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**Geometrical product specifications  
(GPS) — Filtration —**

**Part 20:  
Linear profile filters: Basic concepts**

*Spécification géométrique des produits (GPS) — Filtrage —  
Partie 20: Filtres de profil linéaires: Concepts de base*  
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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In other circumstances, particularly when there is an urgent market requirement for such documents, a technical committee may decide to publish other types of normative document:

- an ISO Publicly Available Specification (ISO/PAS) represents an agreement between technical experts in an ISO working group and is accepted for publication if it is approved by more than 50 % of the members of the parent committee casting a vote;
- an ISO Technical Specification (ISO/TS) represents an agreement between the members of a technical committee and is accepted for publication if it is approved by 2/3 of the members of the committee casting a vote.

An ISO/PAS or ISO/TS is reviewed after three years in order to decide whether it will be confirmed for a further three years, revised to become an International Standard, or withdrawn. If the ISO/PAS or ISO/TS is confirmed, it is reviewed again after a further three years, at which time it must either be transformed into an International Standard or be withdrawn.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TS 16610-20 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO/TS 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

- *Part 1: Overview and basic concepts*
- *Part 20: Linear profile filters: Basic concepts*
- *Part 22: Linear profile filters: Spline filters*
- *Part 29: Linear profile filters: Spline wavelets*
- *Part 31: Robust profile filters: Gaussian regression filters*
- *Part 32: Robust profile filters: Spline filters*
- *Part 40: Morphological profile filters: Basic concepts*

- Part 41: Morphological profile filters: Disk and horizontal line-segment filters
- Part 49: Morphological profile filters: Scale space techniques

The following parts are under preparation:

- Part 21: Linear profile filters: Gaussian filters
- Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets
- Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets
- Part 30: Robust profile filters: Basic concepts
- Part 42: Morphological profile filters: Motif filters
- Part 60: Linear areal filters: Basic concepts
- Part 61: Linear areal filters: Gaussian filters
- Part 62: Linear areal filters: Spline filters
- Part 69: Linear areal filters: Spline wavelets
- Part 70: Robust areal filters: Basic concepts
- Part 71: Robust areal filters: Gaussian regression filters
- Part 72: Robust areal filters: Spline filters
- Part 80: Morphological areal filters: Basic concepts
- Part 81: Morphological areal filters: Sphere and horizontal planar segment filters
- Part 82: Morphological areal filters: Motif filters
- Part 89: Morphological areal filters: Scale space techniques

## Introduction

This part of ISO/TS 16610 is a geometrical product specification (GPS) Technical Specification and is to be regarded as a global GPS Technical Specification (see ISO/TR 14638). It influences the chain links 3 and 5 of all chains of standards.

For more detailed information about the relation of this part of ISO/TS 16610 to the GPS matrix model, see Annex C.

This part of ISO/TS 16610 develops the basic concepts of linear filters, which include spline filters and spline wavelets, as well as the standardized Gaussian filter (see ISO 11562:1996).

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# Geometrical product specifications (GPS) — Filtration —

## Part 20:

### Linear profile filters: Basic concepts

#### 1 Scope

This part of ISO/TS 16610 sets out the basic concepts of linear profile filters.

#### 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11562:1996<sup>1)</sup>, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Metrological characteristics of phase correct filters*

ISO/TS 16610-1:2006, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

*International vocabulary of basic and general terms in metrology (VIM)*. BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 2nd ed., 1993

#### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in VIM and ISO/TS 16610-1, and the following apply.

##### 3.1

##### **linear profile filter**

profile filter which separates profiles into long wave and short wave components

##### 3.2

##### **phase correct (linear) profile filter**

**linear profile filter** (3.1) which does not cause phase shifts leading to asymmetrical profile distortions

NOTE Phase correct filters are a particular kind of the so-called linear phase filters, because any linear phase filter can be transformed (simply by shifting its weighting function) to a zero phase filter, which is a phase correct filter.

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1) To be replaced by ISO 16610-21.

**3.3 weighting function**

function for calculating the mean line, which indicates for each point the weight attached by the profile in the vicinity of that point

NOTE The transmission characteristic of the mean line is the Fourier transformation of the weighting function.

**3.4 transmission characteristic of a filter**

characteristic that indicates the amount by which the amplitude of a sinusoidal profile is attenuated as a function of its wavelength

NOTE The transmission characteristic is the Fourier transformation of the weighting function.

**3.5 cut-off wavelength**

wavelength of a sinusoidal profile, of which 50 % of the amplitude is transmitted by the profile filter

NOTE 1 Linear profile filters are identified by the filter type and the cut-off wavelength value.

NOTE 2 The cut-off wavelength is the recommended nesting index for linear profile filters.

**3.6 filter bank**

set of high-pass and low-pass filters, arranged in a specified structure

**3.7 multiresolution analysis**

decomposition of a profile by a **filter bank** (3.6) into portions of different scales

NOTE The portions at different scales are also referred to as resolutions.

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**4 Basic concepts**

**4.1 General**

For a filter to conform with this part of ISO/TS 16610, it shall exhibit the characteristics described in 5.1, 5.2, 5.3 and 5.4.

NOTE A concept diagram for linear profile filters is given in Annex A. The relationship to the filtration matrix model is given in Annex B.

The most general linear profile filter is defined by

$$y(x) = \int K(x, \xi)z(\xi)d\xi \tag{1}$$

where

$z(\xi)$  is the unfiltered profile;

$y(x)$  is the filtered profile;

$K(x, \xi)$  is a real symmetric and spatial invariant kernel.

If  $K(x, \xi) = K(x - \xi)$ , the filtering is a convolution

$$y(x) = \int K(x - \xi)z(\xi)d\xi \tag{2}$$



and the kernel is called the weighting function of the filter.

However, extracted data are always discrete. Consequently, the filters described here are also discrete. If the weighting function is not discrete (see 4.4, Example 2), it shall be converted into a discrete representation.

#### 4.2 Discrete representation of data

An extracted profile can be represented by a vector. The length  $n$  of this vector is equal to the number of data points. The sampling is assumed to be uniform, i.e. the sampling interval is constant. The  $i$ th data point of the profile is therefore the  $i$ th component of the vector.

$$(a_1 a_2 \dots a_i \dots a_{n-1} a_n) \quad (3)$$

#### 4.3 Discrete representation of the linear profile filter

A linear profile filter is represented by a square matrix. The dimension of this matrix is equal to the number of data points to be filtered. If the filter is non-periodic, the matrix is a constant diagonal (Toeplitz) matrix:

$$\begin{pmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ & c' & b' & a & b & c & & \\ & & c' & b' & a & b & c & \\ & & & c' & b' & a & b & c \\ & & & & c' & b' & a & b & c \\ & & & & & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \quad (4)$$

Otherwise, if the filter is periodic, the matrix is a circulant matrix.

$$\begin{pmatrix} a & b & c & \dots & \dots & c' & b' \\ b' & a & b & c & \dots & \dots & c' \\ c' & b' & a & b & c & \dots & \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \dots & c' & b' & a & b & c & \\ c & \dots & \dots & c' & b' & a & b \\ b & c & \dots & \dots & c' & b' & a \end{pmatrix} \quad (5)$$

If the filter is phase correct, the matrix representing the filter is symmetrical, i.e.  $b = b', c = c', \dots$  (generally  $a_{ij} = a_{ji}$ ) is valid. The sum of the matrix elements  $a_{ij}$  of each row  $i$  is constant, and for low-pass filters equals one, i.e.

$$\sum_j a_{ij} = 1 \quad (6)$$

NOTE In the case of a symmetrical matrix, the sum of the matrix elements  $a_{ij}$  of each column  $j$  is also constant, and equals one, i.e.  $\sum_i a_{ij} = 1$  is valid, too.

#### 4.4 Discrete representation of the weighting function

Given that each row of the matrix representation of the filter is identical after being shifted accordingly, the matrix elements may be represented by one single row. i.e.

$$a_{ij} = s_k \text{ with } k = i - j \quad (7)$$

The values  $s_k$  form a vector  $s$  of a dimension equal to the length of the input or output data vector, respectively. This vector is the discrete representation of the weighting function of the filter.

NOTE 1 The length of the weighting function is usually much smaller than the length of the data set. In this case,  $s$  contains zeros at each end.

EXAMPLE 1 The moving average filter is frequently used for easy smoothing of a data set, which is not necessarily an optimal method. In the following example of a filter with a discrete weighting function, where a length of three has been taken, the weighting function is given by

$$\left( \dots, 0, 0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, \dots \right) \tag{8}$$

NOTE 2 The weighting function is often also called the impulse response function, because it is the output data set of the filter, if the input data set is only a single unity impulse  $(\dots, 0, 0, 0, 1, 0, 0, 0, \dots)$ .

If the weighting function is given as a continuous function, it shall be sampled in order to obtain a discrete data set. The sampling interval used shall be equal to the sampling interval of the extracted data. It is mandatory to re-normalize the sampled data of the weighting function subsequently, in order to fulfil the condition that they shall sum to unity, thus avoiding bias effects (for details concerning bias effects, see [2]).

EXAMPLE 2 The Gaussian filter in accordance with ISO 11562:1996 is an example of a continuous weighting function  $s(x)$  defined by the equation

$$s(x) = \frac{1}{\alpha \lambda_c} \exp \left[ -\pi \left( \frac{x}{\alpha \lambda_c} \right)^2 \right] \tag{9}$$

where

$x$  is the distance from the centre (maximum) of the weighting function;

$\lambda_c$  is the cut-off wavelength;

$\alpha$  is a constant given by the following equation:

$$\alpha = \sqrt{\frac{\log 2}{\pi}} = 0,4697\dots \tag{10}$$

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The graph of this weighting function is shown in Figure 1.

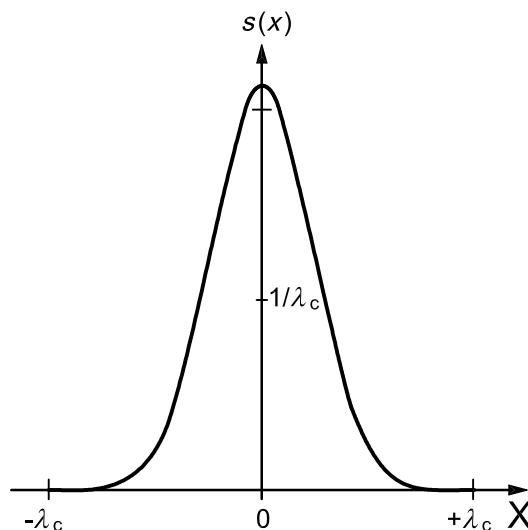


Figure 1 — Example of a continuous weighting function (Gaussian filter)

The sampled data  $s_k$  of the weighting function after a re-normalization are given by

$$s_k = \frac{1}{C} \exp \left[ -\pi \left( \frac{\Delta x}{\alpha \lambda_c} \right)^2 k^2 \right] \quad (11)$$

with the sampling interval  $\Delta x$ , and the normalization constant

$$C = \sum_k \exp \left[ -\pi \left( \frac{\Delta x}{\alpha \lambda_c} \right)^2 k^2 \right] \quad (12)$$

## 5 Linear profile filters

### 5.1 Filter equations

If the filter is represented by the matrix  $S$ , the input data by the vector  $z$  and the output data by the vector  $w$ , then the filtering process is described by the linear equation

$$w = Sz \quad (13)$$

This equation is the filter equation. If  $S^{-1}$  is the inverse matrix of the filter matrix  $S$ , then

$$z = S^{-1}w \quad (14)$$

is a valid filter equation, too.

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NOTE 1 The filter can be defined by the matrix  $S$  or by the inverse matrix  $S^{-1}$ , whichever leads to a simpler definition. However, the weighting function is only given by the rows of the matrix  $S$ .

NOTE 2 The inverse matrix sometimes does not exist, in which case the filtering process is not invertible, i.e. data reconstruction is impossible. Such a filter is called unstable. The stability of a filter can be seen from its transfer function (see 5.3). An unstable filter has a transfer function  $H(\omega)$ , which is zero for at least one frequency  $\omega$ .

EXAMPLE The matrix of the moving average filter mentioned above

$$\frac{1}{3} \begin{pmatrix} \ddots & \ddots & \ddots & & & & \\ & 1 & 1 & 1 & & & \\ & & 1 & 1 & 1 & & \\ & & & 1 & 1 & 1 & \\ & & & & 1 & 1 & 1 \\ & & & & & \ddots & \ddots & \ddots \end{pmatrix} \quad (15)$$

is not invertible, and therefore the filter is unstable. If the filter is changed to a moving average filter ( $\alpha < 1/2$ )

$$\frac{1}{1+2\alpha} \begin{pmatrix} \ddots & \ddots & \ddots & & & & \\ & \alpha & 1 & \alpha & & & \\ & & \alpha & 1 & \alpha & & \\ & & & \alpha & 1 & \alpha & \\ & & & & \alpha & 1 & \alpha \\ & & & & & \ddots & \ddots & \ddots \end{pmatrix} \quad (16)$$

it becomes stable.

The inverse matrix  $S^{-1}$  is a constant diagonal matrix or a circulant matrix, if  $S$  is, respectively, a constant diagonal matrix or a circulant matrix. The inverse matrix  $S^{-1}$  is symmetrical, if  $S$  is symmetrical.