

Designation: D 6415/D 6415M – 06

# Standard Test Method for Measuring the Curved Beam Strength of a Fiber-Reinforced Polymer-Matrix Composite<sup>1</sup>

This standard is issued under the fixed designation D 6415/D 6415M; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

# 1. Scope

1.1 This test method determines the curved beam strength of a continuous fiber-reinforced composite material using a  $90^{\circ}$  curved beam specimen (Fig. 1 and Fig. 2). The curved beam consists of two straight legs connected by a  $90^{\circ}$  bend with a 6.4-mm [0.25 mm] inner radius. An out-of-plane (through-the-thickness) tensile stress is produced in the curved region of the specimen when force is applied. This test method is limited to use with composites consisting of layers of fabric or layers of unidirectional fibers.

1.2 This test method may also be used to measure the interlaminar tensile strength if a unidirectional specimen is used where the fibers run continuously along the legs and around the bend.

1.3 This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.

1.4 The values stated in either SI units or inch-pound units are to be regarded separately as standard. Within the text the inch-pound units are shown in brackets. The values stated in each system are not exact equivalents; therefore, each system must be used independently of the other. Combining values from the two systems may result in nonconformance with the standard.

# 2. Referenced Documents

2.1 ASTM Standards: <sup>2</sup>

- D 792 Test Methods for Density and Specific Gravity (Relative Density) of Plastics by Displacement
- D 883 Terminology Relating to Plastics
- D 3171 Test Methods for Constituent Content of Composite Materials

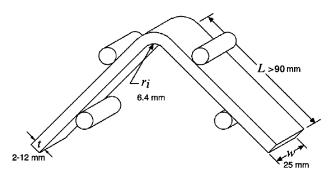


FIG. 1 Test Specimen Geometry (SI units)

D 3878 Terminology for Composite Materials

- D 5229/D 5229M Test Method for Moisture Absorption Properties and Equilibrium Conditioning of Polymer Matrix Composite Materials
- D 5687/D 5687M Guide for Preparation of Flat Composite Panels with Processing Guidelines for Specimen Preparation
- **E** 4 Practices for Force Verification of Testing Machines
- E 6 Terminology Relating to Methods of Mechanical Testing
- E 122 Practice for Calculating Sample Size to Estimate, With a Specified Tolerable Error, the Average for a Characteristic of a Lot or Process
- E 177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods
- E 456 Terminology Relating to Quality and Statistics

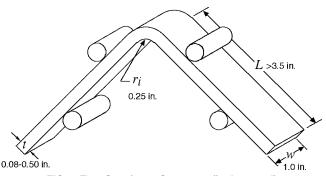


FIG. 2 Test Specimen Geometry (inch-pound)

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<sup>&</sup>lt;sup>1</sup> This test method is under the jurisdiction of ASTM Committee D30 on Composite Materials and is the direct responsibility of D30.06 on Interlaminar Properties.

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<sup>&</sup>lt;sup>2</sup> For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

E 1309 Guide for Identification of Fiber-Reinforced Polymer-Matrix Composite Materials in Databases

E 1434 Guide for Recording Mechanical Test Data of Fiber-Reinforced Composite Materials in Databases

E 1471 Guide for Identification of Fibers, Fillers, and Core Materials in Computerized Material Property Databases

### 3. Terminology

3.1 *Definitions*—Terminology D 3878 defines terms relating to high-modulus fibers and their composites. Terminology D 883 defines terms relating to plastics. Terminology E 6 defines terms relating to mechanical testing. Terminology E 456 and Practice E 177 define terms relating to statistics. In the event of a conflict between terms, Terminology D 3878 shall have precedence over the other terminologies.

3.2 Definitions of Terms Specific to This Standard:

NOTE 1—If the term represents a physical quantity, its analytical dimensions are stated immediately following the term (or letter symbol) in fundamental dimension form, using the following ASTM standard symbology for fundamental dimensions, shown within square brackets: [M] for mass, [L] for length, [T] for time,  $[\theta]$  for thermodynamic temperature, and [nd] for nondimensional quantities. Use of these symbols is restricted to analytical dimensions when used with square brackets, as the symbols may have other definitions when used without the brackets.

3.2.1 applied moment,  $M [ML^2T^{-2}]$ , *n*—the moment applied to the curved test section of the specimen.

3.2.2 curved beam strength, CBS  $[ML^{1}T^{-2}]$ , *n*—the moment per unit width, *M/w*, applied to the curved test section which causes a sharp decrease in applied load or delamination(s) to form.

3.2.3 *interlaminar tensile strength*,  $F^{3u}$  [*ML*<sup>-1</sup>*T*<sup>-2</sup>], *n*—the strength of the composite material in the out-of-plane (through-the-thickness) direction.

3.3 Symbols:

3.3.1 *CBS* = curved beam strength (see 3.2.2). 3.3.2 CV = coefficient of variation statistic of a sample

population for a given property (in percent). 3.3.3  $d_x$ ,  $d_y$  = horizontal and vertical distances between two adjacent top and bottom loading bars, respectively.

3.3.4 D = diameter of the cylindrical loading bars on the four-point-bending fixture.

3.3.5  $E_{i}$ ,  $E_{\theta}$  = moduli in the radial and tangential directions, respectively.

3.3.6  $F^{3u}$  = interlaminar tensile strength (see 3.2.3).

3.3.7 g = parameter used in strength calculation.

3.3.8  $l_b$  = distance between the centerlines of the bottom loading bars on the four-point-bending fixture.

3.3.9  $l_o$  = distance along the specimen's leg between the centerlines of a top and bottom loading bar.

3.3.10  $l_t$  = distance between the centerlines of the top loading bars on the four-point-bending fixture.

3.3.11 M = applied moment (see 3.2.1).

3.3.12 P = total force applied to the four-point-bending fixture.

3.3.13  $P^{\text{max}}$  = maximum force applied to the four-pointbending fixture before failure.

3.3.14  $P_b$  = force applied to the specimen by a single loading bar.

3.3.15 r,  $\theta$  = cylindrical coordinates of any point in the curved segment.

3.3.16  $r_i$ ,  $r_o$  = inner and outer radii of curved segment.

3.3.17  $r_m$  = radial position of the maximum interlaminar (radial) tensile stress.

3.3.18  $S_{n-1}$  = standard deviation statistic of a sample population for a given property.

3.3.19 t = average thickness of specimen.

 $3.3.20 \ w =$  width of the specimen.

3.3.21  $x_1$  = test result for an individual specimen from the sample population for a given property.

3.3.22  $\bar{x}$  = mean or average (estimate of mean) of a sample population for a given property.

3.3.23  $\Delta$  = relative displacement between the top and bottom halves of the four-point-bending fixture.

3.3.24  $\kappa$  = parameter used in strength calculation.

3.3.25  $\rho$  = parameter used in strength calculation.

3.3.26  $\phi$  = angle from horizontal of the specimen legs in degrees.

3.3.27  $\phi_i$  = angle from horizontal of the specimen legs at the start of the test in degrees (0.5 × angle between the legs). 3.3.28  $\sigma_r$  = radial stress component in curved segment.

# 4. Summary of Test Method

4.1 A 90° curved-beam test specimen is used to measure the curved beam strength of a continuous-fiber-reinforced composite material (Fig. 1 and Fig. 2). The curved beam strength represents the moment per unit width which causes a delamination(s) to form. If the curved beam is unidirectional with all fibers running continuously along the legs and around the bend and an appropriate failure mode is observed, an interlaminar (through-the-thickness) tensile strength may also be calculated. The curved beam is uniform thickness and consists of two straight legs connected by a 90° bend with a 6.4-mm [0.25-in.] inner radius. The curved beam is loaded in four-point bending to apply a constant bending moment across the curved test section. An out-of-plane tensile stress is produced in the curved region of the specimen to cause the failure.

#### 5. Significance and Use

5.1 Out-of-plane stress analyses are not easily performed. Failure criteria are varied and poorly validated. Interlaminar allowables are not readily available. However, stress analysts routinely encounter structural details in which they cannot ignore the out-of-plane loads. This test method is designed to produce out-of-plane structural failure data for structural design and analysis, quality assurance, and research and development. For unidirectional specimens, this test method is designed to produce interlaminar tensile strength data. Factors that influence the curved beam strength and should therefore be reported include the following: material, methods of material preparation, methods of processing and specimen fabrication, specimen preparation, specimen conditioning, environment of testing, speed of testing, time at temperature, void content, and volume percent reinforcement.

#### 6. Interferences

6.1 Failure in non-unidirectional specimens may be initiated from matrix cracks or free edge stresses. Consequently, the interlaminar strength calculated from non-unidirectional specimens may be in error.

6.2 The stress state of a curved beam in four-point bending is complex. Circumferential tensile stresses are produced along the inner surface, and circumferential compressive stresses are produced on the outer surface. The radial tensile stress ranges from zero at the inner and outer surfaces to a peak in the middle third of the thickness. Consequently, the failure should be carefully observed to ensure that a delamination(s) is produced across the width before the failure data are used.

6.3 Since stresses are nonuniform and the critical stress state occurs in a small region, the location of architectural characteristics of the specimen (for example, fabric weave, and tow intersections) may affect the curved beam strength.

6.4 Nonlaminated, 3-D reinforced, or textile composites may fail by different mechanisms than laminates. The most critical damage may be in the form of matrix cracking or fiber failure, or both, rather than delaminations.

6.5 *Material and Specimen Preparation*—Poor material fabrication practices, lack of control of fiber alignment, and damage induced by improper coupon machining are known causes of high material data scatter in composites in general. Important aspects of specimen preparation that contribute to data scatter include thickness variation, curve geometry, surface roughness, and failure to maintain the dimensions specified in section 8.2

6.6 The curved beam and interlaminar strengths measured using this test method are extremely sensitive to reinforcement volume and void content. Consequently, the test results may reflect manufacturing quality as much as material properties. Both reinforcement volume and void content shall be reported.

6.7 Specimens with low bending stiffness, or high values of interlaminar strength, or both, may exhibit excessive bending of the specimen legs during flexural loading. This can create large errors in the calculated bending moment, resulting in unconservative strength calculations. A recommended limitation on crosshead displacement is provided in Section 12. Although outside of the scope of this test method, a doubler may be added to the legs to reduce the flexure.

#### 7. Apparatus

7.1 *Testing Machine*—The testing machine shall be in conformance with Practices E 4, and shall satisfy the following requirements:

7.1.1 *Testing Machine Configuration*—The testing machine shall have both an essentially stationary head and a movable head.

7.1.2 *Drive Mechanism*—The testing machine drive mechanism shall be capable of imparting to the movable head a controlled velocity with respect to the stationary head. The velocity of the movable head shall be capable of being regulated in accordance with .

7.1.3 Force Indicator—The testing machine force-sensing device shall be capable of indicating the total force being carried by the test specimen. This device shall be essentially free from inertia lag at the specified rate of testing and shall indicate the force with an accuracy over the force range(s) of interest of within  $\pm 1$  % of the indicated value.

7.1.4 *Grips*—Each head of the testing machine shall have a means to hold half of the four-point-bending fixture firmly in place. A convenient means of providing an attachment point for each fixture half is through the use of a metal "T" in each grip. The lower part of the "T" is clamped in the grips, and the top part of the "T" provides a flat attachment surface for each fixture half.

7.2 Four-Point-Bending Fixture—A four-point-bending test apparatus as shown in Fig. 3 shall be used to load the specimen. Machine drawings for example fixtures are shown in the appendix. Other designs that perform the necessary functions are acceptable. The cylindrical loading bars shall have diameters. D, of 6 to 10 mm [0.25 to 0.40 in.] and be mounted on roller bearings. The distance between the bar centers shall be  $100 \pm 2 \text{ mm} [4.00 \pm 0.05 \text{ in.}] (l_b)$  for the bottom fixture and  $75 \pm 2 \text{ mm} [3.00 \pm 0.05 \text{ in.}] (l_t)$  for the top fixture.

7.3 Displacement Indicator—The relative axial displacement between the upper and lower fixtures may be estimated as the crosshead travel provided the deformation of the testing machine and support fixture is less than 2 % of the crosshead travel. If not, this displacement shall be obtained from a properly calibrated external gage or transducer located between the two fixtures. The displacement indicator shall indicate the displacement with an accuracy of  $\pm 1$  % of the thickness of the specimen.

7.4 Force Versus Displacement (P Versus  $\Delta$ ) Record—An X-Y plotter, or similar device, shall be used to make a permanent record during the test of force versus displacement. Alternatively, the data may be stored digitally and postprocessed.

7.5 *Micrometers*—The micrometer(s) shall use a 4 to 6 mm [0.16 to 0.25 in.] ball-interface on irregular surfaces such as the bag-side of a laminate, and a flat anvil interface on machined or very-smooth tooled surfaces. The accuracy of the instruments shall be suitable for reading to within 1 % of the sample width and thickness. For typical specimen geometries, an instrument with an accuracy of  $\pm 25 \ \mu m \ [\pm 0.001 \ in.]$  is desirable for both thickness and width measurements.

7.6 *Calipers*—The caliper(s) shall use a knife-edge interface on the curved surfaces of the specimen and a flat anvil

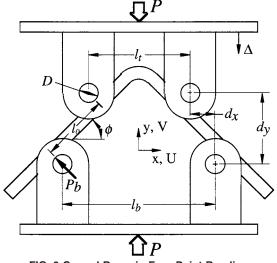


FIG. 3 Curved Beam in Four-Point Bending

interface on machined or very-smooth tooled surfaces. The accuracy of the instruments shall be suitable for reading to within 1 % of the sample width and thickness. For typical specimen geometries, an instrument with an accuracy of  $\pm 25$  µm [ $\pm 0.001$  in.] is desirable for both thickness and width measurements.

7.7 Conditioning Chamber—When conditioning materials at non-laboratory environments, a temperature/vapor-level controlled environmental conditioning chamber is required that shall be capable of maintaining the required temperature to within  $\pm$  3°C [ $\pm$  5°CF] and the required relative humidity level to be within  $\pm$  3%. Chamber conditions shall be monitored either on an automated continuous basis or on a manual basis at regular intervals.

7.8 *Environmental Test Chamber*—An environmental test chamber is required for test environments other than ambient testing laboratory conditions. This chamber shall be capable of maintaining the gage section of the test specimen at the required test environment during the mechanical test.

### 8. Sampling and Test Specimens

8.1 *Sampling*—Test at least five specimens per condition unless valid results can be gained through the use of fewer specimens, such as the case of a designed experiment. For statistically significant data, the procedures outlined in Practice E 122 should be consulted. Report the method of sampling.

#### 8.2 Geometry

8.2.1 Dimensions—Specimen geometry is shown in Fig. 1 and Fig. 2. The laminate shall have a cross section of constant thickness. The thickness shall be 2 to 12 mm [0.08 to 0.50 in.] The width shall be  $25 \pm 1$  mm [1.00  $\pm$  0.04 in.] wide with an inner radius of  $6.4 \pm 0.2$  mm [0.25  $\pm$  0.01 in.] at the bend. The loading leg length shall be a minimum of 90 mm [3.5 in.] and short enough to prevent contact with the fixture base. The variation in thickness for any given specimen shall not exceed 5% of the nominal thickness. The angle between the two loading legs shall be  $90 \pm 3^\circ$ . This angle is often different from 90° because of specimen "spring back" upon removal from the tool after curing.

# 8.2.2 Stacking Sequence

8.2.2.1 *Curved Beam Strength Measurement*—Any stacking sequence that can be manufactured to the specified dimensions may be used.

8.2.2.2 Interlaminar Strength Measurement—Specimens shall have a unidirectional stacking sequence with the fibers running circumferentially around the curved region. For comparison screening of interlaminar strength, a specimen with an appropriate number of plies to produce a thickness of  $4.2 \pm 0.2$  mm [0.17  $\pm$  0.008 in.] is suggested.

8.3 *Specimen Preparation*—Guide D 5687/D 5687M provides recommended specimen preparation practices and should be followed where practical.

8.3.1 A male tool is recommended for lay-up and cure to obtain a more precise inner radius. A male/female tool combination or a completely enclosed mold can also be used. Control of fiber alignment is critical. Improper fiber alignment will affect the measured properties. Erratic fiber alignment will also increase the coefficient of variation. Report the panel fabrication method.

8.3.2 Machining-Specimen preparation is extremely important for this specimen. Take precautions when cutting specimens from large panels to avoid notches, undercuts, rough or uneven surfaces, or delaminations due to inappropriate machining methods. Obtain final dimensions by waterlubricated precisions sawing, milling, or grinding. The use of diamond-tipped tooling (as well as water-jet cutting) has been found to be extremely effective for many material systems. Edges should be flat and parallel within the specified tolerances. Record and report the specimen cutting methods. The machined edges of the specimens may be polished as necessary to provide smooth surfaces to aid visually detecting delaminations during the test. Alternatively, the edges in the curved region may be coated with a thin white layer such as watersoluble typewriter correction fluid to aid delamination detection

8.3.3 *Labelling*—Label the coupons so that they will be distinct from each other and traceable back to the raw material, and will neither influence the test nor be affected by it.

# 9. Calibration

9.1 The accuracy of all measuring equipment shall have certified calibrations that are current at the time of use of the equipment.

# **10.** Conditioning

10.1 Standard Conditioning Procedure—Unless a different environment is specified as part of the experiment, condition the test specimens in accordance with Procedure C of D 5229/ D 5229M and store and test at standard laboratory atmosphere  $(23 \pm 3^{\circ}C [73 \pm 5^{\circ}F] \text{ and } 50 \pm 5 \%$  relative humidity). Report nonstandard conditioning and nonstandard test environments.

10.2 Oven Drying—If strength measurements are desired for laminates in an oven-dried condition, use Procedure D of Test Method D 5229/D 5229M.

# 11. Procedure

11.1 Parameters to be Specified Prior to Test:

11.1.1 The specimen sampling method, specimen geometry, and conditioning travelers (if required).

11.1.2 The properties and data reporting format desired.

NOTE 2—Determine specific material property, accuracy, and data reporting requirements prior to test for proper selection of instrumentation and data recording equipment. Estimate the maximum forces and displacements to aid in transducer selection, calibration of equipment, and determination of equipment settings.

11.1.3 The environmental conditioning test parameters.

11.1.4 Sampling method, specimen geometry, and test parameters used to determine density (if required) and constituent volumes.

11.2 General Instructions:

11.2.1 Report any deviations from this test method, whether intentional or inadvertent.

11.2.2 If specific gravity and density are to be reported, then obtain these samples from the curved region of the panel being tested. Specific gravity and density may be evaluated by means of Test Method D 792.

11.2.3 Obtain samples from the curved region of the panel being tested for measurement of reinforcement volume and

void content. Volume percent of the constituents may be evaluated by one of the procedures of Test Methods D 3171.

11.2.4 Condition the specimens as required. Store the specimens in the conditioned environment until test time, if the test environment is different than the conditioning environment.

11.2.5 Following final specimen machining and any conditioning, but before all testing, measure the specimen width, w, at two locations in the curved region. Measure the thickness of each specimen at several points around the curved region and along each leg. A knife-edge caliper must be used to measure the thickness in the curved region. The variation in thickness shall not exceed 5 % of the nominal thickness. Record the average thickness for each leg and for the curved region. The accuracy of all measurements shall be within 1% of the dimension. Record the dimensions to three significant figures in units of millimeters [inches]. Measure the angle between the inside surface of the two legs to calculate  $\phi$ i (0.5× angle between the legs).

11.3 Speed of Testing, Set the displacement rate so as to produce a failure within 1 to 610 min. If the maximum displacement at failure cannot be reasonably estimated, initial trials should be conducted using standard speeds until the failure load and maximum displacement of the system are known, and speed of testing can be adjusted. The suggested standard head displacement rate is 0.50 mm/min [0.020 in./ min].

11.4 *Test Environment*—If possible, test the specimen under the same fluid exposure level used for conditioning. However, cases such as elevated temperature testing of a moist specimen place unrealistic requirements on the capabilities of common environmental chambers. In such cases the mechanical test environment may need to be modified, for example, by testing at elevated temperature with no fluid exposure control, but with a specified limit on time to failure from withdrawal from the conditioning chamber. Record any modifications to the test environment.

11.5 *Fixture Installation*—Mount the four-point-bending fixture in the testing machine. Align the fixture halves such that all loading bars are parallel to each another. The distance between centerlines of the upper and lower loading bars on the right side must be equal to the distance between the centerlines on the left side, initially and during the test. Report any deviations from this test method, whether intentional or inadvertent.

11.6 Specimen Installation—Place the specimen in the fixture making sure that the specimen is roughly centered between the loading bars. The specimen edges shall be perpendicular to the loading bars so that each of the four loading bars makes contact across the entire width of the specimen. Because of the geometry of the fixture and specimen, the specimen will center itself between the loading bars when force is applied.

11.7 Apply force to the specimen in displacement control at a constant crosshead (or servo-hydraulic ram) displacement rate as specified in section 11.3.

11.8 *Data Recording*—Record force versus head displacement data continuously, or at frequent regular intervals. For this test method, a sampling rate of 3 to 10 data recordings per second and a target minimum of 300 data points per test are recommended. Record the maximum force, the failure force, and the head displacement at, or as near as possible to, the moment of delamination. Example curves of force versus displacement are shown in Fig. 4 and Fig. 5.

11.9 *Failure Modes*—As force is increased, the specimen will begin to flex open. Monitor the edges to determine when

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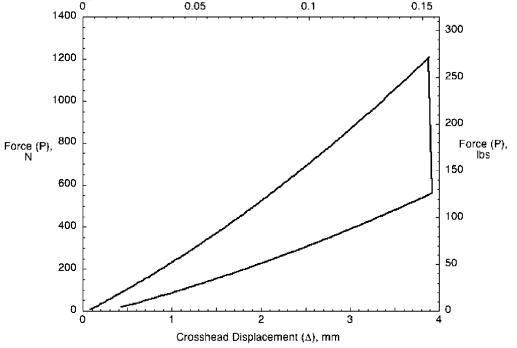


FIG. 4 Typical P/A Response for a Unidirectional Specimen

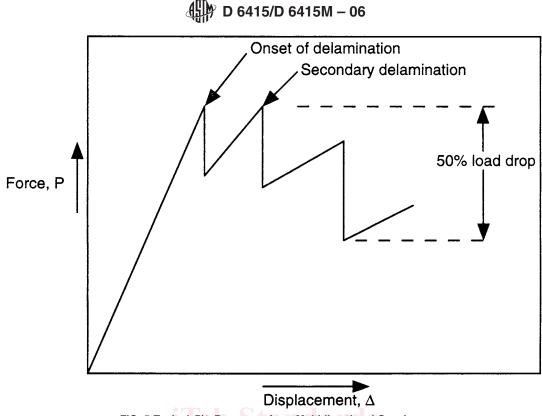


FIG. 5 Typical  $P/\Delta$  Response for a Multidirectional Specimen

delaminations form. A brittle white paint may be applied to the specimen edges to aid in detecting matrix cracks or delaminations. Delamination formation is typically accompanied by a sharp decrease in force. Typical P/ $\Delta$  curves are shown for a unidirectional specimen (for use in calculating interlaminar strength) and for a multidirectional specimen (for use in calculating CBS) in Fig. 4 and Fig. 5, respectively. 11.10 Terminate the test after the force has decreased to less than half of the peak force. Record the force and displacement for the duration of the test to capture any secondary delaminations.

#### 12. Validation

12.1 Strength values shall not be calculated for any specimen that forms damage or breaks at some obvious flaw, unless such flaw constitutes a variable being studied. Retests shall be performed for any specimen on which values are not calculated.

12.2 If a significant number of specimens in a sample population exhibit damage originating outside the curved region, the means of force introduction into the specimen shall be re-examined. Factors considered should include fixture condition and alignment, specimen geometry and alignment in the fixture, specimen surface characteristics, and gaps in the contact zones between the specimen and fixture.

12.3 If the cross head displacement exceeds 5 mm [0.2 in.] prior to failure, a significant error may occur in the bending moment calculation due to flexure of the specimen legs. Alternative test methods or bonding of doublers to the legs may need to be considered but such modifications are outside the scope of this test method.

# 13. Calculation

13.1 Calculate the curved beam strength (moment/width) from the total force, *P*, at the first force drop (corresponding to the initial delamination). The interlaminar strength must be determined from elasticity equations for a curved beam segment with cylindrical anisotropy. Since a force couple acts on each leg, the test section is in pure bending (that is, force resultant is zero). Consequently, the moment is the only required loading input for the stress equations.

NOTE 3—For multidirectional laminated specimens, the use of peak rather than the total force, P, at the first force drop may be of engineering interest in calculating *CBS* from Eq 1, but this deviation should be clearly noted in the test report.

13.2 Curved Beam Strength—Referring to Fig. 1 and Fig. 2, the applied moment on the curved section of the specimen is the product of the force exerted by one of the cylindrical loading bars,  $P_b$ , and the distance,  $l_0$ , between two bars along a leg (Eq 1). Calculate the bar force and distance from the total force, P, at the first force drop (corresponding to the initial delamination) and the geometries of the loading fixture and test specimen.

$$CBS = \frac{M}{w} = \frac{P_b I_0}{w} = \left(\frac{P}{2w\cos(\phi)}\right) \left(\frac{d_x}{\cos(\phi)} + (D+t)\tan(\phi)\right)$$
(1)

The curved beam strength is given in Eq 1 where  $\phi$  is the angle in degrees of the loading arm from horizontal,  $d_x$  is the horizontal distance between the centerlines of two top and bottom adjacent rollers  $(l_b - l_t)/2$ , D is the diameter of the cylindrical loading bars, and t is the specimen thickness.

13.2.1 Since  $\phi$  can change significantly during loading, the value of  $\phi$  at failure can be used to obtain a more accurate