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Statistical interpretation of data —

Part 4: **Detection and treatment of outliers**

Interprétation statistique des données —

Partie 4: Détection et traitement des valeurs aberrantes

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Foreword

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ISO 16269-4 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*.

ISO 16269 consists of the following parts, under the general title *Statistical interpretation of data*:

— *Part 4: Detection and treatment of outliers*

— *Part 6: Determination of statistical tolerance intervals*

— *Part 7: Median — Estimation and confidence intervals*

— *Part 8: Determination of prediction intervals*

Introduction

Identification of outliers is one of the oldest problems in interpreting data. Causes of outliers include measurement error, sampling error, intentional under- or over-reporting of sampling results, incorrect recording, incorrect distributional or model assumptions of the data set, and rare observations, etc.

Outliers can distort and reduce the information contained in the data source or generating mechanism. In the manufacturing industry, the existence of outliers will undermine the effectiveness of any process/product design and quality control procedures. Possible outliers are not necessarily *bad* or *erroneous*. In some situations, an outlier may carry essential information and thus it should be identified for further study.

The study and detection of outliers from measurement processes leads to better understanding of the processes and proper data analysis that subsequently results in improved inferences.

In view of the enormous volume of literature on the topic of outliers, it is of great importance for the international community to identify and standardize a sound subset of methods used in the identification and treatment of outliers. The implementation of this part of ISO 16269 enables business and industry to recognize the data analyses conducted across member countries or organizations.

Six annexes are provided. Annex A provides an algorithm for computing the test statistic and critical values of a procedure in detecting outliers in a data set taken from a normal distribution. Annexes B, D and E provide the tables needed to implement the recommended procedures. Annex C provides the tables and statistical theory that underlie the construction of modified box plots in outlier detection. Annex F provides a structured guide and flow chart to the procedures recommended in this part of ISO 16269.

Statistical interpretation of data —

Part 4: Detection and treatment of outliers

1 Scope

This part of ISO 16269 provides detailed descriptions of sound statistical testing procedures and graphical data analysis methods for detecting outliers in data obtained from measurement processes. It recommends sound robust estimation and testing procedures to accommodate the presence of outliers.

This part of ISO 16269 is primarily designed for the detection and accommodation of outlier(s) from univariate data. Some guidance is provided for multivariate and regression data.

2 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

2.1
sample
data set
subset of a population made up of one or more sampling units

NOTE 1 The sampling units could be items, numerical values or even abstract entities depending on the population of interest.

NOTE 2 A sample from a **normal** (2.22), a **gamma** (2.23), an **exponential** (2.24), a **Weibull** (2.25), a **lognormal** (2.26) or a **type I extreme value** (2.27) population will often be referred to as a normal, a gamma, an exponential, a Weibull, a lognormal or a type I extreme value sample, respectively.

2.2
outlier
member of a small subset of observations that appears to be inconsistent with the remainder of a given **sample** (2.1)

NOTE 1 The classification of an observation or a subset of observations as outlier(s) is relative to the chosen model for the population from which the data set originates. This or these observations are not to be considered as genuine members of the main population.

NOTE 2 An outlier may originate from a different underlying population, or be the result of incorrect recording or gross measurement error.

NOTE 3 The subset may contain one or more observations.

2.3
masking
presence of more than one **outlier** (2.2), making each outlier difficult to detect

2.4
some-outside rate
probability that one or more observations in an uncontaminated sample will be wrongly classified as **outliers** (2.2)

2.5
outlier accommodation method
method that is insensitive to the presence of **outliers** (2.2) when providing inferences about the population

2.6
resistant estimation
estimation method that provides results that change only slightly when a small portion of the data values in a **data set** (2.1) is replaced, possibly with very different data values from the original ones

2.7
robust estimation
estimation method that is insensitive to small departures from assumptions about the underlying probability model of the data

NOTE An example is an estimation method that works well for, say, a **normal distribution** (2.22), and remains reasonably good if the actual distribution is skew or heavy-tailed. Classes of such methods include the L-estimation [weighted average of **order statistics** (2.10)] and M-estimation methods (see Reference [9]).

2.8
rank
position of an observed value in an ordered set of observed values

NOTE 1 The observed values are arranged in ascending order (counting from below) or descending order (counting from above).

NOTE 2 For the purposes of this part of ISO 16269, identical observed values are ranked as if they were slightly different from one another.

2.9
depth
⟨box plot⟩ smaller of the two **ranks** (2.8) determined by counting up from the smallest value of the **sample** (2.1), or counting down from the largest value

NOTE 1 The depth may not be an integer value (see Annex C).

NOTE 2 For all summary values other than the **median** (2.11), a given depth identifies two (data) values, one below the median and the other above the median. For example, the two data values with depth 1 are the smallest value (minimum) and largest value (maximum) in the given **sample** (2.1).

2.10
order statistic
statistic determined by its ranking in a non-decreasing arrangement of random variables

[ISO 3534-1:2006, definition 1.9]

NOTE 1 Let the observed values of a random sample be $\{x_1, x_2, \ldots, x_n\}$. Reorder the observed values in non-decreasing order designated as $x_{(1)} \leqslant x_{(2)} \leqslant \ldots \leqslant x_{(k)} \leqslant \ldots \leqslant x_{(n)}$; then $x_{(k)}$ is the observed value of the kth order statistic in a sample of size n.

NOTE 2 In practical terms, obtaining the order statistics for a **sample** (2.1) amounts to sorting the data as formally described in Note 1.

2.11
median
sample median
median of a set of numbers
Q_2
[$(n+1)/2$]th **order statistic** (2.10), if the sample size n is odd; sum of the [$n/2$]th and the [$(n/2)+1$]th order statistics divided by 2, if the sample size n is even

[ISO 3534-1:2006, definition 1.13]

NOTE The sample median is the second quartile (Q_2).

2.12
first quartile
sample lower quartile
Q_1
for an odd number of observations, **median** (2.11) of the smallest $(n-1)/2$ observed values; for an even number of observations, median of the smallest $n/2$ observed values

NOTE 1 There are many definitions in the literature of a sample quartile, which produce slightly different results. This definition has been chosen both for its ease of application and because it is widely used.

NOTE 2 Concepts such as hinges or **fourths** (2.19 and 2.20) are popular variants of quartiles. In some cases (see Note 3 to 2.19), the first quartile and the **lower fourth** (2.19) are identical.

2.13
third quartile
sample upper quartile
Q_3
for an odd number of observations, median of the largest $(n-1)/2$ observed values; for an even number of observations, median of the largest $n/2$ observed values

NOTE 1 There are many definitions in the literature of a sample quartile, which produce slightly different results. This definition has been chosen both for its ease of application and because it is widely used.

NOTE 2 Concepts such as hinges or **fourths** (2.19 and 2.20) are popular variants of quartiles. In some cases (see Note 3 to 2.20), the third quartile and the upper fourth (2.20) are identical.

2.14
interquartile range
IQR
difference between the **third quartile** (2.13) and the **first quartile** (2.12)

NOTE 1 This is one of the widely used statistics to describe the spread of a data set.

NOTE 2 The difference between the **upper fourth** (2.20) and the **lower fourth** (2.19) is called the fourth-spread and is sometimes used instead of the interquartile range.

2.15
five-number summary
the minimum, **first quartile** (2.12), **median** (2.11), **third quartile** (2.13), and maximum

NOTE The five-number summary provides numerical information about the location, spread and range.

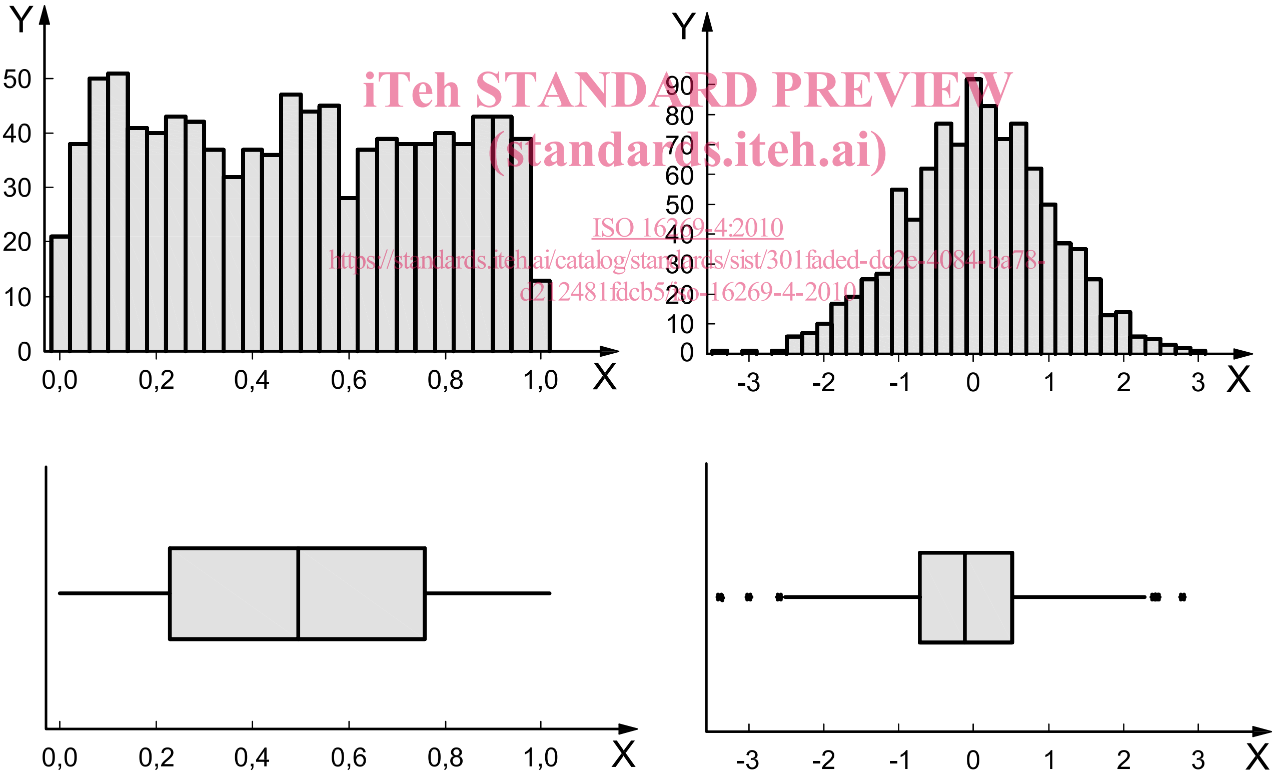
2.16
box plot

horizontal or vertical graphical representation of the **five-number summary** (2.15).

NOTE 1 For the horizontal version, the **first quartile** (2.12) and the **third quartile** (2.13) are plotted as the left and right sides, respectively, of a box, the **median** (2.11) is plotted as a vertical line across the box, the whiskers stretching downwards from the first quartile to the smallest value at or above the **lower fence** (2.17) and upwards from the third quartile to the largest value at or below the **upper fence** (2.18), and value(s) beyond the lower and upper fences are marked separately as **outlier(s)** (2.2). For the vertical version, the first and third quartiles are plotted as the bottom and the top, respectively, of a box, the median is plotted as a horizontal line across the box, the whiskers stretching downwards from the first quartile to the smallest value at or above the lower fence and upwards from the third quartile to the largest value at or below the upper fence and value(s) beyond the lower and upper fences are marked separately as outlier(s).

NOTE 2 The box width and whisker length of a box plot provide graphical information about the location, spread, skewness, tail lengths, and outlier(s) of a sample. Comparisons between box plots and the density function of a) uniform, b) bell-shaped, c) right-skewed, and d) left-skewed distributions are given in the diagrams in Figure 1. In each distribution, a histogram is shown above the boxplot.

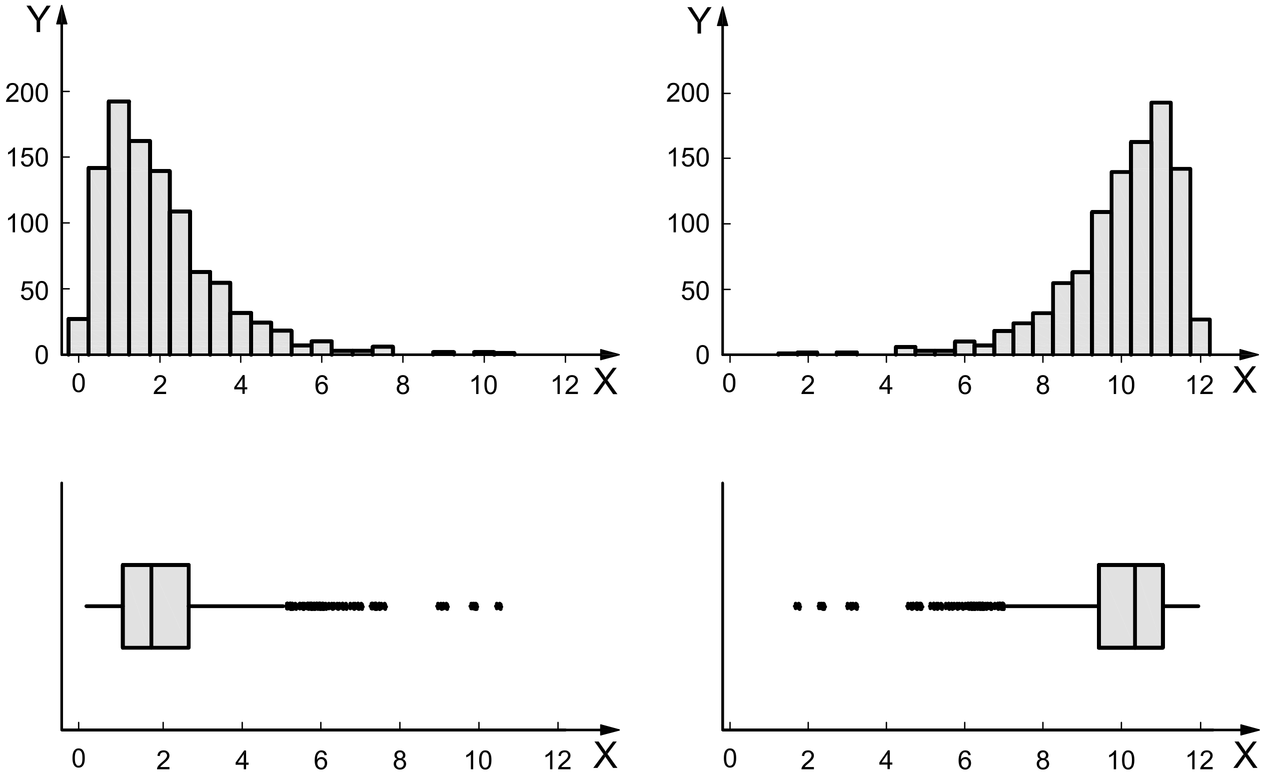
NOTE 3 A box plot constructed with its **lower fence** (2.17) and **upper fence** (2.18) evaluated by taking k to be a value based on the sample size n and the knowledge of the underlying distribution of the sample data is called a modified box plot (see example, Figure 2). The construction of a modified box plot is given in 4.4.



a) Uniform distribution

b) Bell-shaped distribution

Figure 1 (*continued*)



c) Right-skewed distribution

d) Left-skewed distribution

Key

X data values

Y frequency

In each distribution, a histogram is shown above the box plot.

Figure 1 — Box plots and histograms for a) uniform, b) bell-shaped, c) right-skewed, and d) left-skewed distributions

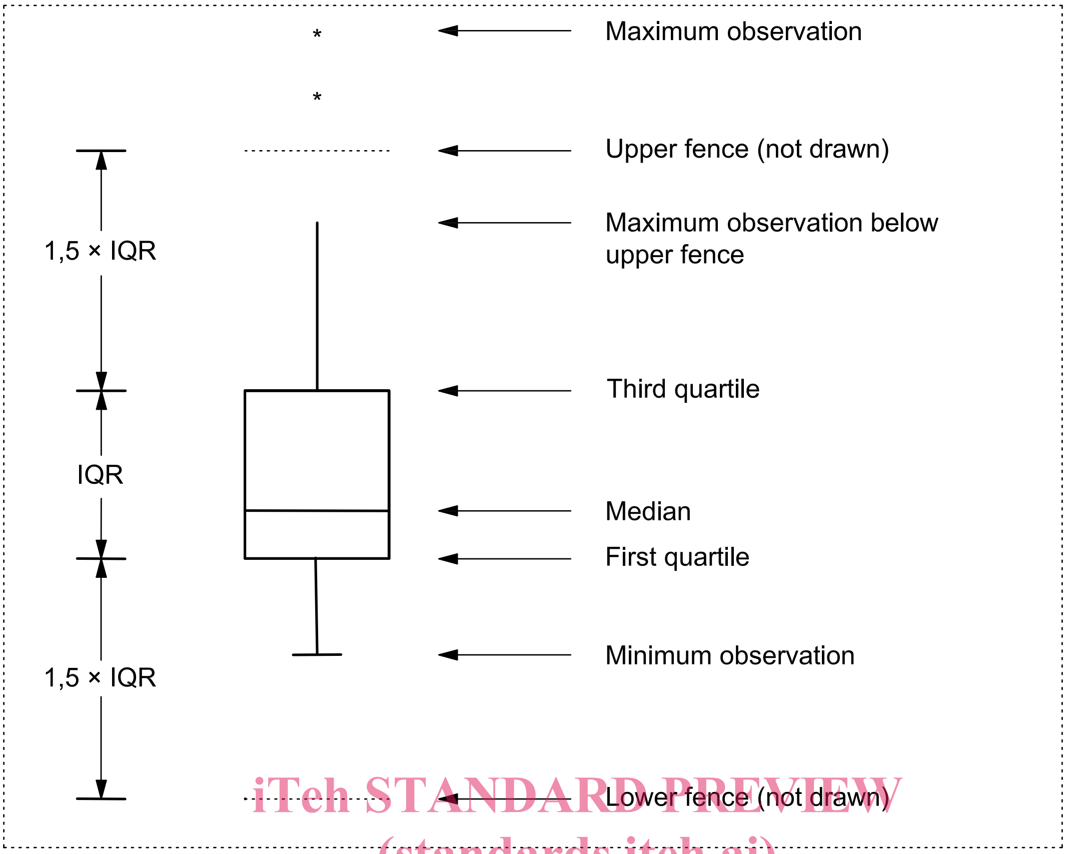


Figure 2 — Modified box plot with lower and upper fences

2.17
lower fence
lower outlier cut-off
lower adjacent value
value in a **box plot** (2.16) situated k times the **interquartile range** (2.14) below the **first quartile** (2.12), with a predetermined value of k

NOTE In proprietary statistical packages, the lower fence is usually taken to be $Q_1 - k(Q_3 - Q_1)$ with k taken to be either 1,5 or 3,0. Classically, this fence is called the “inner lower fence” when k is 1,5, and “outer lower fence” when k is 3,0.

2.18
upper fence
upper outlier cut-off
upper adjacent value
value in a box plot situated k times the **interquartile range** (2.14) above the **third quartile** (2.13), with a predetermined value of k

NOTE In proprietary statistical packages, the upper fence is usually taken to be $Q_3 + k(Q_3 - Q_1)$, with k taken to be either 1,5 or 3,0. Classically, this fence is called the “inner upper fence” when k is 1,5, and the “outer upper fence” when k is 3,0.

2.19
lower fourth

$x_{L:n}$

for a set $x_{(1)} \leqslant x_{(2)} \leqslant \ldots \leqslant x_{(n)}$ of observed values, the quantity 0,5 $[x_{(i)} + x_{(i+1)}]$ when $f = 0$ or $x_{(i+1)}$ when $f > 0$, where i is the integral part of $n/4$ and f is the fractional part of $n/4$

NOTE 1 This definition of a lower fourth is used to determine the recommended values of k_L and k_U given in Annex C and is the default or optional setting in some widely used statistical packages.

NOTE 2 The lower fourth and the **upper fourth** (2.20) as a pair are sometimes called hinges.

NOTE 3 The lower fourth is sometimes referred to as the **first quartile** (2.12).

NOTE 4 When $f = 0$, 0,5 or 0,75, the lower fourth is identical to the first quartile. For example:

Sample size n	i = **integral part of** n**/4**	f = **fractional part of** n**/4**	**First quartile**	**Lower fourth**
9	2	0,25	$[x_{(2)} + x_{(3)}]/2$	$x_{(3)}$
10	2	0,50	$x_{(3)}$	$x_{(3)}$
11	2	0,75	$x_{(3)}$	$x_{(3)}$
12	3	0	$[x_{(3)} + x_{(4)}]/2$	$[x_{(3)} + x_{(4)}]/2$

2.20
upper fourth

$x_{U:n}$

for a set $x_{(1)} \leqslant x_{(2)} \leqslant \ldots \leqslant x_{(n)}$ of observed values, the quantity 0,5 $[x_{(n-i)} + x_{(n-i+1)}]$ when $f = 0$ or $x_{(n-i)}$ when $f > 0$, where i is the integral part of $n/4$ and f is the fractional part of $n/4$

NOTE 1 This definition of an upper fourth is used to determine the recommended values of k_L and k_U given in Annex C and is the default or optional setting in some widely used statistical packages.

NOTE 2 The **lower fourth** (2.19) and the upper fourth as a pair are sometimes called hinges.

NOTE 3 The upper fourth is sometimes referred to as the **third quartile** (2.13).

NOTE 4 When $f = 0$, 0,5 or 0,75, the upper fourth is identical to the third quartile. For example:

Sample size n	i = **integral part of** n**/4**	f = **fractional part of** n**/4**	**Third quartile**	**Upper fourth**
9	2	0,25	$[x_{(7)} + x_{(8)}]/2$	$x_{(7)}$
10	2	0,50	$x_{(8)}$	$x_{(8)}$
11	2	0,75	$x_{(9)}$	$x_{(9)}$
12	3	0	$[x_{(9)} + x_{(10)}]/2$	$[x_{(9)} + x_{(10)}]/2$

2.21
Type I error
rejection of the null hypothesis when in fact it is true

[ISO 3534-1:2006, definition 1.46]

NOTE 1 A Type I error is an incorrect decision. Hence, it is desired to keep the probability of making such an incorrect decision as small as possible.

NOTE 2 It is possible in some situations (for example, testing the binomial parameter p) that a pre-specified significance level such as 0,05 is not attainable due to discreteness in outcomes.

2.22
normal distribution
Gaussian distribution
continuous distribution having the probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

where $-\infty < x < \infty$ and with parameters $-\infty < \mu < \infty$ and $\sigma > 0$

[ISO 3534-1:2006, definition 2.50]

NOTE 1 The location parameter μ is the mean and the scale parameter σ is the standard deviation of the normal distribution.

NOTE 2 A normal sample is a random **sample** (2.1) taken from a population that follows a normal distribution.

2.23
gamma distribution
continuous distribution having the probability density function

$$f(x) = \frac{x^{\alpha-1}\exp(-x/\beta)}{\beta^{\alpha}\Gamma(\alpha)}$$

where $x > 0$ and parameters $\alpha > 0$, $\beta > 0$

[ISO 3534-1:2006, definition 2.56]

NOTE 1 The gamma distribution is used in reliability applications for modelling time to failure. It includes the **exponential distribution** (2.24) as a special case as well as other cases with failure rates that increase with age.

NOTE 2 The mean of the gamma distribution is $\alpha\beta$. The variance of the gamma distribution is $\alpha\beta^2$.

NOTE 3 A gamma sample is a random **sample** (2.1) taken from a population that follows a gamma distribution.

2.24
exponential distribution
continuous distribution having the probability density function

$$f(x) = \beta^{-1}\exp(-x/\beta)$$

where $x > 0$ and with parameter $\beta > 0$

[ISO 3534-1:2006, definition 2.58]

NOTE 1 The exponential distribution provides a baseline in reliability applications, corresponding to the case of “lack of ageing” or memory-less property.

NOTE 2 The mean of the exponential distribution is β. The variance of the exponential distribution is β^2.

NOTE 3 An exponential sample is a random **sample** (2.1) taken from a population that follows an exponential distribution.

2.25
Weibull distribution
type III extreme-value distribution
continuous distribution having the distribution function

$$F(x) = 1 - \exp\left\{-\left(\frac{x-\theta}{\beta}\right)^{\kappa}\right\}$$

where $x > \theta$ with parameters $-\infty < \theta < \infty$, $\beta > 0$, $\kappa > 0$

[ISO 3534-1:2006, definition 2.63]

NOTE 1 In addition to serving as one of the three possible limiting distributions of extreme order statistics, the Weibull distribution occupies a prominent place in diverse applications, particularly reliability and engineering. The Weibull distribution has been demonstrated to provide usable fits to a variety of data sets.

NOTE 2 The parameter θ is a location or threshold parameter in the sense that it is the minimum value that a Weibull variate can achieve. The parameter β is a scale parameter (related to the standard deviation of a Weibull variate). The parameter κ is a shape parameter.

NOTE 3 A Weibull sample is a random **sample** (2.1) taken from a population that follows a Weibull distribution.

2.26
lognormal distribution
continuous distribution having the probability density function

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{(\ln x - \mu)^2}{2\sigma^2}\right\}$$

where $x > 0$ and with parameters $-\infty < \mu < \infty$ and $\sigma > 0$

[ISO 3534-1:2006, definition 2.52]

2.27
type I extreme-value distribution
Gumbel distribution
continuous distribution having the distribution function

$$F(x) = \exp\left\{-\mathrm{e}^{-(x-\mu)/\sigma}\right\}$$

where $-\infty < x < \infty$ and with parameters $-\infty < \mu < \infty$ and $\sigma > 0$

NOTE Extreme-value distributions provide appropriate reference distributions for the extreme **order statistics** (2.10) $x_{(1)}$ and $x_{(n)}$.

[ISO 3534-1:2006, definition 2.61]