# GUIDE TO THE EXPRESSION OF UNCERTAINTY IN MEASUREMENT

CORRECTED AND REPRINTED, 1995

| | |
|---|---|
| BIPM | International Bureau of Weights and Measures |
| IEC | International Electrotechnical Commission |
| IFCC | International Federation of Clinical Chemistry |
| ISO | International Organization for Standardization |
| IUPAC | International Union of Pure and Applied Chemistry |
| IUPAP | International Union of Pure and Applied Physics |
| OIML | International Organization of Legal Metrology |

BIPM
Bureau international des poids et mesures
Pavillon de Breteuil
F-92312 Sèvres Cedex
France

IEC
International Electrotechnical Commission
3, rue de Varembé
Case postale 131
CH-1211 Genève 20
Switzerland

IFCC
International Federation of Clinical Chemistry
Technical Secretariat
Centre du Médicament
Université de Nancy 1
30, rue Lionnois
F-54000 Nancy
France

ISO
International Organization for Standardization
1, rue de Varembé
Case postale 56
CH-1211 Genève 20
Switzerland

IUPAC
International Union of Pure and Applied Chemistry
Bank Court Chambers
2-3 Pound Way
Templars Square, Cowley
Oxford OX4 3YF
United Kingdom

IUPAP
International Union of Pure and Applied Physics
Secretariat
Vittens gata 11
S-421 65 V. Frölunda
Sweden

OIML
International Organization of Legal Metrology
11, rue Turgot
F-75009 Paris
France

# GUIDE TO THE EXPRESSION OF UNCERTAINTY IN MEASUREMENT

This *Guide* establishes general rules for evaluating and expressing uncertainty in measurement that are intended to be applicable to a broad spectrum of measurements. The basis of the *Guide* is Recommendation 1 (CI-1981) of the Comité International des Poids et Mesures (CIPM) and Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties. The Working Group was convened by the Bureau International des Poids et Mesures (BIPM) in response to a request of the CIPM. The CIPM Recommendation is the only recommendation concerning the expression of uncertainty in measurement adopted by an intergovernmental organization.

This *Guide* was prepared by a joint working group consisting of experts nominated by the BIPM, the International Electrotechnical Commission (IEC), the International Organization for Standardization (ISO), and the International Organization of Legal Metrology (OIML).

The following seven organizations supported the development of this *Guide*, which is published in their name:

| | |
|---|---|
| BIPM | Bureau International des Poids et Mesures |
| IEC | International Electrotechnical Commission |
| IFCC | International Federation of Clinical Chemistry |
| ISO | International Organization for Standardization |
| IUPAC | International Union of Pure and Applied Chemistry |
| IUPAP | International Union of Pure and Applied Physics |
| OIML | International Organization of Legal Metrology |

Users of this *Guide* are invited to send their comments and requests for clarification to any of the seven supporting organizations, the mailing addresses of which are given on the inside front cover.

Guide to the Expression of Uncertainty in Measurement

First edition 1995
ISBN 92-67-10188-9

Printed in Switzerland

# Contents

## Annexes

# Foreword

In 1977, recognizing the lack of international consensus on the expression of uncertainty in measurement, the world's highest authority in metrology, the Comité International des Poids et Mesures (CIPM), requested the Bureau International des Poids et Mesures (BIPM) to address the problem in conjunction with the national standards laboratories and to make a recommendation.

The BIPM prepared a detailed questionnaire covering the issues involved and distributed it to 32 national metrology laboratories known to have an interest in the subject (and, for information, to five international organizations). By early 1979 responses were received from 21 laboratories [1].[1] Almost all believed that it was important to arrive at an internationally accepted procedure for expressing measurement uncertainty and for combining individual uncertainty components into a single total uncertainty. However, a consensus was not apparent on the method to be used. The BIPM then convened a meeting for the purpose of arriving at a uniform and generally acceptable procedure for the specification of uncertainty; it was attended by experts from 11 national standards laboratories. This Working Group on the Statement of Uncertainties developed Recommendation INC-1 (1980), Expression of Experimental Uncertainties [2]. The CIPM approved the Recommendation in 1981 [3] and reaffirmed it in 1986 [4].

The task of developing a detailed guide based on the Working Group Recommendation (which is a brief outline rather than a detailed prescription) was referred by the CIPM to the International Organization for Standardization (ISO), since ISO could better reflect the needs arising from the broad interests of industry and commerce.

Responsibility was assigned to the ISO Technical Advisory Group on Metrology (TAG 4) because one of its tasks is to coordinate the development of guidelines on measurement topics that are of common interest to ISO and the six organizations that participate with ISO in the work of TAG 4: the International Electrotechnical Commission (IEC), the partner of ISO in worldwide standardization; the CIPM and the International Organization of Legal Metrology (OIML), the two worldwide metrology organizations; the International Union of Pure and Applied Chemistry (IUPAC) and the International Union of Pure and Applied Physics (IUPAP), the two international unions that represent chemistry and physics; and the International Federation of Clinical Chemistry (IFCC).

TAG 4 in turn established Working Group 3 (ISO/TAG 4/WG 3) composed of experts nominated by the BIPM, IEC, ISO, and OIML and appointed by the Chairman of TAG 4. It was assigned the following terms of reference:

> To develop a guidance document based upon the recommendation of the BIPM Working Group on the Statement of Uncertainties which provides rules on the expression of measurement uncertainty for use within standardization, calibration, laboratory accreditation, and metrology services;
>
> The purpose of such guidance is
> - to promote full information on how uncertainty statements are arrived at;
> - to provide a basis for the international comparison of measurement results.

---

[1] See the bibliography on page 94 *et seq*.

# 0 Introduction

**0.1** When reporting the result of a measurement of a physical quantity, it is obligatory that some quantitative indication of the quality of the result be given so that those who use it can assess its reliability. Without such an indication, measurement results cannot be compared, either among themselves or with reference values given in a specification or standard. It is therefore necessary that there be a readily implemented, easily understood, and generally accepted procedure for characterizing the quality of a result of a measurement, that is, for evaluating and expressing its *uncertainty*.

**0.2** The concept of *uncertainty* as a quantifiable attribute is relatively new in the history of measurement, although *error* and *error analysis* have long been a part of the practice of measurement science or metrology. It is now widely recognized that, when all of the known or suspected components of error have been evaluated and the appropriate corrections have been applied, there still remains an uncertainty about the correctness of the stated result, that is, a doubt about how well the result of the measurement represents the value of the quantity being measured.

**0.3** Just as the nearly universal use of the International System of Units (SI) has brought coherence to all scientific and technological measurements, a worldwide consensus on the evaluation and expression of uncertainty in measurement would permit the significance of a vast spectrum of measurement results in science, engineering, commerce, industry, and regulation to be readily understood and properly interpreted. In this era of the global marketplace, it is imperative that the method for evaluating and expressing uncertainty be uniform throughout the world so that measurements performed in different countries can be easily compared.

**0.4** The ideal method for evaluating and expressing the uncertainty of the result of a measurement should be:

- *universal*: the method should be applicable to all kinds of measurements and to all types of input data used in measurements.

The actual quantity used to express uncertainty should be:

- *internally consistent*: it should be directly derivable from the components that contribute to it, as well as independent of how these components are grouped and of the decomposition of the components into subcomponents;

- *transferable*: it should be possible to use directly the uncertainty evaluated for one result as a component in evaluating the uncertainty of another measurement in which the first result is used.

Further, in many industrial and commercial applications, as well as in the areas of health and safety, it is often necessary to provide an interval about the measurement result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the quantity subject to measurement. Thus the ideal method for evaluating and expressing uncertainty in measurement should be capable of readily providing such an interval, in particular, one with a coverage probability or level of confidence that corresponds in a realistic way with that required.

**0.5** The approach upon which this guidance document is based is that outlined in Recommendation INC-1 (1980) [2] of the Working Group on the Statement of Uncertainties, which was convened by the BIPM in response to a request of the CIPM (see Foreword). This approach, the justification of which is discussed in annex E, meets all of the requirements outlined above. This is not the case for most other methods in current use. Recommendation INC-1 (1980) was approved and reaffirmed by the CIPM in its own Recommendations 1 (CI-1981) [3] and 1 (CI-1986) [4]; the English translations of these CIPM Recommendations are reproduced in annex A (see A.2 and A.3, respectively). Because Recommendation INC-1 (1980) is the foundation upon which this document rests, the English translation is reproduced in 0.7 and the French text, which is authoritative, is reproduced in A.1.

**0.6** A succinct summary of the procedure specified in this guidance document for evaluating and expressing uncertainty in measurement is given in clause 8 and a number of examples are presented in detail in annex H. Other annexes deal with general terms in metrology (annex B); basic statistical terms and concepts (annex C); "true" value, error, and uncertainty (annex D); practical suggestions for evaluating uncertainty components (annex F); degrees of freedom and levels of confidence (annex G); the principal mathematical symbols used throughout the document (annex J); and bibliographical references (annex K). An alphabetical index concludes the document.

## 0.7 Recommendation INC-1 (1980)

Expression of experimental uncertainties

1. The uncertainty in the result of a measurement generally consists of several components which may be grouped into two categories according to the way in which their numerical value is estimated:

   A. those which are evaluated by statistical methods,

   B. those which are evaluated by other means.

There is not always a simple correspondence between the classification into categories A or B and the previously used classification into "random" and "systematic" uncertainties. The term "systematic uncertainty" can be misleading and should be avoided.

Any detailed report of the uncertainty should consist of a complete list of the components, specifying for each the method used to obtain its numerical value.

2. The components in category A are characterized by the estimated variances $s_i^2$ (or the estimated "standard deviations" $s_i$) and the number of degrees of freedom $\nu_i$. Where appropriate, the covariances should be given.

3. The components in category B should be characterized by quantities $u_j^2$, which may be considered as approximations to the corresponding variances, the existence of which is assumed. The quantities $u_j^2$ may be treated like variances and the quantities $u_j$ like standard deviations. Where appropriate, the covariances should be treated in a similar way.

4. The combined uncertainty should be characterized by the numerical value obtained by applying the usual method for the combination of variances. The combined uncertainty and its components should be expressed in the form of "standard deviations."

5. If, for particular applications, it is necessary to multiply the combined uncertainty by a factor to obtain an overall uncertainty, the multiplying factor used must always be stated.

# GUIDE TO THE EXPRESSION OF UNCERTAINTY IN MEASUREMENT

## 1 Scope

**1.1** This *Guide* establishes general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields – from the shop floor to fundamental research. Therefore, the principles of this *Guide* are intended to be applicable to a broad spectrum of measurements, including those required for:

- maintaining quality control and quality assurance in production;
- complying with and enforcing laws and regulations;
- conducting basic research, and applied research and development, in science and engineering;
- calibrating standards and instruments and performing tests throughout a national measurement system in order to achieve traceability to national standards;
- developing, maintaining, and comparing international and national physical reference standards, including reference materials.

**1.2** This *Guide* is primarily concerned with the expression of uncertainty in the measurement of a well-defined physical quantity – the measurand – that can be characterized by an essentially unique value. If the phenomenon of interest can be represented only as a distribution of values or is dependent on one or more parameters, such as time, then the measurands required for its description are the set of quantities describing that distribution or that dependence.

**1.3** This *Guide* is also applicable to evaluating and expressing the uncertainty associated with the conceptual design and theoretical analysis of experiments, methods of measurement, and complex components and systems. Because a measurement result and its uncertainty may be conceptual and based entirely on hypothetical data, the term "result of a measurement" as used in this *Guide* should be interpreted in this broader context.

**1.4** This *Guide* provides general rules for evaluating and expressing uncertainty in measurement rather than detailed, technology-specific instructions. Further, it does not discuss how the uncertainty of a particular measurement result, once evaluated, may be used for different purposes, for example, to draw conclusions about the compatibility of that result with other similar results, to establish tolerance limits in a manufacturing process, or to decide if a certain course of action may be safely undertaken. It may therefore be necessary to develop particular standards based on this *Guide* that deal with the problems peculiar to specific fields of measurement or with the various uses of quantitative expressions of uncertainty. These standards may be simplified versions of this *Guide* but should include the detail that is appropriate to the level of accuracy and complexity of the measurements and uses addressed.

NOTE – There may be situations in which the concept of uncertainty of measurement is believed not to be fully applicable, such as when the precision of a test method is determined (see reference [5], for example).

# 2 Definitions

## 2.1 General metrological terms

The definition of a number of general metrological terms relevant to this *Guide*, such as "measurable quantity," "measurand," and "error of measurement," are given in annex B. These definitions are taken from the *International vocabulary of basic and general terms in metrology* (abbreviated VIM) [6]. In addition, annex C gives the definitions of a number of basic statistical terms taken mainly from International Standard ISO 3534-1 [7]. When one of these metrological or statistical terms (or a closely related term) is first used in the text, starting with clause 3, it is printed in boldface and the number of the subclause in which it is defined is given in parentheses.

Because of its importance to this *Guide*, the definition of the general metrological term "uncertainty of measurement" is given both in annex B and 2.2.3. The definitions of the most important terms specific to this *Guide* are given in 2.3.1 to 2.3.6. In all of these subclauses and in annexes B and C, the use of parentheses around certain words of some terms means that these words may be omitted if this is unlikely to cause confusion.

## 2.2 The term "uncertainty"

The concept of uncertainty is discussed further in clause 3 and annex D.

**2.2.1** The word "uncertainty" means doubt, and thus in its broadest sense "uncertainty of measurement" means doubt about the validity of the result of a measurement. Because of the lack of different words for this *general concept* of uncertainty and the specific quantities that provide *quantitative measures* of the concept, for example, the standard deviation, it is necessary to use the word "uncertainty" in these two different senses.

**2.2.2** In this *Guide*, the word "uncertainty" without adjectives refers both to the general concept of uncertainty and to any or all quantitative measures of that concept. When a specific measure is intended, appropriate adjectives are used.

**2.2.3** The formal definition of the term "uncertainty of measurement" developed for use in this *Guide* and in the current VIM [6] (VIM entry 3.9) is as follows:

> **uncertainty (of measurement)**
> parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand
>
> NOTES
>
> 1 The parameter may be, for example, a standard deviation (or a given multiple of it), or the half-width of an interval having a stated level of confidence.
>
> 2 Uncertainty of measurement comprises, in general, many components. Some of these components may be evaluated from the statistical distribution of the results of series of measurements and can be characterized by experimental standard deviations. The other components, which also can be characterized by standard deviations, are evaluated from assumed probability distributions based on experience or other information.
>
> 3 It is understood that the result of the measurement is the best estimate of the value of the measurand, and that all components of uncertainty, including those arising from systematic effects, such as components associated with corrections and reference standards, contribute to the dispersion.

**2.2.4** The definition of uncertainty of measurement given in 2.2.3 is an operational one that focuses on the measurement result and its evaluated uncertainty. However, it is not inconsistent with other concepts of uncertainty of measurement, such as

- a measure of the possible error in the estimated value of the measurand as provided by the result of a measurement;

- an estimate characterizing the range of values within which the true value of a measurand lies (VIM, first edition, 1984, entry 3.09).

Although these two traditional concepts are valid as ideals, they focus on *unknowable* quantities: the "error" of the result of a measurement and the "true value" of the measurand (in contrast to its estimated value), respectively. Nevertheless, whichever *concept* of uncertainty is adopted, an uncertainty component is always *evaluated* using the same data and related information. (See also E.5.)

## 2.3 Terms specific to this *Guide*

In general, terms that are specific to this *Guide* are defined in the text when first introduced. However, the definitions of the most important of these terms are given here for easy reference.

NOTE – Further discussion related to these terms may be found as follows: for 2.3.2, see 3.3.3 and 4.2; for 2.3.3, see 3.3.3 and 4.3; for 2.3.4, see clause 5 and equations (10) and (13); and for 2.3.5 and 2.3.6, see clause 6.

### 2.3.1 standard uncertainty

uncertainty of the result of a measurement expressed as a standard deviation

### 2.3.2 Type A evaluation (of uncertainty)

method of evaluation of uncertainty by the statistical analysis of series of observations

### 2.3.3 Type B evaluation (of uncertainty)

method of evaluation of uncertainty by means other than the statistical analysis of series of observations

### 2.3.4 combined standard uncertainty

standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities

### 2.3.5 expanded uncertainty

quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand

NOTES

1 The fraction may be viewed as the coverage probability or level of confidence of the interval.

2 To associate a specific level of confidence with the interval defined by the expanded uncertainty requires explicit or implicit assumptions regarding the probability distribution characterized by the measurement result and its combined standard uncertainty. The level of confidence that may be attributed to this interval can be known only to the extent to which such assumptions may be justified.

3 Expanded uncertainty is termed *overall uncertainty* in paragraph 5 of Recommendation INC-1 (1980).

### 2.3.6 coverage factor

numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty

NOTE – A coverage factor, $k$, is typically in the range 2 to 3.

# 3 Basic concepts

Additional discussion of basic concepts may be found in annex D, which focuses on the ideas of "true" value, error, and uncertainty and includes graphical illustrations of these concepts; and in annex E, which explores the motivation and statistical basis for Recommendation INC-1 (1980) upon which this *Guide* rests. Annex J is a glossary of the principal mathematical symbols used throughout the *Guide*.

## 3.1 Measurement

**3.1.1** The objective of a **measurement** (B.2.5) is to determine the **value** (B.2.2) of the **measurand** (B.2.9), that is, the value of the **particular quantity** (B.2.1, note 1) to be measured. A measurement therefore begins with an appropriate specification of the measurand, the **method of measurement** (B.2.7), and the **measurement procedure** (B.2.8).

NOTE – The term "true value" (see annex D) is not used in this *Guide* for the reasons given in D.3.5; the terms "value of a measurand" (or of a quantity) and "true value of a measurand" (or of a quantity) are viewed as equivalent.

**3.1.2** In general, the **result of a measurement** (B.2.11) is only an approximation or **estimate** (C.2.26) of the value of the measurand and thus is complete only when accompanied by a statement of the **uncertainty** (B.2.18) of that estimate.

**3.1.3** In practice, the required specification or definition of the measurand is dictated by the required **accuracy of measurement** (B.2.14). The measurand should be defined with sufficient completeness with respect to the required accuracy so that for all practical purposes associated with the measurement its value is unique. It is in this sense that the expression "value of the measurand" is used in this *Guide*.

EXAMPLE – If the length of a nominally one-metre long steel bar is to be determined to micrometre accuracy, its specification should include the temperature and pressure at which the length is defined. Thus the measurand should be specified as, for example, the length of the bar at 25,00 °C and 101 325 Pa (plus any other defining parameters deemed necessary, such as the way the bar is to be supported). However, if the length is to be determined to only millimetre accuracy, its specification would not require a defining temperature or pressure or a value for any other defining parameter.

NOTE – Incomplete definition of the measurand can give rise to a component of uncertainty sufficiently large that it must be included in the evaluation of the uncertainty of the measurement result (see D.1.1, D.3.4, and D.6.2).

**3.1.4** In many cases, the result of a measurement is determined on the basis of series of observations obtained under **repeatability conditions** (B.2.15, note 1).

**3.1.5** Variations in repeated observations are assumed to arise because **influence quantities** (B.2.10) that can affect the measurement result are not held completely constant.

**3.1.6** The mathematical model of the measurement that transforms the set of repeated observations into the measurement result is of critical importance because, in addition to the observations, it generally includes various influence quantities that are inexactly known. This lack of knowledge contributes to the uncertainty of the measurement result, as do the variations of the repeated observations and any uncertainty associated with the mathematical model itself.

**3.1.7** This *Guide* treats the measurand as a scalar (a single quantity). Extension to a set of related measurands determined simultaneously in the same measurement requires replacing the scalar measurand and its **variance** (C.2.11, C.2.20, C.3.2) by a vector measurand and **covariance matrix** (C.3.5). Such a replacement is considered in this *Guide* only in the examples (see H.2, H.3, and H.4).

## 3.2 Errors, effects, and corrections

**3.2.1** In general, a measurement has imperfections that give rise to an **error** (B.2.19) in the measurement result. Traditionally, an error is viewed as having two components, namely, a **random** (B.2.21) component and a **systematic** (B.2.22) component.

NOTE – Error is an idealized concept and errors cannot be known exactly.

**3.2.2** Random error presumably arises from unpredictable or stochastic temporal and spatial variations of influence quantities. The effects of such variations, hereafter termed *random effects*, give rise to variations in repeated observations of the measurand. Although it is not possible to compensate for the random error of a measurement result, it can usually be reduced by increasing the number of observations; its **expectation** or **expected value** (C.2.9, C.3.1) is zero.

NOTES

1 The experimental standard deviation of the arithmetic mean or average of a series of observations (see 4.2.3) is *not* the random error of the mean, although it is so designated in some publications. It is instead a measure of the *uncertainty* of the mean due to random effects. The exact value of the error in the mean arising from these effects cannot be known.

2 In this *Guide*, great care is taken to distinguish between the terms "error" and "uncertainty." They are not synonyms, but represent completely different concepts; they should not be confused with one another or misused.

**3.2.3** Systematic error, like random error, cannot be eliminated but it too can often be reduced. If a systematic error arises from a recognized effect of an influence quantity on a measurement result, hereafter termed a *systematic effect*, the effect can be quantified and, if it is significant in size relative to the required accuracy of the measurement, a **correction** (B.2.23) or **correction factor** (B.2.24) can be applied to compensate for the effect. It is assumed that, after correction, the expectation or expected value of the error arising from a systematic effect is zero.

NOTE – The uncertainty of a correction applied to a measurement result to compensate for a systematic effect is *not* the systematic error, often termed bias, in the measurement result due to the effect as it is sometimes called. It is instead a measure of the *uncertainty* of the result due to incomplete knowledge of the required value of the correction. The error arising from imperfect compensation of a systematic effect cannot be exactly known. The terms "error" and "uncertainty" should be used properly and care taken to distinguish between them.

**3.2.4** It is assumed that the result of a measurement has been corrected for all recognized significant systematic effects and that every effort has been made to identify such effects.

EXAMPLE – A correction due to the finite impedance of a voltmeter used to determine the potential difference (the measurand) across a high-impedance resistor is applied to reduce the systematic effect on the result of the measurement arising from the loading effect of the voltmeter. However, the values of the impedances of the voltmeter and resistor, which are used to estimate the value of the correction and which are obtained from other measurements, are themselves uncertain. These uncertainties are used to evaluate the component of the uncertainty of the potential difference determination arising from the correction and thus from the systematic effect due to the finite impedance of the voltmeter.

NOTES

1 Often, measuring instruments and systems are adjusted or calibrated using measurement standards and reference materials to eliminate systematic effects; however, the uncertainties associated with these standards and materials must still be taken into account.

2 The case where a correction for a known significant systematic effect is not applied is discussed in the note to 6.3.1 and in F.2.4.5.

## 3.3 Uncertainty

**3.3.1** The uncertainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand (see 2.2). The result of a measurement after correction for recognized systematic effects is still only an *estimate* of the value of the measurand because of the uncertainty arising from random effects and from imperfect correction of the result for systematic effects.

NOTE – The result of a measurement (after correction) can unknowably be very close to the value of the measurand (and hence have a negligible error) even though it may have a large uncertainty. Thus the uncertainty of the result of a measurement should not be confused with the remaining unknown error.

**3.3.2** In practice, there are many possible sources of uncertainty in a measurement, including:

a) incomplete definition of the measurand;
b) imperfect realization of the definition of the measurand;
c) nonrepresentative sampling – the sample measured may not represent the defined measurand;
d) inadequate knowledge of the effects of environmental conditions on the measurement or