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Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy

Part 3: Statistical methods

Technologie du combustible nucléaire — Étalonnage et détermination du volume de cuve pour la comptabilité des matières nucléaires

Partie 3: Méthodes statistiques

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 18213-3 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, Subcommittee SC 5, *Nuclear fuel technology*.

ISO 18213 consists of the following parts, under the general title *Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy*:

— *Part 1: Procedural overview*

— *Part 2: Data standardization for tank calibration*

— *Part 3: Statistical methods*

— *Part 4: Accurate determination of liquid height in accountancy tanks equipped with dip tubes, slow bubbling rate*

— *Part 5: Accurate determination of liquid height in accountancy tanks equipped with dip tubes, fast bubbling rate*

— *Part 6: Accurate in-tank determination of liquid density in accountancy tanks equipped with dip tubes*

Introduction

This part of ISO 18213 describes statistical procedures suitable for the treatment of tank calibration and volume measurement data for nuclear materials accountancy tanks. It is one part of a six-part International Standard that deals with the acquisition, analysis, standardization and use of calibration data to determine liquid volumes in process tanks for accountability purposes, and is intended for use in conjunction with other parts of ISO 18213. Other parts of ISO 18213 and their topics are ISO 18213-1 (procedural overview), ISO 18213-2 (data standardization), ISO 18213-4 (slow bubbling rate), ISO 18213-5 (fast bubbling rate), and ISO 18213-6 (in-tank determination of liquid density).

To someone without formal statistical training, the methods of ISO 18213-3 might appear to be unnecessarily complex. However, within the context of the data standardization model presented in other parts of ISO 18213, the statistical methods presented herein have been kept as simple as possible. Data collection, data standardization and statistical analysis go hand-in-hand. In order for one to meet the target uncertainty limits established for accountability purposes, it is necessary that the data standardization model be consistent with the measurement (instrument) capability and that the statistical error model likewise be compatible with the data standardization model. It makes no sense to use a highly refined data standardization model with crude measurement instruments. Conversely, the advantage of highly refined and precise measurement instruments is lost if a crude data standardization model is used in the subsequent analysis. Using a more refined measurement instrument, for example, does not improve results if the data standardization model fails, for example, to take proper account of the effects of temperature variation.

Similarly, it makes no sense to use a sophisticated statistical model with either crude measurements or a crude data standardization model. Conversely, an overly simple statistical model, or one that is inconsistent with the underlying data standardization model, yields poor results even when used with high-quality instrumentation and a refined data standardization model. Because of the important role volume determinations play in its overall accountability program, a facility typically devotes significant resources to instrumentation for tank calibration and volume determination. However, refined state-of-the-art measurement capability by itself is not sufficient to meet target uncertainty limits. Resources are also required to develop a data standardization model and statistical methods with quality comparable to that of the plant's measurement capability. The resources required for data analysis are typically much fewer than those allocated for instrumentation, but they are equally as important. In any event, adequate resources are required to engage someone with the necessary training to guide the development and application of computational and statistical methods that are comparable in sophistication to the measurements to which they are applied.

The statistical methods presented in this part of ISO 18213 are closely tied to the comprehensive state-of-the-art data standardization methodology presented in other parts of ISO 18213 and are therefore designed to be applicable over a wide range of measurement systems and operating conditions. As noted in the introduction to ISO 18213-1, it is not always necessary, or even possible, for the operator to develop the full model for all tanks in a given facility. Under these circumstances, the methods presented herein provide the framework for developing a "reduced" calibration model, including suitable estimates of uncertainty, that is consistent with the "reduced" standardization model developed for a particular tank.

Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy

Part 3: Statistical methods

1 Scope

This part of ISO 18213 presents statistical procedures that can be applied to tank calibration and volume measurement data for nuclear materials accountancy tanks. In particular, this part of ISO 18213 presents

a) several diagnostic plots that can be used to evaluate and compare tank calibration data;

b) a procedure for estimating the uncertainties of tank calibration measurements (i.e., determinations of height and volume);

c) a model for estimating either a tank's calibration equation or its inverse (the measurement equation), together with related uncertainties, from a set of standardized tank calibration data (i.e., from a series of standardized height-volume determinations);

d) a method for computing uncertainty estimates for determinations of liquid volume.

It is intended that the methods in this part of ISO 18213 be used within the context of the other parts of ISO 18213. Specifically, the methods presented in this part of ISO 18213 are tailored to the general methodology described in ISO 18213-1 and to appropriate related algorithms in ISO 18213-2, ISO 18213-4, ISO 18213-5 or ISO 18213-6. Although the methodology in this part of ISO 18213 is intended for application specifically within the context of the other parts of ISO 18213, the methods are more widely applicable. In particular, the statistical model presented in Clause 6 for estimating the tank's measurement equation from a set of standardized calibration data can be applied, regardless of whether or not these data are acquired in accordance with the methods of ISO 18213. A similar statement holds for (propagation) methods of variance estimation: it is intended that the results in this part of ISO 18213 be applied to the specific models for which they were derived, but the methods themselves are more widely applicable.

This part of ISO 18213 provides a facility with the option to develop equivalent plant- or tank-specific methods of statistical analysis as an alternative. However, if a facility adopts ISO 18213 and chooses not to develop equivalent alternative methods of statistical analysis, it is necessary to use the methods of this part of ISO 18213.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 18213-1:2007, *Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy — Part 1: Procedural overview*

ISO 18213-4:2008, *Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy — Part 4: Accurate determination of liquid height in accountancy tanks equipped with dip tubes, slow bubbling rate*

ISO 18213-5:2008, *Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy — Part 5: Accurate determination of liquid height in accountancy tanks equipped with dip tubes, fast bubbling rate*

ISO 18213-6:2008, *Nuclear fuel technology — Tank calibration and volume determination for nuclear materials accountancy — Part 6: Accurate in-tank determination of liquid density in accountancy tanks equipped with dip tubes*

3 Symbols

The symbols used in this part of ISO 18213 are defined below. The symbols are listed in the first column of the table, approximately in order of appearance. Some symbols are introduced in groups, such as in connection with a particular equation. The ordering of symbols within such a group may differ from their appearance in the text if doing so makes the information easier to use. The location at which each symbol first appears is given in the corresponding row of the second column. The definition or usage of each symbol is presented in the third column.

Symbol	First reference	Definition/Usage
Y	5.2.1	response variable (either height or volume, height by convention)
X	5.2.1	control variable (either volume or height, volume by convention)
i	5.2.2	subscript that denotes either calibration increment number or observation number
Y_i	5.2.2	standardized elevation of a point in the tank above some pre-established reference point, typically associated with the standardized volume determined from the liquid added during the first i increments of a calibration run
X_i	5.2.2	standardized volume of the tank determined from the total volume of liquid added during the first i calibration increments, i.e., the standardized volume of the tank below Y_i
j	5.2.2	subscript
x_j	5.2.2	standardized volume of the jth increment of calibration liquid
(X_i, Y_i)	5.2.2	standardized volume-height data pair for the ith calibration increment
f or $f(...)$	5.2.2	generic function, the tank calibration equation, by convention
$\hat{Y} = \alpha + \beta X + \varepsilon$	5.2.3	equation that expresses height as a linear function of volume
α, β	5.2.3	equation parameters
ε	5.2.3	residual (height), the difference between the observed value of the response variable (Y) and the corresponding predicted value ($\alpha + \beta X$), $Y - \alpha - \beta X$
a, b	5.2.3	estimates of α, β
$\hat{Y}$	5.2.3	predicted response (height by convention) derived from some functional relationship between height and volume, $\hat{Y} = a + bX$
$Y_i - a - bX_i$	5.2.3	estimated residual, the estimated difference between observed and estimated values of the response variable for the ith calibration increment, $Y_i - \hat{Y}_i$

$Y = f(X)$	5.2.3	tank calibration equation
Δ	5.2.4	difference operator
ΔY	5.2.4	change (difference) in the response variable (height), typically between two calibration increments
ΔX	5.2.4	change (difference) in the control variable (volume), typically between two calibration increments
m_i	5.2.4	computed slope (change in height per unit change in volume) of calibration equation for the ith calibration increment, $\Delta Y_i/\Delta X_i$
f_1, f_2	5.2.5.1	generic functions, typically used to denote calibration equations or segments thereof
$\hat{f}$	5.2.5.2	estimate of the function f, the estimated calibration equation by convention
T_i	5.3.1	temperature, in either the tank or the prover, of the ith increment of calibration liquid
t_i	5.3.2	time associated with the ith calibration increment, e.g., time at start of increment
Δt_i	5.3.2	time required to complete the ith calibration increment, $t_i - t_{i-1}$
T_{m}	6.1, Eq. (6)	measured temperature of tank liquid
T_{r}	6.1, Eq. (6)	reference temperature established for calibration
H_{M}	6.1, Eq. (6)	height of a point in the tank at measured temperature T_{m}
H_{r}	6.1, Eq. (6)	height of a point in the tank at reference temperature T_{r}
ΔP	6.1, Eq. (6)	observed difference in pressure between the submerged bubbling probe and the reference probe
c_{M}	6.1, Eq. (6)	“corrections” that compensate for differences between the observed pressure at the manometer and the actual pressure at the tip of the submerged probe
ρ_{M}	6.1, Eq. (6)	average density of the liquid in the tank at the measured temperature T_{m}
$\rho_{\mathrm{a},s}$	6.1, Eq. (6)	average density of the air in the tank above the liquid surface at the prevailing pressure
g	6.1, Eq. (6)	local value of the acceleration due to gravity
α_{ex}	6.1, Eq. (6)	coefficient of linear thermal expansion for the dip tubes
ΔT_{m}	6.1, Eq. (6)	difference between the measured and reference temperatures, $T_{\mathrm{m}} - T_{\mathrm{r}}$
var(...)	6.1	variance operator, e.g., **var**(H_{r}) denotes the variance of H_{r} and **var**(ΔP) denotes the variance of ΔP, etc.
$\hat{f}^{-1}$	6.1	estimate of f^{-1}
f^{-1}	7.1	inverse of f, the measurement equation, by convention

$H = f(V)$	7.1	generic expression for the calibration equation
$V = f^{-1}(H)$	7.1	generic expression for the measurement equation
h or $h(...)$	7.2.1, Eq. (10)	generic function, f^{-1}, by convention
ε	7.2.1, Eq. (10)	residual, the difference between the observed value of the response variable (Y) and the corresponding predicted value $h(X)$, $Y - h(X)$
ε_i	7.2.1	residual difference between the observed value of the response variable (Y_i) and the corresponding predicted value $h(X_i)$ for the ith calibration increment, $Y_i - h(X_i)$
$\hat{h}$	7.2.1	estimate of h, typically $\hat{h} = \hat{f}^{-1}$
s	7.2.1, Eq. (11)	subscript
c_s	7.2.1, Eq. (11)	“cut point,” point in the (height) range of the measurement equation
S	7.2.1, Eq. (11)	number of segments (intervals) into which the range of the measurement equation is partitioned by cut points
c_0	7.2.1, Eq. (11)	left-hand endpoint of the first segment, usually 0
c_S	7.2.1, Eq. (11)	the right-hand endpoint of the largest segment, usually the largest value of the control variable, i.e., $c_S = X_{max}$
h_s	7.2.1, Eq. (12)	function defined over the interval (c_{s-1}, c_s), i.e., function defined for values between c_{s-1} and c_s, where s ranges from 1 to S
β_i	7.2.1	model parameters (β_0 denotes the intercept)
n	7.2.1, Eq. (16)	total number of observations, i.e., total number of height-volume data pairs (X_i, Y_i)
$p + 1$	7.2.1, Eq. (16)	number of parameters in the specified model
$\mathbf{Y}$	7.2.1, Eq. (16)	$n \times 1$ vector of (response variable) observations
$\mathbf{H}$	7.2.1, Eq. (16)	$n \times (p + 1)$ design matrix
$\boldsymbol{\beta}$	7.2.1, Eq. (16)	$(p + 1) \times 1$ vector of model parameters
$\boldsymbol{\varepsilon}$	7.2.1, Eq. (16)	$n \times 1$ vector of residual differences, i.e., $n \times 1$ vector of fitting errors
σ (σ^2)	7.2.1	standard deviation (variance) of the components of $\boldsymbol{\varepsilon}$
h_1, h_2, h_3	7.2.1	generic functions
$\boldsymbol{\theta}$	7.2.2	$(p + 1) \times 1$ vector of perturbations to the vector of model parameters, $\boldsymbol{\beta}$
$\boldsymbol{\theta}_j$	7.2.2	$(p + 1) \times 1$ vector of perturbations to the vector of model parameters, $\boldsymbol{\beta}$, attributable to the jth run
$\theta_{j,k}$	7.2.2	kth component of $\boldsymbol{\theta}_j$
$\boldsymbol{\beta}_j$	7.2.2	$(p + 1) \times 1$ vector of model parameters for the jth run, $\boldsymbol{\beta}_j = \boldsymbol{\beta} + \boldsymbol{\theta}_j$

$\beta_{j,k}$	7.2.2	kth component of $\boldsymbol{\beta}_j$
E(...)	7.2.2	expectation operator
$\boldsymbol{\theta}'$	7.2.2, Eq. (18)	transpose of the vector $\boldsymbol{\theta}$
$\boldsymbol{\Phi}^2$	7.2.2, Eq. (18)	variance-covariance matrix of (the components of) $\boldsymbol{\theta}$
$(X_{j,i}, Y_{j,i})$	7.2.2	ith standardized height-volume data pair from jth calibration run
n_j	7.2.2, Eq. (19)	total number of observations, i.e., total number of height-volume data pairs $(X_{j,i}, Y_{j,i})$, from jth calibration run
$\mathbf{Y}_j$	7.2.2, Eq. (19)	$n_j \times 1$ vector of (response variable) observations from the jth calibration run
$\mathbf{H}_j$	7.2.2, Eq. (19)	$n_j \times (p + 1)$ design matrix for the jth calibration run
$\boldsymbol{\varepsilon}_j$	7.2.2, Eq. (19)	$n_j \times 1$ vector of residual differences (fitting errors) for the jth calibration run
r	7.2.2, Eq. (19)	number of calibration runs
$\mathbf{H}'_j$	7.2.2, Eq. (21)	transpose of the matrix $\mathbf{H}_j$
$\sigma_j \left(\sigma_j^2\right)$	7.2.2, Eq. (21)	standard deviation (variance) of the components of $\boldsymbol{\varepsilon}_j$
I	7.2.2, Eq. (21)	$n_j \times n_j$ identity matrix
$Y_{j,i}$	7.2.2, Eq. (22)	ith component of the vector $\mathbf{Y}_j$
$\mathbf{h}'_{j,i}$	7.2.2, Eq. (22)	ith row of the design matrix $\mathbf{H}_j$
$\hat{\boldsymbol{\beta}}_j, \hat{\boldsymbol{\varepsilon}}_j^2, \hat{\sigma}_j^2$, etc.	7.3.2	respective estimators of $\boldsymbol{\beta}_j, \boldsymbol{\varepsilon}_j^2, \sigma_j^2$, etc.
$\varepsilon_{j,i}$	7.3.2	ith component of $\boldsymbol{\varepsilon}_j$
$\hat{h}_j$	7.3.2, Eq. (26)	estimated measurement equation from data of the jth calibration run
$\hat{\boldsymbol{\beta}}$	7.3.3.1	estimator of $\boldsymbol{\beta}$
X_0	7.3.3.2	specified (unobserved) value of the control variable (volume by convention)
Y_0	7.3.3.2	value of the response variable (height) at X_0
$\mathbf{h}'_0$	7.3.3.2	row of the design matrix, $\mathbf{H}$, that corresponds to X_0
$\hat{\bar{Y}}_0$	7.3.3.2	predicted (mean) value of the response variable Y_0 at X_0, $\mathbf{h}'_0\hat{\boldsymbol{\beta}}$
$\hat{\boldsymbol{\Phi}}^2, \hat{\boldsymbol{\theta}}_j$	7.3.3.3	respective estimators of $\boldsymbol{\Phi}^2$ and $\boldsymbol{\theta}_j$
$\widehat{\mathbf{var}}(\ldots)$	7.3.3.3	estimated variance, e.g., $\widehat{\mathbf{var}}\left(\hat{\boldsymbol{\beta}}\right)$ denotes the estimated variance of $\hat{\boldsymbol{\beta}}$, etc.
$\hat{\sigma}^2$	7.3.3.4	estimator of σ^2
$\hat{\boldsymbol{\varepsilon}}_j$	7.3.3.4	estimator of $\boldsymbol{\varepsilon}_j$

$\hat{\varepsilon}_{j,i}$	7.3.3.4	ith component of $\hat{\boldsymbol{\varepsilon}}_j$, estimate of the ith component of $\boldsymbol{\varepsilon}_j$
n	7.3.3.4	total number of observations from all runs, $\sum_j n_j$
ε_0	7.4, Eq. (37)	prediction error for a new (future) value of Y_0, $\varepsilon_0 = Y_0 - h(X_0) = Y_0 - \mathbf{h}'_0(\boldsymbol{\beta} + \boldsymbol{\theta})$
$\hat{Y}_0$	7.4	estimated (predicted) value of Y_0
α	7.5.2.1, Eq. (40)	specified confidence level (typically 0,025 or 0,05)
$\hat{\sigma}_0 \left(\hat{\sigma}_0^2\right)$	7.5.2.1, Eq. (40)	estimated standard deviation (variance) of $\hat{\hat{Y}}_0$, given by Equation (35)
ν	7.5.2.1, Eq. (40)	(approximate) degrees of freedom for the variance estimate $\hat{\sigma}_0^2$
$t_{\alpha/2}(\nu)$	7.5.2.1, Eq. (40)	100(1 – α/2) % point from the t-distribution with parameter (degrees of freedom) ν
S_1^2, S_2^2	7.5.2.1	quantities used to compute degrees of freedom, ν
ν_1, ν_2	7.5.2.1	component degrees of freedom
V, W	7.5.2.1	quantities used to compute degrees of freedom, ν
X	7.5.2.2	an arbitrary unspecified value of the control variable (volume)
$\hat{\hat{Y}}_X$	7.5.2.2	predicted (mean) value of the response variable (height) at X, $\mathbf{h}'_X\hat{\boldsymbol{\beta}}$
$\mathbf{h}'_X$	7.5.2.2	row of the design matrix **H** that corresponds to X
$\hat{\sigma}_X \left(\hat{\sigma}_X^2\right)$	7.5.2.2	estimated standard deviation (variance) of $\hat{\hat{Y}}_X$
ν	7.5.2.2	(approximate) degrees of freedom for the variance estimate, $\hat{\sigma}_X^2$
$F_\alpha(p+1, \nu)$	7.5.2.2	100(1 – α) % point from the F-distribution with parameters (p + 1) and ν
$\hat{\boldsymbol{\beta}}_{\text{new}}$	7.5.2.3	estimator of β computed from the data of a new calibration run
$\hat{\hat{Y}}_{X,\text{new}}$	7.5.2.3	predicted (mean) value of the response variable (height) at X obtained from the new calibration equation, $\mathbf{h}'_X\hat{\boldsymbol{\beta}}_{\text{new}}$
$\hat{\sigma}_{X,\text{new}} \left(\hat{\sigma}_{X,\text{new}}^2\right)$	7.5.2.3	estimated standard deviation (variance) of $\hat{\hat{Y}}_{X,\text{new}}$
ν_1, ν_2	7.5.2.3	(approximate) degrees of freedom for components of the variance estimate $\hat{\sigma}_X^2$
$\hat{\sigma}_X (\hat{\sigma}_X^2)$	7.5.3.2	estimated standard deviation (variance) of $\hat{Y}_X$
H_0	8.2.1	a standardized reference height (at reference temperature T_r), $H_0 = X_0$
V_0	8.2.1	standardized reference volume that corresponds to the height $H_0 = X_0$, $V_0 = Y_0$
$\mathbf{var}\left(V_{0,\text{pred}}\right)$	8.2.1, Eq. (55)	variance of the (mean) predicted volume obtained from the measurement equation at H_0
$\mathbf{var}\left(V_{0,\text{new}}\right)$	8.2.1, Eq. (55)	measurement component of variance of a (new) volume determination at H_0

$\mathbf{var}\left(V_{0,\text{trans}}\right)$	8.2.1, Eq. (55)	component of variance of a (new) predicted volume resulting from the “transfer” of uncertainty in H_0 through the measurement equation
$\partial\hat{h}(H_0)/\partial(H_0)$	8.2.1, Eq. (57)	derivative of the estimated measurement equation $\hat{h}$, taken with respect to H and evaluated at $H = H_0$
V_M	8.2.1	the volume at temperature T_m of the standardized reference volume, V_0
T_3	8.2.2.2	specified temperature
V_3, ρ_3	8.2.2.2	respective density and volume, at temperature T_3, of a liquid that has volume V_M and density ρ_M at temperature T_m
V_1, V_2	8.3	specified standardized volumes
ΔV	8.3	difference between two specified volumes, $V_1 - V_2$
$\mathbf{h}'_1$, $\mathbf{h}'_2$	8.3	row vectors that correspond respectively to the standardized height H_1 and H_2

4 Data required

This part of ISO 18213 applies generally to data acquired during the process of data collection and analysis for tank calibration and volume determination as outlined in ISO 18213-1. Specific procedures apply either to particular subsets of these data or at various stages in the process pertaining to their acquisition, analysis, interpretation and use. The data to which a particular statistical procedure applies and the stage in the process at which the procedure should be used are identified in the subclause(s) where that procedure is discussed.

5 Diagnostic plots

5.1 Overview

Diagnostic plots are among the most powerful tools available for analyzing and verifying volume measurement data. Plots are particularly useful for identifying anomalous observations and measurements in a set of tank calibration data. They are also quite useful for comparing the (standardized) data from two calibration runs and for comparing two estimates of the tank's calibration equation. Plots that may be used to evaluate a set of calibration (height and volume) data are presented in 5.2. Plots of auxiliary data (time, temperature) are presented in 5.3. Examples of all plots are given in Annex A.

5.2 Calibration data

5.2.1 General comments

The plots discussed in 5.2 may be based on either the tank's calibration equation or its measurement equation. For plots based on the calibration equation, the response variable (denoted by Y) represents the height or elevation and the control variable (denoted by X) represents volume or increment number. For plots based on the measurement equation (the inverse of the calibration equation), the interpretation of Y and X is reversed: Y denotes volume and X denotes height. Both plot orientations can be useful in a particular application and both are illustrated in Annex A. For convenience, only the term “calibration equation” is used in 5.2, with the understanding that the discussion also applies to the measurement equation.

The plots of 5.2 can be constructed from data that are presented in various forms. A specific plot is typically constructed from the standardized data from a particular calibration run, but it can also be constructed from the corresponding raw data, or from “data points” obtained by evaluating the tank's estimated calibration (or measurement) equation at a number of selected points. Although it is possible to construct plots from raw calibration data, the analysis of raw data is generally not recommended because meaningful comparisons are difficult, particularly if measurement conditions vary significantly during the period(s) of data collection.

Finally, it is often useful for comparative purposes to superpose or overlay several plots on a set of common axes. A plot obtained by superposing profile variation plots of the calibration data from several individual runs, for example (see 5.2.3), is very useful for examining run-to-run variations. Likewise, overlaying a profile variation plot of the data from a new calibration run on that from a previous estimate of the calibration equation can be very helpful for verifying that the tank's calibration equation has not changed since the previous calibration.

5.2.2 Cumulative (Y vs. X) plots

A cumulative plot displays the relationship between the height or elevation of points in a tank above some pre-selected reference point, Y, and the corresponding volume of the tank, X, below these points. A cumulative plot shows the general features (shape) of the height-volume relationship for the tank.

As noted in 5.2.1, a cumulative plot can be constructed

a) from the standardized data from a particular calibration run,

b) from the corresponding raw data (generally not recommended), or

c) from a previously defined calibration equation for the tank, expressed in functional or tabular form.

In the first two cases, the response of the tank's measurement system, e.g., “liquid height,” is plotted for each increment of the calibration run against a measure of the total amount of calibration liquid, e.g., cumulative volume, required to reach this height. In the latter case, the plotted “data” are obtained by evaluating the tank's estimated calibration or measurement equation at a number of selected points.

In the notation of ISO 18213-2, Y_i denotes the (standardized) elevation, above some pre-established reference point, of a point in the tank determined by the liquid added during the first i increments of a calibration run. Similarly, X_i denotes the (standardized) total volume of the tank below that point, as determined from the volume of liquid added during the first i calibration increments, as given in Equation (1):

$$X_i = \sum_j x_j \tag{1}$$

where x_j denotes the standardized volume of the jth incremental addition of calibration liquid. A cumulative plot is obtained by plotting the standardized volume-height pairs (X_i, Y_i) derived from the raw data of a particular calibration run. Methods for computing the standardized values X_i and Y_i from a set of raw calibration data are described in ISO 18213-2.

It can be useful for comparative purposes to overlay plots of the standardized data from several calibration runs. It is also possible to include a cumulative plot derived from a previously defined calibration equation, f, in the overlay plot. This is done by plotting the points [X_i, $f(X_i)$] obtained by evaluating the function at a suitable number of points.

One variation of the cumulative plot (and all other plots discussed in 5.2) is to plot the response variable against increment number, i, instead of against cumulative volume, X_i. However, when several plots are overlaid, a valid comparison is possible only when all data are plotted on a common scale.

Cumulative plots show the general features of the tank's profile (i.e., its height-volume relationship). They can also reveal gross differences in the data from several calibration runs, or among the data of several calibration runs and some pre-established calibration equation. However, the plotting range on the vertical scale is generally too large to provide adequate resolution for detecting

— small differences in tank profile, or

— outlying points in a set of calibration data.

Variations in tank profile and anomalous data points are more easily detected with the aid of the profile variation and incremental slope plots discussed in 5.2.3 and 5.2.4, respectively.

5.2.3 Profile variation [$(Y - a - bX)$ vs. X] plots

A profile variation plot shows the difference between the observed height, Y, and an estimate of height computed from an equation that expresses height as a linear function of volume, $\hat{Y}=a+bX$, versus the volume, X, of the tank below the corresponding height. In other words, the profile variation plot shows the variation in the free (unobstructed) cross-sectional area of the tank about its average free cross-sectional area. A profile variation plot provides greater resolution in the vertical ("height") scale than the cumulative plot, thereby revealing greater detail about the free cross-sectional area of the tank.

Like the cumulative plot, the profile variation plot can be constructed from

a) the standardized data from a particular calibration run,

b) the corresponding raw data (generally not recommended), or

c) a previously defined calibration equation for the tank, expressed in a functional form.

In the first two cases, a profile variation plot is obtained by plotting, for each increment of a calibration run, the "residual height" against the corresponding cumulative volume. The residual heights are the differences between observed heights and corresponding estimates computed from a linear function chosen to describe or "fit" the relationship between height and cumulative volume in the selected data. In the later case, the plotted "data" are obtained by first evaluating the tank's estimated calibration or measurement equation at a number of selected points.

In the notation of 5.2.2, where Y denotes standardized (liquid) height and X denotes the corresponding standardized cumulative volume, the profile variation plot is obtained by plotting the following points:

$$(Y_i - \hat{Y}_i, X_i) = (Y_i - a - bX_i, X_i) \qquad (2)$$

for all pairs of observations (X_i, Y_i) obtained during the calibration run. In Equation (2), a and b are estimates of the coefficients α and β in the linear relationship $Y = \alpha + \beta X + \varepsilon$ employed to describe the data.

The main objective of the profile variation plot is to increase the resolution in the vertical (height) dimension, and the method used to estimate α and β is secondary to this objective. The coefficients a and b may be taken as least squares regression estimates of the intercept and slope for a straight-line fit to the calibration data (X_i, Y_i). Alternatively, a and b may be taken as the slope and intercept of a line that passes through some initial point, e.g., the second or third, and a terminal point, e.g., the next-to-last, of the run. It is generally advisable to avoid the first and last points because they tend to be more anomalous than other points in the run.

As with cumulative plots, it can be useful for comparative purposes to overlay profile variation plots of the standardized data from several calibration runs. It is possible to include a profile variation plot from a previously defined calibration equation, f, in the overlay plot by evaluating the function $Y = f(X)$ at suitable points, X_i, and computing $Y_i - a - bX_i$ for each. It is also possible to make profile plots from the raw data, but this is not recommended for the reasons cited in 5.2.1.

When data from several calibration runs or tank calibration equations are being compared, the linear coefficients a and b should be determined from the aggregated data from all runs or equations of interest to ensure that all data are plotted on a common scale.

5.2.4 Incremental slope ($\Delta Y/\Delta X$ vs. X) plots

The incremental slope plot displays the incremental changes in the slope of the calibration function, i.e., changes in height between successive calibration increments with respect to the corresponding incremental changes in volume, plotted relative to the volume of the tank below the associated height. In other words, an incremental slope plot displays the rate of change in liquid height in the tank per unit change in volume for each volume increment in a calibration run. Incremental slope plots reveal great detail, so they are very useful for detecting small changes in tank profile that would not be revealed by cumulative or profile variation plots.