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Gas cylinders — Methods for establishing acceptance/rejection criteria for flaws in seamless steel and aluminium alloy cylinders at time of periodic inspection and testing

Bouteilles à gaz — Méthodes d'établissement des critères d'acceptation/de rejet des défauts dans les bouteilles en acier et en alliages d'aluminium, sans soudure, lors des contrôles et essais périodiques

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Foreword

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In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 22694 was prepared by Technical Committee ISO/TC 58, *Gas cylinders*, Subcommittee SC 4, *Operational requirements for gas cylinders*.

Introduction

Seamless steel cylinders and seamless aluminium-alloy cylinders used to transport high pressure gases are required to meet safety requirements based on ISO standards and the requirements of national authorities. These requirements cover the design, materials, manufacturing, initial inspection and testing, and periodic inspection and testing of the cylinders. As part of these requirements, the cylinders need to be periodically inspected and tested at regular intervals during their lifetime.

Periodic inspection and testing has traditionally been performed by a combination of visual inspection (internal and external) and hydrostatic pressure testing (sometimes including volumetric expansion measurements during pressurization). Using these traditional methods of retesting, the cylinders are rejected due to excessive volumetric expansion, excessively large surface flaws detected by visual examination, leaking or bursting. The maximum allowable size of surface flaws to cause rejection of the cylinders was essentially qualitative and was established from past service experience. None of the rejection criteria were based on quantitative assessment of the cylinder's performance or mechanical characteristics.

However, recently, methods of periodic inspection and testing the cylinders using ultrasonic inspection have been developed. These new retesting methods permit the quantitative determination of the cylinder wall thickness and the size of the flaws that are present in the cylinders. The ISO standards for periodic inspection and the requirements of certain national authorities permit the use of ultrasonic test methods for retesting seamless steel and aluminium-alloy cylinders. These ultrasonic test methods permit the quantitative determination of the size of any flaws that are detected in the cylinders. However, to use the ultrasonic test methods, it is required that quantitative "allowable flaw sizes" be established to set acceptance/rejection limits for the cylinders at the time of periodic inspection and testing.

NOTE The main conclusions and acceptance/rejection criteria are based on those provided by the United States Department of Transportation (DOT-designed cylinders) that have a working pressure to test-pressure ratio of 3:5. Application to ISO-designed cylinders, which use a working pressure to test-pressure ratio of 2:3, needs a further calculation.

Gas cylinders — Methods for establishing acceptance/rejection criteria for flaws in seamless steel and aluminium alloy cylinders at time of periodic inspection and testing

1 Scope

The aim of this Technical Report is to establish a technical basis for developing quantitative, allowable flaw sizes and for setting acceptance/rejection limits for cylinders at the time of periodic inspection and testing based on the performance and mechanical properties of the cylinders.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

API RP 579, *Recommended Practice for Fitness-for-Service*

3 Terms and definitions

For the purposes of this document, the following terms, acronyms and definitions apply.

3.1
allowable flaw size
largest flaw that will not grow to the critical flaw size during the periodic inspection and testing interval of the cylinder

3.2
burst
opening of the cylinder due to the internal pressure with substantial extension of the flaw

3.3
cluster of pits
small, approximately round, flaws that are close together in a limited area

3.4
corrosion
general loss of wall thickness of either the interior or exterior surface of the cylinder, or localized corrosion which may form a narrow longitudinal or circumferential line or strip, or isolated craters or pits that are almost connected in a line

3.5
crack
split in the metal

3.6
critical flaw size
CFS
flaw size that causes the cylinder to fail at a designated pressure

3.7
cut/gouge
sharp impression on the exterior of the cylinder where metal has been removed or redistributed and whose depth exceeds 5 % of the cylinder wall thickness

3.8
failure by plastic collapse
failure of the cylinder containing a flaw due to internal pressure in the cylinder by failure of the remaining ligament below the flaw without substantial extension of the flaw

3.9
fatigue crack growth rate
average flaw growth amount for each cycle of pressure loading

3.10
fracture
unstable extension of a flaw in the cylinder

3.11
fracture toughness
generic term for measure of resistance to extension of a crack

3.12
leak
release of gas pressure from the cylinder without significant extension of the flaw

NOTE This can occur due to internal pressure or due to corrosion.

3.13
local thin area
LTA
area of reduced wall thickness, the length and width of which are approximately equal

NOTE LTAs can be circular or rectangular.

3.14
notch
nominally a two-dimensional, long, narrow flaw with the width much smaller than the length

3.15
periodic inspection and testing
reference to a visual and/or ultrasonic examination and/or pressure test

3.16
residual strength factor
RSF
ratio of the failure pressure of a cylinder containing a flaw to the failure pressure of the same cylinder without a flaw

4 Cylinder symbols

A flaw area

a flaw depth

a_i	initial flaw depth
C	flaw width (circumferential dimension of the flaw)
D	nominal outside diameter of the cylinder
da/dN	fatigue crack growth rate
ID	inside cylinder diameter
K_{IC} (J)	fracture toughness obtained from J integral test method
L	flaw length (longitudinal dimension of flaw)
M_p	stress magnification factor for "part through flaw"
M_t	Folias stress magnification factor for "through-wall flaw"
N	number of pressure cycles
p_b	failure pressure for cylinder without a flaw
p_f	failure pressure for a cylinder with a flaw
p_f/p_b	residual strength factor[1)]
p_h	cylinder test pressure
p_s	cylinder working pressure
R_e	minimum guaranteed value of yield strength
R_m	actual value of tensile strength as determined by tensile test
$(R_m + R_e)/2$	flow stress
R_t	remaining wall thickness ratio (t_{mm}/t)
t	measured minimum wall thickness
t_a	actual wall thickness at the flaw
t_d	calculated minimum design wall thickness
t_{mm}	minimum ligament (material below the flaw) thickness

5 Technical approach

In this Technical Report, the performance of selected cylinders was evaluated based on the principles of structural integrity analysis. The effect of various types and sizes of flaws on the performance of seamless steel and aluminium-alloy cylinders was evaluated by analytical modelling that was verified by using data from other studies that involved testing of steel and aluminium-alloy cylinders containing artificially induced flaws.

1) Residual strength factor is sometimes referred to as the failure pressure ratio.

The periodic inspection of seamless cylinders requires that allowable flaw sizes be established for each type of flaw. Typical flaws that can occur in high-pressure seamless gas cylinders during service are cuts or gouges, cracks, general corrosion, local corrosion (LTA) and chain/line/pitting corrosion. To establish allowable flaw sizes, an assessment of typical flaws that (e.g. of an LTA) occur in seamless cylinders was carried out using the analytical procedures described in the API *Recommended Practice for Fitness-for-Service* (API RP 579, hereafter referred to as API 579). The analytical assessments were subsequently verified by experimental testing.

In using these procedures, first the critical flaw size(s) (CFS) are determined. The CFS is defined as the size (e.g. depth and length or area) of the flaw that will cause the cylinders to fail at a specified pressure, such as the test pressure or the working pressure of the cylinder. API 579 was used to calculate the CFS for a range of cylinder sizes and strength levels. Next, the allowable flaw sizes are determined by adjusting the CFS to account for any time-dependent degradation that can occur in service, such as crack growth by fatigue or corrosion.

Firstly, to determine the CFS, the procedures described in API 579 were used to predict, by analysis, the effect of various sizes of LTAs, pits, notches and cracks on the calculated cylinder burst pressure for selected sizes and strength levels of cylinders. Then, to verify the API 579 analysis procedures, experimental data from a number of hydrostatic burst tests on selected cylinders with various sizes of flaws were compared with the analytical results. These results showed that the analysis conducted according to API 579 reliably estimated the actual measured burst pressure of the cylinders for all flaw sizes and types.

CFSs were determined for various types of flaws at (1) the designated working pressure and (2) the hydrostatic test pressure of the cylinder. This establishes the CFS (depth versus area or length) for each type of flaw in any cylinder. The CFS calculated at the designated working pressure predicts the size of the flaws that could be expected to cause the cylinder to fail in service. The CFS calculated at the hydrostatic test pressure predicts the size of flaws that could be expected to cause the cylinder to fail during the hydrostatic pressure test.

After calculating the CFS to cause failure of the cylinders at both the test and working pressures, the allowable flaw sizes to be used as the acceptance or rejection criteria during periodic inspection and testing were established for a wide range of cylinder types and strength levels. This was done by modifying (reducing) the size of the CFS for each cylinder by adjusting for time-dependent degradation, such as fatigue crack growth or corrosion that may occur during the use of the cylinder. In the preparation of this Technical Report, only the effects of fatigue crack growth were evaluated. The fatigue procedure that was used to make this adjustment involved 3 500 cycles (approximately once per day filling for the 10 year retest interval) at the cylinder's designated working pressure. This resulted in the allowable flaw size that may be used to establish the acceptance or rejection criterion for the cylinders during periodic inspection and testing. The final acceptance or rejection criteria that are used during periodic inspection and testing may also take into account other factors, such as the capability of the inspection instruments and procedures.

The allowable flaw sizes are based on the assumption that there is no free moisture in the interior of the cylinder and consequently no risk of corrosion.

6 Modelling and analysis of flaw sizes

6.1 Basis and theory

The approach used to define allowable flaw sizes for seamless cylinders was to determine the effect of various types and sizes of flaws on the performance of the cylinders. In particular, the reduction in the failure pressure of the cylinders containing flaws was determined by analytical modelling. These analytical results were then verified by using data from studies involving the experimental testing of selected cylinders containing flaws.

To evaluate the significance of flaws in cylinders, the principles of structural integrity analysis are used. Several general theoretical, empirical or semi-empirical methods of analysis have been developed to model flaws in pressure vessels, such as cylinders, and to evaluate the significance of the flaws. The purpose of these methods of analysis is to determine how much the failure pressure of a cylinder containing a flaw is reduced compared to a similar cylinder that does not contain any flaws. Failure of the cylinder may occur by bursting, by fracture, by leaking or other failure modes. These methods of analysis can be used to make an assessment of the current state of the cylinder, that is, the current failure pressure of the cylinder. These methods of analysis can also be used to determine a projected future state of the cylinder due to increases in the size of the flaws over time by such mechanisms as fatigue, corrosion, stress corrosion, or other time-dependent degradation.

After reviewing the methods of analysis that have been developed to evaluate the significance of flaws in pressure vessels, the methods of analysis described in API 579 were chosen to evaluate the cylinders used in the preparation of this Technical Report and to develop CFS and allowable flaw sizes for seamless cylinders. The fitness-for-service method of analysis provides a quantitative evaluation of cylinders containing flaws to determine their suitability for continued use.

The fitness-for-service method of analysis can be used to evaluate all types of flaws commonly found in cylinders. Methods of analysis are available for analysing various types of flaws such as general corrosion, localised corrosion area (LTA), widespread pitting, localized pitting, cracks and crack-like flaws. Brittle fracture, fatigue cracking and environmental cracking can be evaluated.

6.2 Summary of the fitness-for-service method of analysis

6.2.1 Application of API 579 fitness-for-service method of analysis

The application of the API 579 fitness-for-service method of analysis requires the following steps:

1) identification of the type of flaw (crack, LTA, pit, etc.) and the type of damage that caused the flaw (corrosion, fatigue, cracking, cuts, gouges, etc.);

2) identification of the failure mode (brittle fracture, burst, leak);

3) selection of the specific method of analysis (fracture analysis, burst analysis, leak analysis, etc.);

4) obtaining the necessary data (material properties, applied stresses, flaw characterization and size, etc.);

5) selection of the level of assessment;

6) selection of the appropriate acceptance criteria;

7) evaluation of the remaining life of cylinder due to enlargement of the flaws.

6.2.2 Step 1, identification of type of flaw

The types of flaws that can occur in seamless steel cylinders and aluminium-alloy cylinders have been identified in ISO 6406, and ISO 10461 respectively. The types of flaws that have been identified are cracks, notches, gouges, general corrosion, localised corrosion area (e.g. corrosion of an LTA), pitting corrosion including isolated pit and multiple pits (i.e. line corrosion), arc burns and fire damage.

However, in this Technical Report, the only flaws evaluated are

— LTA,

— cracks,

— notches,

— general/local corrosion, and

— corrosion pits.

Therefore, in this Technical Report, the only types of damage evaluated are from flaws caused either by corrosion or mechanical damage subsequently propagated by fatigue.

6.2.3 Step 2, identification of failure mode

The failure modes that can cause seamless cylinders to fail in service are burst or leak type. Cylinders can fail by bursting when a flaw of sufficient size is present in the cylinder wall. The failure stress is dependent on the material's fracture toughness and flow strength properties. For relatively high-toughness material, which is the case for the high-pressure cylinders, the cylinder burst stress is primarily controlled by flow stress. Cylinders can fail by leaking when the flaw is sufficiently deep such that the remaining wall ruptures. Cylinders can fail in service by fracturing or fragmenting when the combination of a sufficiently large flaw and a high enough wall stress exceeds the fracture toughness of the cylinder.

6.2.4 Step 3, selection of specific method of analysis

Each of the different failure modes can be reliably evaluated by the fitness-for-service analysis procedures. However, each failure mode (burst or leak) must be analysed by a different analytical model. The selection of which failure mode is most likely to occur depends on the cylinder design, its material properties, and the size of the flaws in the cylinder. The only failure mode evaluated was failure by bursting due to the internal pressure in the cylinders.

6.2.5 Step 4, obtaining necessary data

The data required to conduct the fitness-for-service analysis of flaws in cylinders are (1) the material properties (i.e. yield strength, tensile strength, fracture toughness, etc.); (2) the applied stress due to the pressure in the cylinder; and (3) the size, shape and location of the flaws to be evaluated. Only applied stresses caused by the internal pressure in the cylinders are considered. When exact values of some of the necessary data are not available for the specific cylinder being evaluated, the necessary data may have to be assumed or generic data for a typical cylinder may have to be used.

6.2.6 Step 5, selection of level of assessment

The selection of the level of assessment depends on the available data and on the accuracy of the evaluation that is required. For example, the API 579 methods of analysis, Section 5 (Assessment of Local Metal Loss) permit three levels of assessment depending on the available data and on the accuracy of the evaluation that is required.

The Level 1 assessment requires a minimum amount of data on the flaw size, the applied stress and the material properties. This level of assessment is the easiest to use, but the predicted failure pressure of a cylinder with a specified flaw size may be significantly less than the actual measured failure pressure of the flawed cylinder.

The Level 2 assessment requires additional, more detailed data than the Level 1 assessment for the flaw size, the applied stress and the material properties. This level of assessment uses more calculations that are complex and gives a more exact prediction of the failure pressure of the cylinder. That is, the predicted failure pressure of a cylinder with a specified flaw size is closer to the actual measured failure pressure of the flawed cylinder.

The Level 3 assessment requires the use of advanced stress analysis and material modelling procedures and exact measurements of the flaw size. This level of assessment generally results in a good prediction of the failure pressure of the cylinder. That is, the predicted failure pressure of a cylinder with a specified flaw size is very close to the actual measured failure pressure of the flawed cylinder. However, because of the increased demands for additional data and the increased complexity of the calculations, the Level 3 assessment is used only in very demanding and specialized applications.

With regard to seamless cylinders, the Level 1 assessment procedures, which are conservative and concise, were used for the flaw size analysis.

6.2.7 Step 6, selection of appropriate basis for the acceptance criteria

The next step in using the fitness-for-service assessment procedures is the choice of the basis for the acceptance criteria. The basis for the acceptance criteria is chosen for each specific case that is analysed. The acceptance criteria may be (1) the maximum allowable stress, (2) the RSF, or (3) the failure assessment diagram.

The maximum allowable stress criterion is used where the design is based on a specified fraction of the yield strength or tensile strength. This is the criterion used to specify the wall thickness in the design of new cylinders. This criterion has limited use in the fitness-for-service analysis because suitable maximum allowable stress levels cannot easily be established for cylinders containing flaws. The only place where the criterion can be used is in evaluation of general corrosion where the stress in the remaining wall can be calculated and related to the maximum allowable wall stress.

The RSF can be used for the analysis of most types of flaws in cylinders. The acceptance criterion is then specified as a fixed value of RSF. This was the criterion primarily used in the preparation of this Technical Report.

For crack-like flaws, it is necessary to use the failure assessment diagram criterion.

Cylinders containing crack-like flaws can fail either by unstable fracture or by plastic collapse. Plastic collapse occurs in cylinders with relatively large flaws that are made from high-toughness materials. Most seamless steel gas cylinders containing crack-like flaws fail by the plastic collapse mechanism. The steel cylinders containing flaws that were evaluated in this Technical Report failed by plastic collapse. (This statement is valid for the flaws that are in the acceptance range in this Technical Report.)

6.2.8 Step 7, evaluation of remaining life of cylinder

After an assessment is made of the present state of the cylinder (i.e. the predicted failure pressure of the cylinder) containing a flaw, the fitness-for-service method of analysis may also be used to assess the remaining life of the cylinder, if required. The remaining-life assessment is used to account for any increase in the size of existing flaws during the anticipated service, for example by corrosion or fatigue. This assessment is used (1) to establish presently allowable flaw sizes and (2) to define appropriate retest intervals. An assessment of the effect of fatigue on the size of existing flaws in cylinders was made to establish allowable flaws sizes for setting retest requirements.

7 Experimental results

Steel and aluminium-alloy cylinders containing machined flaws were tested by monotonic or cyclical pressurization until failure occurred as part of a project being conducted by ISO/TC 58/SC 4/WG 1 on rejection criteria for metal cylinders.

Cylinders tested by monotonic pressurization contained machined flaws mostly on the exterior of the cylinder (OD flaws). A few of the cylinders that were tested by monotonic pressurization had flaws machined on the inside surface (ID flaws). The cylinders that contained OD machined flaws had flaws that simulated notches, round LTA, rectangular LTA, and pits (small round flaws). All the machined ID flaws simulated round LTA-type flaws. The simulated flaws on the cylinders that were tested by cyclic pressurization all had OD notch-type flaws. The results of these tests were used (1) to verify that the API 579 method of analysis can be reliably used to predict the failure pressure of cylinders containing flaws, (2) to verify the calculated CFS for cylinders and (3) to verify the calculated allowable flaw sizes for cylinders. The test results for the steel cylinders tested as part of the ISO/TC 5/SC 4/WG 1 programme are shown in Tables A.1, A.2, A.3, and A.4 of Annex A.

As part of earlier work conducted by ISO/TC 58/SC 3/WG 14 on toughness and acceptance levels of steel cylinders with strength levels from less than 750 MPa to more than 1 250 MPa, several hundred monotonic hydrostatic, flawed-cylinder burst tests were conducted to evaluate the fracture performance of a wide range of steel cylinders [5]. Each test cylinder had a longitudinal notch machined in the external wall of the cylinder (see Figures 1a and 1b).

a) Circumferential view **b) Longitudinal view**

Key

r_1 tip radius (0,2 mm)

r_2 radius (35 mm)

L flaw length

a flaw depth

Figure 1 — Longitudinal notch geometry used in the tested steel cylinders

In the ISO/TC 58/SC 3/WG 14 test programme [5], the cylinders tested ranged in tensile strength from 700 MPa to 1 400 MPa. The cylinders tested were divided into five groups of materials based on the tensile strength range of the material. The cylinders ranged in outer diameter (D) from 140 mm to 240 mm, in thickness (t_d) from 3,8 mm to 14,4 mm, and had flaw sizes (longitudinal machined notches) that ranged in depth from 20 % to 90 % of the actual wall thickness and in length from four times the cylinder wall thickness to 20 times the cylinder wall thickness.

In the ISO/TC 58/SC 3/WG 14 test programme [5], steel cylinders were tested to failure by monotonic pressurization. In the fitness-for-service analysis using API 579 procedures to calculate critical flaw sizes, the critical flaw sizes are calculated at specified pressures. For the analysis that was done, the specified pressures chosen were (1) the working pressure and (2) the test pressure. Therefore, to verify the analysis used (i.e. to determine if the analysis is reliable), the calculated values are compared with the experimental values. For this purpose, the only relevant experimental data is test data that was done at the same pressure as the pressure used in the analysis, that is, either their working pressure or the test pressure. Thus, only experimental data points where the cylinders failed at either working pressure or test pressure were chosen. The other test data where the cylinders failed at different pressure were not used as they were not relevant. The selected test results for the steel cylinders tested as part of the ISO/TC 58/SC 3/WG 14 programme that were used are shown in Table A.5 and A.6 of Annex A.

Similarly, using the same concept, ISO TC 58/SC 3/WG 19 has developed data for aluminium-alloy cylinders.

8 Verification of the flaw size analysis

8.1 Seamless steel cylinders

The API 579 fitness-for-service method of analysis provides a sound technical basis for evaluating the significance of flaws in any type of pressure vessel. To demonstrate that these methods of analysis can be applied reliably to the evaluation of flaws in seamless cylinders, a limited number of seamless steel cylinders containing flaws of different types and sizes were tested hydrostatically to failure by bursting. To verify that the API 579 method of analysis reliably predicts the performance of cylinders containing flaws, the results of these burst tests were compared with the burst pressure predicted by the API 579 analysis results.

The preliminary analysis showed that the failure of the steel cylinders that were tested could be evaluated by calculating the RSF for the cylinders containing flaws. For these cylinders, the fracture toughness was sufficiently high that failure of the cylinders containing flaws was by bursting when the stress in the cylinder wall caused failure by plastic collapse as the internal pressure increased.

For this verification analysis, both LTA-type flaws and notch-type flaws were evaluated. An LTA-type flaw is a flaw that represents a typical area of wall thickness reduction due to corrosion in the cylinder. The notch-type flaw used in this Technical Report is V-shaped, and the length of the flaw is many times greater than the width of the flaw. This type of flaw represents a crack-like flaw in the cylinder. For the examples analysed here, the API 579 Level 1 assessment method was found to be adequate. The stress in the cylinder wall at the location of the flaw was only caused by the internal pressure in the cylinder.

To verify the use of the API 579 procedures, the RSF was calculated for each cylinder that was tested. The RSF is defined here as the failure pressure ratio (p_f / p_b) where p_f is the failure pressure of the cylinder containing the flaw and p_b is the failure pressure of the same type and size of cylinder that does not contain a flaw.

For RSF or p_f / p_b, the ratio for Level 1 is calculated as:

$$\text{RSF} = R_t / [1-(1/M_t)(1-R_t)] \quad (1)$$

where

M_t is the Folias stress magnification factor for through-wall flaw

$$= (1 + 0{,}48\ \lambda^2)^{1/2} \quad (2)$$

where

$$\lambda = 1{,}285\ L / (D \cdot t)^{1/2} \quad (3)$$

R_t is the remaining thickness ratio $= t_{mm} / t$ (4)

The following provides a theoretical background for Equation 1.

The failure hoop stress in the presence of a flaw is given by the following equation:

$$\sigma_f = \sigma_{flow} / M_p \quad (5)$$

where M_p is the stress magnification factor for part through flaw.

M_p is given by the following equation:

$$M_p = [1-a/(t \cdot M_t)] / (1-a/t) \quad (6)$$

where M_t is the stress magnification factor for through-wall flaw of length L.

M_t can be obtained from Equation (2) above.

The ratio, σ_f / σ_{flow}, is defined as RSF.

Therefore from Equation (5),

$$\text{RSF} = 1/ M_p = [(1 - a/t) / (1-(a/t \cdot M_t))] \quad (7)$$

$$R_t = t_{mm} / t = (t - a) / t = 1-a/t \quad (8)$$

$$a/t = 1- R_t \quad (9)$$

Substituting a/t in terms of R_t in Equation (7) results in Equation (1).