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Ophthalmic optics and instruments — Reporting aberrations of the human eye

Optique et instruments ophtalmiques — Méthodes de présentation des aberrations de l'œil humain

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Foreword

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Ophthalmic optics and instruments — Reporting aberrations of the human eye

1 Scope

This International Standard specifies standardized methods for reporting aberrations of the human eye.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 8429, *Optics and optical instruments — Ophthalmology — Graduated dial scale*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply. Symbols used are summarized in Table 1.

3.1
line of sight
line from the point of interest in object space to the centre of the entrance pupil of the eye and continuing from the centre of the exit pupil to the retinal point of fixation (generally the foveola)

3.2
Zernike polynomial function
one of a complete set of functions defined and orthogonal over the unit circle, the product of three terms, a normalization term, a radial term and a meridional term, parameterized by a dimensionless radial parameter, ρ, and a dimensionless meridional parameter, θ, designated by a non-negative radial integer index, n, and a signed meridional index, m, and given by the equation

$$Z_n^m = N_n^m R_n^{|m|}(\rho) M(m\theta) \tag{1}$$

where

N_n^m	is the normalization term;
$R_n^{\|m\|}$	is the radial term;
$M(m\theta)$	is the meridional term;
the parameter ρ	is a real number continuous over its range of 0 to 1,0;
the parameter θ	is a real number continuous over its range of 0 to 2π.

NOTE For a given value of radial index n, the meridional index m may only take the values $-n$, $-n+2$, …, $n-2$ and n.

3.2.1
radial term
Zernike polynomial function term with indices n and m given by the equation

$$R_n^{|m|}(\rho) = \sum_{s=0}^{0,5(n-|m|)} \frac{(-1)^s (n-s)!}{s!\left[0,5(n+|m|)-s\right]!\left[0,5(n-|m|)-s\right]!} \rho^{n-2s} \quad (2)$$

where s is an integer summation index incremented by one unit

3.2.2
radial parameter
ρ
dimensionless number taking values between 0 and 1, its value at any radial distance, r, from the aperture centre being given by the expression

$$\rho = \frac{r}{a} \quad (3)$$

where a is the value of the aperture radius

3.2.3
meridional term
Zernike polynomial function term with index m given by the equations

$$M(m\theta) = \cos(m\theta) \quad \text{if } m \geqslant 0 \quad (4)$$

$$M(m\theta) = \sin(|m|\theta) \quad \text{if } m < 0 \quad (5)$$

NOTE The meridional term is also known as the *azimuthal* term.

3.2.4
meridional parameter
θ
angular value taking values between 0 and 2π (0° and 360°), expressed in the coordinate system defined in Clause 4

NOTE This is also called the *azimuthal* angle.

3.2.5
normalization term
Zernike polynomial function term with indices n and m, equal to 1,0 for “un-normalized” functions (3.2.7) and for “normalized” functions (3.2.6) by the equation

$$N_n^m = \sqrt{(2-\delta_{0,m})(n+1)} \quad (6)$$

where $\delta_{0,m} = 1$ if $m = 0$, $\delta_{0,m} = 0$ if $m \neq 0$.

3.2.6
normalized Zernike polynomial function
Zernike polynomial function whose normalization term takes the form given in 3.2.5 for “normalized” functions defined as orthogonal in the sense that it satisfies the following equation

$$\int_0^1 \rho d\rho \int_0^{2\pi} Z_n^m Z_{n'}^{m'} d\theta = \pi \delta_{n,n'} \delta_{m,m'} \quad (7)$$

where

$\delta_{n,n'} = 1$ if $n = n'$, $\delta_{n,n'} = 0$ if $n \neq n'$;

$\delta_{m,m'} = 1$ if $m = m'$, $\delta_{m,m'} = 0$ if $m \neq m'$.

3.2.7
un-normalized Zernike polynomial function
Zernike polynomial function whose normalization term is equal to 1,0 and defined as orthogonal in the sense that it satisfies the equation

$$\left(2-\delta_{0,m}\right)\left(n+1\right)\int_0^1 \rho d\rho \int_0^{2\pi} Z_n^m Z_{n'}^{m'} d\theta = \pi \delta_{n,n'} \delta_{m,m'} \tag{8}$$

where

$\delta_{n,n'} = 1$ if $n = n'$, $\delta_{n,n'} = 0$ if $n \neq n'$;

$\delta_{m,m'} = 1$ if $m = m'$, $\delta_{m,m'} = 0$ if $m \neq m'$;

$\delta_{0,m} = 1$ if $m = 0$, $\delta_{0,m} = 0$ if $m \neq 0$.

3.2.8
order
value of the radial index n of a Zernike polynomial function

3.3
Zernike coefficient
member of a set of real numbers, c_n^m, which is multiplied by its associated Zernike function to yield a term that is subsequently used in a sum of terms to give a value equal to the best estimate of the surface, $S(\rho,\theta)$, that has been fitted with Zernike terms, such a sum being represented by

$$S\left(\rho,\theta\right) = \sum_{\text{all } n \text{ and } m} c_n^m Z_n^m \tag{9}$$

NOTE 1 Each set of Zernike coefficients has associated with it the aperture diameter that was used to generate the set from surface elevation data. The set is incomplete without this aperture information.

NOTE 2 Annex A gives information on a method to find Zernike coefficients from wavefront slope (gradient) data.

3.3.1
normalized Zernike coefficient
Zernike coefficient generated using normalized Zernike functions and so designed to be used with them to reconstruct a surface

NOTE Normalized Zernike coefficients have dimensional units of length.

3.3.2
un-normalized Zernike coefficient
Zernike coefficient generated using un-normalized Zernike functions and so designed to be used with them to reconstruct a surface

NOTE Un-normalized Zernike coefficients have dimensional units of length.

3.4
wavefront error (of an eye)
$W(x,y)$ **or** $W(r,\theta)$
optical path-length (i.e. physical distance times refractive index) between a plane wavefront in the eye's entrance pupil and the wavefront of light exiting the eye from a point source on the retina, and specified as a function (wavefront error function) of the (x,y) (or r,θ) coordinates of the entrance pupil

NOTE 1 Wavefront error is measured in an axial direction (i.e. parallel to the z-axis defined in Clause 4) from the pupil plane towards the wavefront.

NOTE 2 By convention, the wavefront error is set to zero at the pupil centre by subtracting the central value from values at all other pupil locations.

NOTE 3 Wavefront error has physical units of metres (typically reported in micrometres) and pertains to a specified wavelength.

3.5
optical path-length difference
OPD
negative of the **wavefront error** (3.4) at each point in a wavefront representing the correction of the optical path-length needed to correct the wavefront error

3.6
root mean square wavefront error
RMS wavefront error
⟨of an eye⟩ quantity computed as the square root of the variance of the **wavefront error** (3.4) function and defined as

$$RMS_{\mathrm{WFE}} = \sqrt{\frac{\iint\limits_{\text{pupil}} \left[W(x,y)\right]^2 dxdy}{A}} \tag{10}$$

where A is the area of the pupil

or, if the wavefront error function is expressed in terms of normalized Zernike coefficients, a quantity equal to the square root of the sum of the squares of the coefficients with radial indices 2 or greater

$$RMS_{\mathrm{WFE}} = \sqrt{\sum_{n>1,\text{all } m} \left(c_n^m\right)^2} \tag{11}$$

NOTE 1 Piston and average tilt should be excluded from this calculation because they correspond to lateral displacements of the image rather than image degradation *per se*.

NOTE 2 The RMS error can also be found using the discrete set of wavefront error values that were used to generate the Zernike coefficients and standard statistical methods. When this is done it might be found that this RMS value does not exactly match the value found using the formula given above. This is more likely to happen in cases where the locations in the pupil used to sample the wavefront error form a non-uniformly spaced grid. Then the data set does not lead to the formation of discrete, orthogonal Zernike functions.

3.7
higher-order aberrations
those aberrations experienced by the eye in addition to sphero-cylindrical refractive errors and prismatic error and thus, if the wavefront error is expressed in terms of Zernike polynomial function coefficients, those of order 3 and higher

3.8
wavefront gradient
$\partial W(x,y)$
vector giving the values of the gradient of the wavefront, $\partial W(x,y)/\partial x$ and $\partial W(x,y)/\partial y$, at locations x and y and, when expressed in terms of Zernike polynomial coefficients, given by:

$$\frac{\partial W(x,y)}{\partial x} = \sum_{\text{all } n \text{ and } m} c_n^m \frac{\partial Z_n^m(x,y)}{\partial x} \quad \text{and} \quad \frac{\partial W(x,y)}{\partial y} = \sum_{\text{all } n \text{ and } m} c_n^m \frac{\partial Z_n^m(x,y)}{\partial y} \tag{12}$$

NOTE Measured gradient values are referred to by $\beta_x(x,y)$ and $\beta_y(x,y)$ at locations x,y.

Table 1 — Symbols

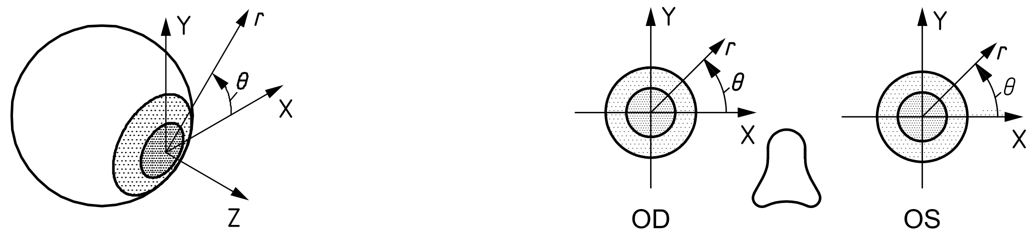
Symbol	Name	Definition given in
$A(m\theta,\alpha)$	meridional term for magnitude/axis Zernike functions	5.1.9
c_n^m	Zernike coefficient	3.3
c_{nm}	Zernike coefficient – magnitude	5.1.9
m	meridional index for Zernike functions	3.2
$M_n^m(m\theta)$	meridional term for Zernike functions	3.2.3
n	radial index for Zernike functions	3.2
N_n^m	normalization term for Zernike functions	3.2.5
$R_n^{\lvert m \rvert}(\rho)$	radial term for Zernike functions	3.2.1
Z_n^m	Zernike function [alternate notation: $Z(n,m)$]	3.2
Z_{nm}	Zernike function – magnitude/axis form	5.1.9
α	axis parameter for magnitude/axis form Zernike functions	5.1.9
ρ	radial parameter for Zernike functions	3.2.2
θ	meridional parameter for Zernike functions	3.2.4
$W(x,y)$	wavefront error	3.4
$\beta x,y$	measured gradient at a location x,y	3.8
$\partial Wx,y$	wavefront gradient at a location x,y	3.8
β_{fit}	gradient fit error	5.3

4 Coordinate system

The coordinate system used to represent wavefront surfaces shall be the standard ophthalmic coordinate system in accordance with ISO 8429 in which the x-axis is local horizontal with its positive sense to the right as the examiner looks at the eye under measurement, the y-axis is local vertical with its positive sense superior with respect to the eye under measurement, the z-axis is the line of sight of the eye under measurement with its positive sense in the direction from the eye toward the examiner. The horizontal and vertical origin of the coordinate system is the centre of the visible pupil of the eye. The coordinate system origin lies in the plane of the exit pupil of the eye (for light originating on the retina and passing out through the pupil). This coordinate system is illustrated in Figure 1.

The sign convention used for wavefront error values reported at any location on a wavefront shall be that used for this coordinate system.

When Zernike coefficients are used to represent a wavefront or to report wavefront error, the sign convention used to describe the individual Zernike functions shall be that used for this coordinate system.



a) Coordinate system **b) Clinician's view of patient**

Key

OD right eye

OS left eye

Figure 1 — Ophthalmic coordinate system (ISO 8429)

5 Representation of wavefront data

5.1 Representation of wavefront data with the use of Zernike polynomial function coefficients

5.1.1 Symbols for Zernike polynomial functions

Zernike polynomial functions shall be designated by the upper case letter Z followed by a superscript and a subscript. The superscript shall be a signed integer representing the meridional index of the function, m. The subscript shall be a non-negative integer representing the radial index of the function, n. Therefore a Zernike polynomial function shall be designated by the form Z_n^m .

If, for reasons of font availability, it is not possible to write superscript and subscripts, the Zernike polynomial functions may be represented as a upper case letter Z followed by parentheses in which the radial index, n, appears first, followed, after a comma, by the meridional index, m, thus $Z(n,m)$.

5.1.2 Radial index

The radial index shall be designated by the lower case letter n.

5.1.3 Meridional index

The meridional index shall be designated by the lower case letter m.

5.1.4 Radial parameter

The radial parameter shall be designated by the Greek letter ρ.

5.1.5 Meridional parameter

The meridional parameter shall be designated by the Greek letter θ.

5.1.6 Coefficients

When a surface is represented by Zernike coefficients, these coefficients shall be designated by the lower case letter c followed by a superscript and a subscript. The superscript shall be a signed integer representing the meridional index of the function, m. The subscript shall be a non-negative integer representing the radial index of the function, n. Therefore, a Zernike coefficient shall be designated by the form c_n^m.

5.1.7 Common names of Zernike polynomial functions

Zernike polynomial functions are often referred to by their common names. These names are given in Table 2 in so far as the functions have been given a common name.

Table 2 — Common names of Zernike polynomial functions

Zernike function	Common name
Z_0^0	piston
Z_1^{-1}	vertical tilt
Z_1^1	horizontal tilt
Z_2^{-2}	oblique astigmatism
Z_2^0	myopic defocus (positive coefficient value) hyperopic defocus (negative coefficient value)
Z_2^2	against the rule astigmatism (positive coefficient value) with the rule astigmatism (negative coefficient value)
Z_3^{-3}	oblique trefoil
Z_3^{-1}	vertical coma – superior steepening (positive coefficient value) vertical coma – inferior steepening (negative coefficient value)
Z_3^1	horizontal coma
Z_3^3	horizontal trefoil
Z_4^{-4}	oblique quatrefoil
Z_4^{-2}	oblique secondary astigmatism
Z_4^0	spherical aberration positive coefficient value – pupil periphery more myopic than centre negative coefficient value – pupil periphery more hyperopic than centre
Z_4^2	with/against the rule secondary astigmatism
Z_4^4	quatrefoil
Z_5^{-1}	secondary vertical coma
Z_5^1	secondary horizontal coma

5.1.8 Comparison of data expressed as Zernike coefficients generated using different aperture sizes

The Zernike coefficient values describing a given wavefront error depend on the aperture size used when they are generated from measurement data. Due to this dependence on pupil diameter, different coefficient values will be found to describe the wavefront error of a given eye if the pupil size changes from one measurement to the next. Therefore, to adequately compare the wavefront error of the same eye at different times or to compare the wavefront errors of two eyes using Zernike coefficients, the compared coefficients shall have been generated using the same pupil diameter even though measurements were taken with different pupil diameters. Zernike coefficients taken at one pupil diameter may be converted into values for a second, smaller pupil diameter using either the method given in Annex B or a similar method.

Wavefront error comparisons using Zernike coefficients found in accordance with this International Standard shall be made between sets of Zernike coefficients that have be converted to a common pupil diameter.

5.1.9 Representation of wavefront error data expressed as Zernike coefficients presented in magnitude/axis form

Zernike terms of the same radial order, n, and having meridional indices, m, with the same magnitude but with opposite signs may be considered to represent the two components of a vector in an angular space with a multiplicity equal to the magnitude of m. It is therefore possible to define Zernike functions that combine the functions defined in 3.2 having the same radial order, n, and meridional indices with the same magnitude into a new set of functions defined by

$$Z_{nm}(\rho,\theta,\alpha) = N_n^m R_n^{|m|}(\rho) A(m\theta,\alpha) \tag{13}$$

where

$R_n^{|m|}(\rho)$ is defined by 3.2.1;

N_n^m is defined by 3.2.5;

$A(m\theta,\alpha) = \cos\left[m(\theta-\alpha)\right]$

and where α is an angular parameter giving the orientation of the vector in space.

A surface, $S(\rho,\theta)$, such as a wavefront error, is expressed using these Zernike functions as

$$S(\rho,\theta) = \sum_{\text{all } n \text{ and } m} c_{nm} Z_{nm}(\rho,\theta,\alpha_{nm})$$

where the coefficients c_{nm} and the angular parameters α_{nm} are related to the coefficients defined in 3.3 by the equations

$$c_{nm} = \sqrt{\left(c_n^{-m}\right)^2 + \left(c_n^m\right)^2} \tag{14}$$

$$\alpha_{nm} = \frac{a\tan\left(\dfrac{c_n^{-m}}{c_n^m}\right)}{|m|} \tag{15}$$

