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Determination of the characteristic limits (decision threshold, detection limit and limits of the confidence interval) for measurements of ionizing radiation — Fundamentals and application

Détermination des limites caractéristiques (seuil de décision, limite de détection et extrémités de l'intervalle de confiance) pour mesurages de rayonnements ionisants — Principes fondamentaux et applications

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Foreword

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International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 11929 was prepared by Technical Committee ISO/TC 85, *Nuclear energy*, Subcommittee SC 2, *Radiation protection*.

This first edition of ISO 11929 cancels and replaces ISO 11929-1:2000, ISO 11929-2:2000, ISO 11929-3:2000, ISO 11929-4:2001, ISO 11929-5:2005, ISO 11929-6:2005, ISO 11929-7:2005 and ISO 11929-8:2005, which have been technically revised, specifically with reference to the type of statistical treatment of the data.

Introduction

The limits to be provided according to this International Standard by means of statistical tests and specified probabilities allow detection possibilities to be assessed for a measurand and for the physical effect quantified by this measurand as follows:

— the “decision threshold” gives a decision on whether or not the physical effect quantified by the measurand is present;

— the “detection limit” indicates the smallest true value of the measurand which can still be detected with the applied measurement procedure; this gives a decision on whether or not the measurement procedure satisfies the requirements and is therefore suitable for the intended measurement purpose;

— the “limits of the confidence interval” enclose, in the case of the physical effect recognized as present, a confidence interval containing the true value of the measurand with a specified probability.

Hereinafter, the limits mentioned are jointly called “characteristic limits”.

Since measurement uncertainty plays an important part in this International Standard, the evaluation of measurements and the treatment of measurement uncertainties are carried out by means of the general procedures according to ISO/IEC Guide 98-3; see also References [1, 2]. This enables the strict separation of the evaluation of the measurements, on the one hand (Clause 5), and the provision and calculation of the characteristic limits, on the other hand (Clause 6). This International Standard is based on Bayesian statistics according to References [6 to 19], such that uncertain quantities and influences, which do not behave randomly in measurements repeated several times or in counting measurements, can also be taken into account.

Equations are provided for the calculation of the characteristic limits of an ionizing radiation measurand via the “standard measurement uncertainty” of the measurand (hereinafter “standard uncertainty”). The standard uncertainties of the measurement, as well as those of sample treatment, calibration of the measuring system and other influences are taken into account. However, the latter standard uncertainties are assumed to be known from previous investigations.

Determination of the characteristic limits (decision threshold, detection limit and limits of the confidence interval) for measurements of ionizing radiation — Fundamentals and application

1 Scope

This International Standard specifies a procedure, in the field of ionizing radiation metrology, for the calculation of the “decision threshold”, the “detection limit” and the “limits of the confidence interval” for a non-negative ionizing radiation measurand, when counting measurements with preselection of time or counts are carried out, and the measurand results from a gross count rate and a background count rate as well as from further quantities on the basis of a model of the evaluation. In particular, the measurand can be the net count rate as the difference of the gross count rate and the background count rate, or the net activity of a sample. It can also be influenced by calibration of the measuring system, by sample treatment and by other factors.

This International Standard also applies, in the same way to:

— counting measurements on moving objects (see B.2);

— measurements with linear-scale analogue count rate measuring instruments (hereinafter called ratemeters, see B.3);

— repeated counting measurements with random influences (see B.4);

— counting measurements on filters during accumulation of radioactive material (see B.5);

— counting spectrometric multi-channel measurements, if particular lines in the spectrum are to be considered and no adjustment calculations, for instance, an unfolding, have to be carried out (see C.2 to C.4);

— counting spectrometric multi-channel measurements if evaluated by unfolding methods (see C.5), in particular, alpha- and gamma-spectrometric measurements (see C.5.5 and C.5.6, respectively).

This International Standard also applies analogously to other measurements of any kind if the same model of the evaluation is involved. In this sense, it is also applicable to measurements with albedo dosimeters [18]. Further practical examples can be found in other International Standards, for example ISO 18589[21], ISO 9696[22], ISO 9697[23], ISO 9698[24], ISO 9699[25], ISO 10703[26], ISO 7503[27] and ISO 28218[28].

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 31-0, *Quantities and units — Part 0: General principles*

ISO 31-9, *Quantities and units — Part 9: Atomic and nuclear physics*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 31-0, ISO 31-9, ISO/IEC Guide 98-3, ISO/IEC Guide 99 and ISO 3534-1 and the following apply.

3.1
measurement procedure
set of operations, described specifically, used in the performance of particular measurements according to a given method

[ISO/IEC Guide 99:2007, 2.6]

3.2
measurand
particular quantity subject to measurement

[ISO/IEC Guide 99:2007, 2.3]

NOTE In this International Standard, a measurand is non-negative and quantifies a nuclear radiation effect. The effect is not present if the true value of the measurand is zero. An example of a measurand is the intensity of an energy line in a spectrum above the background in a spectrometric measurement.

3.3
result of a measurement
value attributed to a measurand, obtained by measurement

[ISO/IEC Guide 99:2007, 2.9]

3.4
uncertainty of measurement
uncertainty
non-negative parameter, which characterizes the dispersion of the values which could reasonably be attributed to the measurand

[ISO/IEC Guide 99:2007, 2.26]

See also ISO/IEC Guide 98-3.

NOTE The uncertainty of a measurement derived according to ISO/IEC Guide 98-3 comprises, in general, many components. Some of these components can be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviations. The other components, which can also be characterized by standard deviations, are evaluated from assumed or known probability distributions based on experience and other information.

3.5
model of evaluation
set of mathematical relationships between all measured and other quantities involved in the evaluation of measurements

NOTE The model of evaluation does not need to be an explicit function; it can also be an algorithm realized by a computer code.

3.6
decision threshold
value of the estimator of the measurand, which when exceeded by the result of an actual measurement using a given measurement procedure of a measurand quantifying a physical effect, one decides that the physical effect is present

NOTE 1 The decision threshold is defined such that in cases where the measurement result, y, exceeds the decision threshold, y^*, the probability that the true value of the measurand is zero is less or equal to a chosen probability, α.

NOTE 2 If the result, y, is below the decision threshold, y^*, the result cannot be attributed to the physical effect; nevertheless it cannot be concluded that it is absent.

3.7
detection limit
smallest true value of the measurand which ensures a specified probability of being detectable by the measurement procedure

NOTE With the decision threshold according to 3.6, the detection limit is the smallest true value of the measurand for which the probability of wrongly deciding that the true value of the measurand is zero is equal to a specified value, β, when, in fact, the true value of the measurand is not zero.

3.8
limits of the confidence interval
values which define a confidence interval containing the true value of the measurand with a specified probability

NOTE A confidence interval is sometimes known as a credible interval or a Bayesian interval. It is characterized in this International Standard by a specified probability $(1-\gamma)$.

3.9
best estimate of the true value of the measurand
expectation value of the probability distribution of the true value of the measurand, given the experimental result and all prior information on the measurand

NOTE The best estimate, among all possible estimates of the measurand on the basis of given information, which is associated with the minimum uncertainty.

3.10
guideline value
value which corresponds to scientific, legal or other requirements and which is intended to be assessed by the measurement procedure

NOTE 1 The guideline value can be given, for example, as an activity, a specific activity or an activity concentration, a surface activity or a dose rate.

NOTE 2 The comparison of the detection limit with a guideline value allows a decision on whether or not the measurement procedure satisfies the requirements set forth by the guideline value and is therefore suitable for the intended measurement purpose. The measurement procedure satisfies the requirement if the detection limit is smaller than the guideline value.

3.11
background effect
measurement effect caused by radiation other than that caused by the object of the measurement itself

EXAMPLE Natural radiation sources.

3.12
net effect
contribution of the possible radiation of a measurement object (for instance, of a radiation source or radiation field) to the measurement effect

3.13
gross effect
measurement effect caused by the background effect and the net effect

3.14
shielding factor
factor describing the reduction of the background count rate by the effect of shielding caused by the measurement object

3.15
relaxation time constant
duration in which the output signal of a linear-scale ratemeter decreases to 1/e times the starting value after stopping the sequence of the input pulses

3.16
background
⟨spectrometric measurements⟩ number of events of no interest in the region of a specific line in the spectrum

NOTE The events can be due to the background effect by the environmental radiation and also to the sample itself (for instance, from other lines).

4 Quantities and symbols

The symbols for auxiliary quantities and the symbols only used in the annexes are not listed. Physical quantities are denoted by upper-case letters but shall be carefully distinguished from their values, denoted by the corresponding lower-case letters. In addition, the special quantities and symbols for unfolding in spectrometric measurements given in C.5.1 and for Bayesian statistics given in F.2.1 are used.

m	number of input quantities
X_i	input quantity $(i = 1, \ldots, m)$
x_i	estimate of the input quantity X_i
$u(x_i)$	standard uncertainty of the input quantity X_i associated with the estimate x_i
$h_1(x_1)$	standard uncertainty $u(x_1)$ as a function of the estimate x_1
Δx_i	width of the region of the possible values of the input quantity X_i
$u_{rel}(w)$	relative standard uncertainty of a quantity Y_k associated with the estimate w
G	model function
Y	random variable as an estimator of the measurand; also used as the symbol for the non-negative measurand itself, which quantifies the physical effect of interest
$\tilde{y}$	true value of the measurand; if the physical effect of interest is not present, then $\tilde{y} = 0$; otherwise, $\tilde{y} > 0$
y	determined value of the estimator Y, estimate of the measurand, primary measurement result of the measurand
y_j	values y from different measurements $(j = 0, 1, 2, \ldots)$
$u(y)$	standard uncertainty of the measurand associated with the primary measurement result y

$\tilde{u}(\tilde{y})$ standard uncertainty of the estimator Y as a function of the true value $\tilde{y}$ of the measurand

$\hat{y}$ best estimate of the measurand

$u(\hat{y})$ standard uncertainty of the measurand associated with the best estimate $\hat{y}$

y^* decision threshold of the measurand

$y^\#$ detection limit of the measurand

$\tilde{y}_i$ approximations of the detection limit $y^\#$

y_r guideline value of the measurand

$y^\triangleleft$, $y^\triangleright$ lower and upper limit of the confidence interval, respectively, of the measurand

ρ_i count rate as an input quantity X_i

ρ_n count rate of the net effect (net count rate)

ρ_g count rate of the gross effect (gross count rate)

ρ_0 count rate of the background effect (background count rate)

n_i number of counted pulses obtained from the measurement of the count rate ρ_i

n_g, n_0 number of counted pulses of the gross effect and of the background effect, respectively

t_i measurement duration of the measurement of the count rate ρ_i

t_g, t_0 measurement duration of the measurement of the gross effect and of the background effect, respectively

r_i estimate of the count rate ρ_i

r_g, r_0 estimate of the gross count rate and of the background count rate, respectively

τ_g, τ_0 relaxation time constant of a ratemeter used for the measurement of the gross effect and of the background effect, respectively

α, β probability of the error of the first and second kind, respectively

$1-\gamma$ probability for the confidence interval of the measurand

k_p, k_q quantiles of the standardized normal distribution for the probabilities p and q, respectively (for instance $p = 1-\alpha$, $1-\beta$ or $1-\gamma/2$)

$\Phi(t)$ distribution function of the standardized normal distribution; $\Phi(k_p) = p$ applies

5 Fundamentals

5.1 General aspects concerning the measurand

A non-negative measurand shall be assigned to the physical effect to be investigated according to a given measurement task. The measurand shall quantify the effect. It assumes the true value $\tilde{y} = 0$ if the effect is not present in a particular case.

Then, a random variable Y, an estimator, shall be assigned to the measurand. The symbol Y is also used in this clause for the measurand itself. A value y of the estimator Y, determined from measurements, is an estimate of the measurand. It shall be calculated as the primary measurement result together with the primary standard uncertainty $u(y)$, of the measurand associated with y. These two values form the primary complete measurement result for the measurand and are obtained in accordance with ISO/IEC Guide 98-3 (see also References [1, 2]) by evaluation of the measurement data and other information by means of a model (of the evaluation), which mathematically connects all the quantities involved (see 5.2). In general, the fact that the measurand is non-negative is not explicitly taken into account in the evaluation. Therefore, y may be negative, especially when the measurand nearly assumes the true value $\tilde{y} = 0$. The primary measurement result, y, differs from the best estimate, $\hat{y}$, of the measurand calculated in 6.5. With $\hat{y}$, the knowledge that the measurand is non-negative is taken into account. The standard uncertainty, $u(\hat{y})$, associated with $\hat{y}$ is smaller than $u(y)$.

5.2 Model

5.2.1 General model

In many cases, the measurand, Y, is a function of several input quantities, X_i, in the form of Equation (1):

$$Y = G(X_1, \ldots, X_m) \tag{1}$$

Equation (1) is the model of the evaluation. Substituting given estimates, x_i, of the input quantities, X_i, in the model function, G, Equation (1) yields the primary measurement result y of the measurand as:

$$y = G(x_1, \ldots, x_m) \tag{2}$$

The standard uncertainty, $u(y)$, of the measurand associated with the primary measurement result, y, is calculated according to Equation (3), if the input quantities, X_i, are independently measured and standard uncertainties, $u(x_i)$, associated with the estimates, x_i, are given, from the relation:

$$u^2(y) = \sum_{i=1}^{m} \left(\frac{\partial G}{\partial X_i} \right)^2 u^2(x_i) \tag{3}$$

In Equation (3), the estimates, x_i, shall be substituted for the input quantities, X_i, in the partial derivatives of G. The determination of the estimates, x_i, and the associated standard uncertainties, $u(x_i)$, and also the numerical or experimental determination of the partial derivatives are in accordance with ISO/IEC Guide 98-3 or References [1, 2]. For a count rate, $X_i = \rho_i$, with the given counting result, n_i, recorded during the measurement of duration, t_i, the specifications $x_i = r_i = n_i / t_i$ and $u^2(x_i) = n_i / t_i^2 = r_i / t_i$ apply (see also F.1). If the input quantities are not independently measured and for more complicated measurement evaluations such as unfolding, see C.5.2.

In 5.2.2, the input quantity, X_1, for instance the gross count rate, is taken as that quantity whose value, x_1, is not given when a true value, $\tilde{y}$, of the measurand, Y, is specified within the framework of the calculation of the decision threshold and the detection limit. Analogously, the input quantity, X_2, is assigned in a suitable way to the background effect. The data of the other input quantities are taken as given from independent previous investigations.

5.2.2 Model in ionizing radiation measurements

In this International Standard, the measurand, Y, with its true value, $\tilde{y}$, relates to a sample of radioactive material and is determined from counting the gross effect and the background effect with preselection of time or counts. In particular, Y can be the net count rate, ρ_n, or the net activity, A, of the sample. The symbols belonging to the counting of the gross effect and of the background effect are marked in the following by the subscripts g and 0, respectively.

In this International Standard, the model is specified as follows:

$$Y = G(X_1, ..., X_m) = (X_1 - X_2 X_3 - X_4) \cdot \frac{X_6 X_8 \cdots}{X_5 X_7 \cdots} = (X_1 - X_2 X_3 - X_4) \cdot W \quad (4)$$

with

$$W = \frac{X_6 X_8 \cdots}{X_5 X_7 \cdots} \quad (5)$$

$X_1 = \rho_g$ is the gross count rate and $X_2 = \rho_0$ is the background count rate. The other input quantities, X_i, are calibration, correction or influence quantities, or conversion factors, for instance the emission or response probability or, in particular, X_3 is a shielding factor and X_4 an additional background correction quantity. If some of the input quantities are not involved, $x_i = 1$ $(i = 3; i > 4)$, $x_4 = 0$ and $u(x_i) = 0$ shall be set for them. For the count rates, $x_1 = r_g = n_g / t_g$ and $u^2(x_1) = n_g / t_g^2 = r_g / t_g$ as well as $x_2 = r_0 = n_0 / t_0$ and $u^2(x_2) = n_0 / t_0^2 = r_0 / t_0$ apply.

By substituting the estimates, x_i, in Equation (4), the primary estimate, y, of the measurand, Y, gives the result:

$$y = G(x_1, ..., x_m) = (x_1 - x_2 x_3 - x_4) \cdot w = (r_g - r_0 x_3 - x_4) \cdot w = \left(\frac{n_g}{t_g} - \frac{n_0}{t_0} x_3 - x_4 \right) \cdot w \quad (6)$$

with

$$w = \frac{x_6 x_8 \cdots}{x_5 x_7 \cdots} \quad (7)$$

With the partial derivatives:

$$\frac{\partial G}{\partial X_1} = W \, ; \; \frac{\partial G}{\partial X_2} = -X_3 W \, ; \; \frac{\partial G}{\partial X_3} = -X_2 W \, ; \; \frac{\partial G}{\partial X_4} = -W \, ; \; \frac{\partial G}{\partial X_i} = \pm \frac{Y}{X_i} \quad (i \geqslant 5) \quad (8)$$

and by substituting the estimates x_i, w and y, Equation (3) yields the standard uncertainty $u(y)$ of the measurand associated with y:

$$\begin{aligned} u(y) &= \sqrt{w^2 \cdot \left[u^2(x_1) + x_3^2 u^2(x_2) + x_2^2 u^2(x_3) + u^2(x_4) \right] + y^2 u_{rel}^2(w)} \\ &= \sqrt{w^2 \cdot \left[r_g / t_g + x_3^2 r_0 / t_0 + r_0^2 u^2(x_3) + u^2(x_4) \right] + y^2 u_{rel}^2(w)} \end{aligned} \quad (9)$$

where

$$u_{rel}^2(w) = \sum_{i=5}^{m} \frac{u^2(x_i)}{x_i^2} \quad (10)$$

is the sum of the squared relative standard uncertainties of the quantities X_5 to X_m. For $m < 5$, the values $w = 1$ and $u_{rel}^2(w) = 0$ apply.

The estimate x_i and the standard uncertainty $u(x_i)$ of X_i $(i = 3, ..., m)$ are taken as known from previous investigations or as values of experience according to other information. In the previous investigations, x_i can be determined as an arithmetic mean value and $u^2(x_i)$ as an empirical variance (see B.4.1). If necessary,

$u^2(x_i)$ can also be calculated as the variance of a rectangular distribution over the region of the possible values of X_i with the width Δx_i. This yields $u^2(x_i) = (\Delta x_i)^2 / 12$.

For the application of the procedure to particular measurements, including spectrometric measurements, see Annexes B and C.

5.3 Calculation of the standard uncertainty as a function of the measurand

5.3.1 General aspects

For the provision and numerical calculation of the decision threshold in 6.2 and of the detection limit in 6.3, the standard uncertainty of the measurand is needed as a function $\tilde{u}(\tilde{y})$ of the true value $\tilde{y} \geqslant 0$ of the measurand. This function shall be determined in a way similar to $u(y)$ within the framework of the evaluation of the measurements by application of ISO/IEC Guide 98-3; see also References [1, 2]. In most cases, $\tilde{u}(\tilde{y})$ shall be formed as a positive square root of a variance function $\tilde{u}^2(\tilde{y})$ calculated first. This function shall be defined, unique and continuous for all $\tilde{y} \geqslant 0$ and shall not assume negative values.

In most cases, $\tilde{u}(\tilde{y})$ can be explicitly specified, provided that $u(x_1)$ is given as a function $h_1(x_1)$ of x_1. In such cases, y shall be formally replaced by $\tilde{y}$ and Equation (2) shall be solved for x_1. With a specified $\tilde{y}$, the value x_1 can also be calculated numerically from Equation (2); for instance, by means of an iteration procedure, which results in x_1 as a function of $\tilde{y}$ and x_2, ..., x_m. This function shall replace x_1 in Equation (3) and in $u(x_1) = h_1(x_1)$, which finally yields $\tilde{u}(\tilde{y})$ instead of $u(y)$. In the case of the model according to Equation (6) and 5.3.2, one shall proceed in this way. Otherwise, 5.3.3 shall be applied, where $\tilde{u}(\tilde{y})$ follows as an approximation by interpolation from the data y_j and $u(y_j)$ of several measurements.

5.3.2 Explicit calculation

When, in the case of the model according to Equation (6), the standard uncertainty, $u(x_1)$, of the gross count rate $X_1 = \rho_g$, is given as a function $h_1(x_1)$ of the estimate, $x_1 = r_g$, either $h_1(x_1) = \sqrt{x_1 / t_g}$ or $h_1(x_1) = x_1 / \sqrt{n_g}$ applies if the measurement duration, t_g (time preselection), or, respectively, the number, n_g, of recorded pulses (preselection of counts) is specified.

The value y shall be formally replaced by $\tilde{y}$. This allows the elimination of x_1 in the general case and, in particular, of n_g with time preselection and of t_g with preselection of counts in Equation (9) by means of Equation (6). These values are not available when $\tilde{y}$ is specified. This yields in the general case according to Equation (6)

$$x_1 = \tilde{y}/w + x_2 x_3 + x_4 \tag{11}$$

By substituting x_1 according to Equation (11) in the given function $h_1(x_1)$, i.e. with $u^2(x_1) = h_1^2(\tilde{y}/w + x_2 x_3 + x_4)$, the following results from Equation (9):

$$\tilde{u}(\tilde{y}) = \sqrt{w^2 \cdot \left[h_1^2(\tilde{y}/w + x_2 x_3 + x_4) + x_3^2 u^2(x_2) + x_2^2 u^2(x_3) + u^2(x_4) \right] + \tilde{y}^2 u_{\mathrm{rel}}^2(w)} \tag{12}$$

With time preselection and because of $x_1 = n_g / t_g$ and $x_2 = r_0$,

$$n_g = t_g \cdot (\tilde{y}/w + r_0 x_3 + x_4) \tag{13}$$

is obtained from Equation (11). Then, with $h_1^2(x_1) = x_1/t_g = n_g/t_g^2$ and by substituting n_g according to Equation (13) and with $u^2(x_2) = r_0/t_0$, Equation (12) leads to

$$\tilde{u}(\tilde{y}) = \sqrt{w^2 \cdot \left[(\tilde{y}/w + r_0 x_3 + x_4)/t_g + x_3^2 r_0/t_0 + r_0^2 u^2(x_3) + u^2(x_4) \right] + \tilde{y}^2 u_{rel}^2(w)} \tag{14}$$

With preselection of counts,

$$t_g = \frac{n_g}{\tilde{y}/w + r_0 x_3 + x_4} \tag{15}$$

is analogously obtained. Then, with $h_1^2(x_1) = x_1/t_g = n_g/t_g^2$ and by substituting t_g according to Equation (15) and with $u^2(x_2) = r_0^2/n_0$, Equation (12) leads to

$$\tilde{u}(\tilde{y}) = \sqrt{w^2 \cdot \left[(\tilde{y}/w + r_0 x_3 + x_4)^2/n_g + x_3^2 r_0^2/n_0 + r_0^2 u^2(x_3) + u^2(x_4) \right] + \tilde{y}^2 u_{rel}^2(w)} \tag{16}$$

Equation (22) has a solution, which is the detection limit, $y^\#$, if, with time preselection, the following condition is satisfied:

$$k_{1-\beta} u_{rel}(w) < 1 \tag{17}$$

or with preselection of counts, the following condition is satisfied:

$$k_{1-\beta} \cdot \sqrt{\frac{1}{n_g} + u_{rel}^2(w)} < 1 \tag{18}$$

Otherwise, it can happen that a detection limit does not exist because of too great an uncertainty of the quantities X_5 to X_m, summarily expressed by $u_{rel}(w)$. The condition according to Equation (17) also applies in the case of Equation (12) if $h_1(x_1)$ increases for growing x_1 more slowly than x_1, i.e. if $h_1(x_1)/x_1 \to 0$ for $x_1 \to \infty$.

5.3.3 Approximations

It is often sufficient to use the following approximations for the function $\tilde{u}(\tilde{y})$, in particular, if the standard uncertainty, $u(x_1)$, is not known as a function $h_1(x_1)$. A prerequisite is that measurement result, y_j, and associated standard uncertainties, $u(y_j)$, calculated according to 5.1 and 5.2 from previous measurements of the same kind, are already available $(j = 0,1,2,...)$. The measurements shall be carried out on different samples with differing activities, but in other respects as far as possible under similar conditions. One of the measurements can be a background effect measurement or a blank measurement with $\tilde{y} = 0$ and, for instance, $j = 0$. Then, $y_0 = 0$ shall be set and $\tilde{u}(0) = u(y_0)$. The measurement currently carried out can be taken as a further measurement with $j = 1$.

The function $\tilde{u}(\tilde{y})$ often shows a rather slow increase. Therefore, the approximation $\tilde{u}(\tilde{y}) = u(y_1)$ is sufficient in some of these cases, especially if the primary measurement result, y_1, of the measurand is not much larger than the associated standard uncertainty $u(y_1)$.

If only $\tilde{u}(0) = u(y_0)$, $y_1 > 0$ and $u(y_1)$ are known, the following linear interpolation often suffices:

$$\tilde{u}^2(\tilde{y}) = \tilde{u}^2(0)(1 - \tilde{y}/y_1) + u^2(y_1)\tilde{y}/y_1 \tag{19}$$

