TECHNICAL SPECIFICATION

ISO/TS 16610-31

First edition
2010-08-15

Geometrical product specifications (GPS) — Filtration —

Part 31:
Robust profile filters: Gaussian regression filters

Spécification géométrique des produits (GPS) — Filtrage —

Partie 31: Filtres de profil robustes: Filtres de régression gaussiens

Reference number
ISO/TS 16610-31:2010(E)

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In other circumstances, particularly when there is an urgent market requirement for such documents, a technical committee may decide to publish other types of document:

— an ISO Publicly Available Specification (ISO/PAS) represents an agreement between technical experts in an ISO working group and is accepted for publication if it is approved by more than 50 % of the members of the parent committee casting a vote;

— an ISO Technical Specification (ISO/TS) represents an agreement between the members of a technical committee and is accepted for publication if it is approved by 2/3 of the members of the committee casting a vote.

An ISO/PAS or ISO/TS is reviewed after three years in order to decide whether it will be confirmed for a further three years, revised to become an International Standard, or withdrawn. If the ISO/PAS or ISO/TS is confirmed, it is reviewed again after a further three years, at which time it must either be transformed into an International Standard or be withdrawn.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TS 16610-31 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO 16610 consists of the following parts, under the general title *Geometrical product specification (GPS) — Filtration*:

— *Part 1: Overview and basic concepts* [Technical Specification]

— *Part 20: Linear profile filters: Basic concepts* [Technical Specification]

— *Part 21: Linear profile filters: Gaussian filters*

— *Part 22: Linear profile filters: Spline filters* [Technical Specification]

— *Part 28: Profile filters: End effects* [Technical Specification]

— *Part 29: Linear profile filters: Spline wavelets* [Technical Specification]

— *Part 30: Robust profile filters: Basic concepts* [Technical Specification]

— *Part 31: Robust profile filters: Gaussian regression filters* [Technical Specification]

— *Part 32: Robust profile filters: Spline filters* [Technical Specification]

— *Part 40: Morphological profile filters: Basic concepts* [Technical Specification]

— *Part 41: Morphological profile filters: Disk and horizontal line-segment filters* [Technical Specification]

— *Part 49: Morphological profile filters: Scale space techniques* [Technical Specification]

The following parts are planned:

— *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*

— *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*

— *Part 42: Morphological profile filters: Motif filters*

— *Part 60: Linear areal filters: Basic concepts*

— *Part 61: Linear areal filters: Gaussian filters*

— *Part 62: Linear areal filters: Spline filters*

— *Part 69: Linear areal filters: Spline wavelets*

— *Part 70: Robust areal filters: Basic concepts*

— *Part 71: Robust areal filters: Gaussian regression filters*

— *Part 72: Robust areal filters: Spline filters*

— *Part 80: Morphological areal filters: Basic concepts*

— *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*

— *Part 82: Morphological areal filters: Motif filters*

— *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO 16610 is a geometrical product specification (GPS) standard and is to be regarded as a global GPS standard (see ISO/TR 14638). It influences the chain link 3 of all chains of standards.

For more detailed information of the relation of this part of ISO 16610 to the GPS matrix model, see Annex C.

This part of ISO 16610 develops the concept of the discrete robust Gaussian regression filter. The robust process reduces the influence of the deep valleys and high peaks. The subject of this part of ISO 16610 is the robust Gaussian regression filter of degree $p = 2$, which has very good robust behaviour and form approximation for functional stratified engineering surfaces.

Geometrical product specifications (GPS) — Filtration —

Part 31:
Robust profile filters: Gaussian regression filters

1 Scope

This part of ISO 16610 specifies the characteristics of the discrete robust Gaussian regression filter for the evaluation of surface profiles with spike discontinuities such as deep valleys and high peaks.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/TS 16610-1:2006, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic terminology*

ISO/TS 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO/TS 16610-30, *Geometrical product specifications (GPS) — Filtration — Part 30: Robust profile filters: Basic concepts*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC Guide 99, ISO/TS 16610-1, ISO/TS 16610-20, ISO/TS 16610-30 and the following apply.

3.1
robust filter
filter that is insensitive to output data against specific phenomena in the input data

3.2
regression filter
M-estimator based on the local polynomial modelling of the profile

3.3
robust Gaussian regression filter
regression filter based on the Gaussian weighting function and a biweight influence function

3.4
biweight influence function
asymmetric function which is scale-invariant, expressed by

$$\psi(x) = \begin{cases} x \times \left(1 - \left(\dfrac{x}{c}\right)^2\right)^2 & \text{for} \quad |x| \leqslant c \\ 0 & \text{for} \quad |x| > c \end{cases} \tag{1}$$

where c is the scale parameter

4 Robust Gaussian regression filter

4.1 Weighting function

The weighting function of the robust Gaussian regression filter depends on the profile values (distance to the reference line) and the location of the weighting function along the profile.

4.2 Filter equation

4.2.1 General

The robust Gaussian regression filter is derived from the general discrete regression filter (see Annex A) by setting the degree to $p = 2$, using the biweight influence function and the Gaussian weighting function according to ISO 16610-21. In the case of $p = 2$, the robust Gaussian regression filter follows form components up to the second degree.

4.2.2 Filter equation for the robust Gaussian regression filter for open profiles

For open profiles, the filter equation for the robust Gaussian regression filter is given by

$$w_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \left(\boldsymbol{X}_k^T \times \boldsymbol{S}_k \times \boldsymbol{X}_k \right)^{-1} \times \boldsymbol{X}_k^T \times \boldsymbol{S}_k \times \boldsymbol{z} \tag{2}$$

The regression function is spanned by the matrix

$$\boldsymbol{X}_k = \begin{bmatrix} 1 & x_{1,k} & x_{1,k}^2 \\ \vdots & \vdots & \vdots \\ 1 & x_{n,k} & x_{n,k}^2 \end{bmatrix} \tag{3}$$

where $x_{l,k} = (l - k) \times \Delta x, \qquad l = 1, \ldots, n$ (4)

The space variant weighting function, $\boldsymbol{S}_k$, is given by

$$\boldsymbol{S}_k = \begin{bmatrix} s_{1,k} \times \delta_1 & 0 & \cdots & 0 \\ 0 & s_{2,k} \times \delta_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & s_{n,k} \times \delta_n \end{bmatrix} \tag{5}$$

with the Gaussian function

$$s_{l,k} = \frac{1}{\gamma \times \lambda_\mathrm{c}} \times \exp\left(-\pi \left(\frac{x_{l,k}}{\gamma \times \lambda c} \right)^2 \right), \qquad l = 1, .., n \tag{6}$$

and the parameter

$$\gamma = \sqrt{\frac{-1 - W\left(-\frac{1}{2 \times \exp(1)} \right)}{\pi}} \approx 0{,}730\ 9 \tag{7}$$

The additional weights

$$\delta_l = \begin{cases} \left(1 - \left(\frac{z_l - w_l}{c} \right)^2 \right)^2 & \text{for} \quad |z_l - w_l| \le c \\ 0 & \text{for} \quad |z_l - w_l| > c \end{cases}, \qquad l = 1, .., n \tag{8}$$

are derived from the biweight influence function with the parameter

$$c = \frac{3}{\sqrt{2} \times \mathrm{erf}^{-1}(0{,}5)} \times \mathrm{median}|\boldsymbol{z} - \boldsymbol{w}| \approx 4{,}447\ 8 \times \mathrm{median}|\boldsymbol{z} - \boldsymbol{w}| \tag{9}$$

The definition for c is equivalent to three times Rq of the surface roughness for Gaussian distributed profiles and is the default case

where

- $W(X)$ is the “Lambert W” function;
- $\mathrm{erf}^{-1}(x)$ is the inverse error function;
- n is the number of values in the profile;
- k is the index of the profile ordinate $k = 1, \ldots, n$;
- $\boldsymbol{z}$ is the vector of dimension n of the profile values before filtering;
- $\boldsymbol{w}$ is the vector of dimension n of the profile values of the filter reference line;
- w_k is the value of the filter mean line at position k;
- λ_c is the cut-off wavelength of the profile filter;
- Δx is the sampling interval.

NOTE 1 Vector $\boldsymbol{w}$ gives the profile values of the long-wave component (reference line). The short-wave component, $\boldsymbol{r}$, can be obtained by the difference vector, $\boldsymbol{r} = \boldsymbol{z} - \boldsymbol{w}$.

NOTE 2 For surfaces with big pores or peaks at the profile boundaries, the robustness can be increased by setting p = 0. In this case, the nominal form is eliminated by using a pre-filtering technique. The filter equation for p = 0 results in

$$w_k = \left(\boldsymbol{X}_k^T \times \boldsymbol{S}_k \times \boldsymbol{X}_k \right)^{-1} \times \boldsymbol{X}_k^T \times \boldsymbol{S}_k \times z = \left(\sum_{l=1}^{n} s_{l,k} \times \delta_l \right)^{-1} \times \sum_{l=1}^{n} \left(s_{l,k} \times \delta_l \times z_l \right)$$

where

$$\boldsymbol{X}_k = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \text{ and } \gamma = \sqrt{\frac{\ln 2}{\pi}}$$

4.2.3 Filter equation for robust Gaussian regression filter for closed profiles

For closed profiles, the filter equation for the robust Gaussian regression filter is given by

$$\tilde{w}_k = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \times \left(\tilde{\boldsymbol{X}}_k^T \times \tilde{\boldsymbol{S}}_k \times \tilde{\boldsymbol{X}}_k \right)^{-1} \times \tilde{\boldsymbol{X}}_k^T \times \tilde{\boldsymbol{S}}_k \times \tilde{\boldsymbol{z}} \tag{10}$$

The regression function is spanned by the matrix

$$\tilde{\boldsymbol{X}}_k = \begin{bmatrix} 1 & \tilde{x}_{1,k} & \tilde{x}_{1,k}^2 \\ \vdots & \vdots & \vdots \\ 1 & \tilde{x}_{n,k} & \tilde{x}_{n,k}^2 \end{bmatrix} \tag{11}$$

with

$$\tilde{x}_{l,k} = \left(\left(l - k + \frac{n}{2} \right) \bmod n - \frac{n}{2} \right) \times \Delta x, \qquad l = 1, \ldots, n \tag{12}$$

The space variant weighting function, $\tilde{\boldsymbol{S}}_k$, is given by

$$\tilde{\boldsymbol{S}}_k = \begin{bmatrix} \tilde{s}_{1,k} \times \tilde{\delta}_1 & 0 & \cdots & 0 \\ 0 & \tilde{s}_{2,k} \times \tilde{\delta}_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{s}_{n,k} \times \tilde{\delta}_n \end{bmatrix} \tag{13}$$

with the Gaussian function

$$\tilde{s}_{l,k} = \frac{1}{\gamma \times \lambda_c} \times \exp\left(-\pi \left(\frac{\tilde{x}_{l,k}}{\gamma \times \lambda_c} \right)^2 \right), \qquad l = 1, \ldots, n \tag{14}$$

and the parameter

$$\gamma = \sqrt{\frac{-1 - W\left(-\frac{1}{2 \times \exp(1)} \right)}{\pi}} \approx 0{,}730\,9 \tag{15}$$

The additional weights

$$\tilde{\delta}_l = \begin{cases} \left(1 - \left(\frac{\tilde{z}_l - \tilde{w}_l}{\tilde{c}} \right)^2 \right)^2 & \text{for} \quad \left| \tilde{z}_l - \tilde{w}_l \right| \leqslant \tilde{c} \\ 0 & \text{for} \quad \left| \tilde{z}_l - \tilde{w}_l \right| > \tilde{c} \end{cases}, \qquad l = 1, \ldots, n \tag{16}$$

are derived from the biweight influence function with the parameter

$$\tilde{c} = \frac{3}{\sqrt{2} \times \mathrm{erf}^{-1}(0{,}5)} \times \mathrm{median}\left|\tilde{\boldsymbol{z}} - \tilde{\boldsymbol{w}}\right| \approx 4{,}447\,8 \times \mathrm{median}\left|\tilde{\boldsymbol{z}} - \tilde{\boldsymbol{w}}\right| \tag{17}$$

The definition for c is equivalent to three times Rq of the surface roughness for Gaussian distributed profiles and is the default case

where

$W(X)$	is the "Lambert W" function;
$\mathrm{erf}^{-1}(x)$	is the inverse error function;
n	is the number of values in the profile;
k	is the index of the profile ordinate $k = 1, \ldots, n$;
$\tilde{\boldsymbol{z}}$	is the vector of dimension n of the profile values before filtering;
$\tilde{\boldsymbol{w}}$	is the vector of dimension n of the profile values of the filter reference line;
$\tilde{w}_k$	is the value of the filter mean line at position k;
λ_c	is the cut-off wavelength of the profile filter;
Δx	is the sampling interval.

NOTE Vector $\tilde{\boldsymbol{w}}$ gives the profile values of the long-wave component (reference line). The short-wave component, $\tilde{\boldsymbol{r}}$, may be obtained by the difference vector, $\boldsymbol{r} = \boldsymbol{Z} - \boldsymbol{W}$.

4.2.4 Transmission characteristics

The weighting function of the robust Gaussian regression filter depends on the profile values and the location along the profile. Therefore, no transmission characteristic can be given.

5 Recommendations for nesting index (cutoff values λ_c)

It is recommended that a nesting index be chosen equivalent to three times the feature width in the profile data set. Otherwise, the nesting index should be chosen from the following series of values:

… 2,5 µm; 8 µm; 25 µm; 80 µm; 250 µm; 0,8 mm; 2,5 mm; 8 mm; 25 mm; …

6 Filter designation

Robust Gaussian regression filters according to this part of ISO 16610 are designated

FPRG

See also ISO/TS 16610-1:2006, Clause 5.