);

$S_{GF,Y}$ is the local sliding parameter (see clause 9).

Equation (4) should be calculated in the case of Method B at the seven local points (Y) defined in 5.3 b) using the values for $\rho_{n,Y}$, U_Y, W_Y and $S_{GF,Y}$ that exists at each point Y. The minimum of the seven h_Y ($\lambda_{GF,Y}$) values shall be used in equation (1).

An example calculation is presented in Annex B.

a) Method A

The local specific lubricant film thickness can be determined in the complete contact area by any appropriate gear computing program. In order to determine the local specific lubricant film thickness, the load distribution, the influence of normal and sliding velocity with changes of meshing phase and the actual service conditions shall be taken into consideration.

b) Method B

This method involves the assumption that the determinant local specific lubricant film thickness occurs on the tooth flank in the area of negative sliding. For simplification the calculation of the local specific lubricant film thickness is limited to certain points on the path of contact. For this purpose the lower point A and upper point E on the path of contact, the lower point B and upper point D of single pair tooth contact, the midway point AB between A and B, the midway point DE between D and E as well as the pitch point C are surveyed.

5.4 Permissible specific lubricant film thickness λ_{GFP}

For the determination of the permissible specific lubricant film thickness λ_{GFP} different procedures are applicable.

a) Method A

For Method A experimental investigations or service experience relating to micropitting on real gears are used.

Running real gears under conditions where micropitting just occurs the minimum specific lubricant film thickness can be calculated according to 5.3 a). This value is equivalent to the limiting specific lubricant film thickness which is used to calculate the micropitting load capacity.

Such experimental investigations may be performed on gears having the same design as the actual gear pair. In this case the gear manufacturing, gear accuracy, operating conditions, lubricant and operating temperature have to be appropriate for the actual gear box.

The cost required for this method is in general only justifiable for the development of new products as well as for gear boxes where failure would have serious consequences.

Otherwise the permissible specific lubricant film thickness λ_{GFP} may be derived from consideration of dimensions, service conditions and performance of carefully monitored reference gears operated with the respective lubricant. The more closely the dimensions and service conditions of the actual gears resemble those of the reference gears, the more effective will be the application of such values for the purpose of design ratings or calculation checks.

b) Method B

The method adapted is validated by carrying out careful comparative studies of well-documented histories of a number of test gears applicable to the type, quality and manufacture of gearing under consideration. The permissible specific lubricant film thickness λ_{GFP} is calculated from the critical specific lubricant film thickness λ_{GFT} which is the result of any standardised test method applicable to evaluate the micropitting load capacity of lubricants or materials by means of defined test gears operated under specified test conditions. λ_{GFT} is a function of the temperature, oil viscosity, base oil and additive chemistry and can be calculated according to equation (2) in the contact point of the defined test gears where the minimum specific lubricant film thickness is to be found and for the test conditions where the failure limit concerning micropitting in the standardised test procedure has been reached.

The test gears as well as the test conditions (for example the test temperature) have to be appropriate for the real gears in consideration.

Any standardised test can be used to determine the data. Where a specific test procedure is not available or required, a number of internationally available standardised test methods for the evaluation of micropitting performance of gears, lubricants and materials are currently available. Some widely used test procedures are the FVA-FZG-micropitting test [7], Flender micropitting test [12], BGA-DU micropitting test [2] and the micropitting test according to [3]. Annex A provides some generalised test data (for reference only) that have been produced using the test procedure according to FVA-Information Sheet 54/7 [7] where a value for λ_{GFP} can be calculated for a generalised reference allowable using equation A.1.

5.5 Recommendation for the minimum safety factor against micropitting $S_{\lambda,min}$

For a given application, adequate micropitting load capacity is demonstrated by the computed value of S_{λ} and being greater than or equal to the value $S_{\lambda,min}$, respectively.

Certain minimum values for the safety factor shall be determined. Recommendations concerning these minimum values are made in the following, but values are not proposed.

An appropriate probability of failure and the safety factor shall be carefully chosen to meet the required reliability at a justifiable cost. If the performance of the gears can be accurately appraised through testing of the actual unit under actual load conditions, a lower safety factor and more economical manufacturing procedures may be permissible:

$$\text{Safety factor} = \frac{\text{Calculated minimum specific film thickness}}{\text{Permissible specific film thickness}}$$

In addition to the general requirements mentioned and the special requirements for specific lubricant film thickness, the safety factor shall be chosen after careful consideration of the following influences.

- reliability of load values used for calculation: If loads or the response of the system to vibration, are estimated rather than measured, a larger safety factor should be used.
- variations in gear geometry and surface texture due to manufacturing tolerances,
- variations in alignment,
- variations in material due to process variations in chemistry, cleanliness and microstructure (material quality and heat treatment),
- variations in lubrication and its maintenance over the service life of the gears.

Depending on the reliability of the assumptions on which the calculations are based (for example load assumptions) and according to the reliability requirements (consequences of occurrence), a corresponding safety factor is to be chosen.

Where gears are produced according to a specification or a request for proposal (quotation), in which the gear supplier is to provide gears or assembled gear drives having specified calculated capacities (ratings) in accordance with this technical report, the value of the safety factor for micropitting is to be agreed upon between the parties.

6 Material parameter G_M

The material parameter G_M accounts for the influence of the reduced modulus of elasticity E_r and the pressure-viscosity coefficient of the lubricant at bulk temperature $\alpha_{\theta M}$.

$$G_M = 10^6 \cdot \alpha_{\theta M} \cdot E_r \tag{5}$$

where

E_r is the reduced modulus of elasticity (see 6.1);

$\alpha_{\theta M}$ is the pressure-viscosity coefficient at bulk temperature (see 6.2).

6.1 Reduced modulus of elasticity E_r

For mating gears of different material and modulus of elasticity E_1 and E_2, the reduced modulus of elasticity E_r can be determined by equation (6). For mating gears of the same material $E = E_1 = E_2$ equation (7) may be used.

$$E_r = 2 \cdot \left(\frac{1-\nu_1^{\,2}}{E_1} + \frac{1-\nu_2^{\,2}}{E_2} \right)^{-1} \tag{6}$$

$$E_\mathrm{r} = \frac{E}{1-\nu^2} \qquad \text{for } E_1 = E_2 = E \text{ and } \nu_1 = \nu_2 = \nu \tag{7}$$

where

E_1 is the modulus of elasticity of pinion (for steel: E = 206000 N/mm^2);

E_2 is the modulus of elasticity of wheel (for steel: E = 206000 N/mm^2);

ν_1 is the Poisson's ratio of pinion (for steel: ν = 0,3);

ν_2 is the Poisson's ratio of wheel (for steel: ν = 0,3).

6.2 Pressure-viscosity coefficient at bulk temperature $\alpha_{\theta M}$

If the data for the pressure-viscosity coefficient at bulk temperature $\alpha_{\theta M}$ for the specific lubricant is not available, it can be approximated by equation (8) (see [9]).

$$\alpha_{\theta M} = \alpha_{38} \cdot \left[1 + 516 \cdot \left(\frac{1}{\theta_M + 273} - \frac{1}{311}\right)\right] \tag{8}$$

where

α_{38} is the pressure-viscosity coefficient of the lubricant at 38 °C;

θ_M is the bulk temperature (see clause 14).

If no values for α_{38} are available then the following approximated values [1] can be used.

$\alpha_{38} = 2{,}657 \cdot 10^{-8} \cdot \eta_{38}^{\ 0{,}1348}$ for mineral oil (9)

$\alpha_{38} = 1{,}466 \cdot 10^{-8} \cdot \eta_{38}^{\ 0{,}0507}$ for PAO - based synthetic non-VI improved oil (10)

$\alpha_{38} = 1{,}392 \cdot 10^{-8} \cdot \eta_{38}^{\ 0{,}1572}$ for PAG - based synthetic oil (11)

where

η_{38} is the dynamic viscosity of the lubricant at 38 °C.

7 Velocity parameter U_Y

The velocity parameter U_Y describes the proportional increase of the specific lubricant film thickness with increasing dynamic viscosity $\eta_{\theta M}$ of the lubricant at bulk temperature and sum of the tangential velocities $v_{\Sigma,Y}$.

$$U_Y = \eta_{\theta M} \cdot \frac{v_{\Sigma,Y}}{2000 \cdot E_\mathrm{r} \cdot \rho_{n,Y}} \tag{12}$$

where

$\eta_{\theta M}$ is the dynamic viscosity of the lubricant at bulk temperature (see 7.2);

$v_{\Sigma,Y}$ is the sum of the tangential velocities (see 7.1);

E_r is the reduced modulus of elasticity (see 6.1);

$\rho_{v,Y}$ is the local normal radius of relative curvature (see clause 10).