TECHNICAL REPORT

ISO/TR 12845

First edition
2010-08-01

Selected illustrations of fractional factorial screening experiments

Illustrations choisies de plans d'expériences factoriels fractionnaires

Reference number
ISO/TR 12845:2010(E)

PDF disclaimer

This PDF file may contain embedded typefaces. In accordance with Adobe's licensing policy, this file may be printed or viewed but shall not be edited unless the typefaces which are embedded are licensed to and installed on the computer performing the editing. In downloading this file, parties accept therein the responsibility of not infringing Adobe's licensing policy. The ISO Central Secretariat accepts no liability in this area.

Adobe is a trademark of Adobe Systems Incorporated.

Details of the software products used to create this PDF file can be found in the General Info relative to the file; the PDF-creation parameters were optimized for printing. Every care has been taken to ensure that the file is suitable for use by ISO member bodies. In the unlikely event that a problem relating to it is found, please inform the Central Secretariat at the address given below.

iTeh STANDARD PREVIEW
(standards.iteh.ai)

ISO/TR 12845:2010
https://standards.iteh.ai/catalog/standards/sist/10d61f6b-8205-44fe-be5b-4694ea90deba/iso-tr-12845-2010

COPYRIGHT PROTECTED DOCUMENT

© ISO 2010

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Published in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 12845 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 7, *Applications of statistical and related techniques for the implementation of Six Sigma*.

Introduction

The Six Sigma[1] and international statistical standards communities share a philosophy of continuous improvement and many analytical tools. The Six Sigma community tends to adopt a pragmatic approach driven by time and resource constraints. The statistical standards community arrives at rigorous documents through long-term international consensus. The disparities in time pressures, mathematical rigor, and statistical software usage have inhibited exchanges, synergy and mutual appreciation between the two groups.

The present document takes one specific statistical tool (two-level fractional factorial design) and develops the topic somewhat generically (in the spirit of International Standards) but then illustrates it through the use of six detailed and distinct applications. The generic description focuses on the commonalities across the designs. The annexes containing the six illustrations follow the basic framework but also identify the nuances and peculiarities in the specific applications. Each example offers at least one “wrinkle” to the problem, which is generally the case for real Six Sigma applications. It is thus hoped that practitioners can identify with at least one of the six examples, if only to remind them of the basic material on fractional factorial designs that was encountered during their Six Sigma training. Each of the six examples is developed and analysed using statistical software of current vintage. The explanations throughout are devoid of mathematical detail – such material can be readily obtained from the many design and analysis of experiments textbooks (such as those given in the Bibliography).

---

1) Six Sigma is a trade mark of Motorola, Inc.

Selected illustrations of fractional factorial screening experiments

1 Scope

This Technical Report describes the steps necessary to use and to analyse two-level fractional factorial designs through illustration with six distinct applications of this methodology.

NOTE 1 Each of these six illustrations is similar in that resource constraints precluded the possibility of naively running full factorial designs. Other commonalities among the six examples are noted [e.g. study objective, two levels for factors, response variable(s), factors affecting the response]. On the other hand, the individual illustrations have some salient features that are distinct.

NOTE 2 The examples suggest the spectrum of possibilities both in application area and in choice of fractional factorial designs. Fractional factorial designs can be used to identify important factors for subsequent investigation (screening design) and can in some cases provide a viable understanding of the process under study. Fractional factorial designs include screening designs and designs that have been popularized by Genichi Taguchi.

NOTE 3 Fractional factorial experiments are sometimes employed by individuals (so-called “black belts” or “green belts”) associated with Six Sigma methods. Six Sigma methods are concerned with problem solving and continuous improvement. A fractional factorial experiment can be a cost-effective tool for obtaining timely improvements of processes and products. Detailed discussions and treatment of other tools employed by Six Sigma practitioners can be identified in various ISO/TC 69/SC 7 documents.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3534-3, *Statistics — Vocabulary and symbols — Part 3: Design of experiments*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 3534-2, ISO 3534-3 and the following apply.

3.1
analysis of variance
ANOVA
technique which subdivides the total variation of a response variable into components associated with defined sources of variation

NOTE Adapted from ISO 3534-3:—[2], definition 3.3.8. (The notes and example are not included here.)

---

[2] To be published. (Revision of ISO 3534-3:1999)

3.2
binomial distribution

discrete distribution having the probability mass function

$$P(X = x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

where $x = 0, 1, \ldots, n$ and with indexing parameters $n = 1, 2, \ldots$, and $0 < p < 1$.

NOTE Adapted from ISO 3534-1:2006, definition 2.46. (The example and notes are not included here.)

3.3
block

collection of experimental units

NOTE Adapted from ISO 3534-3:—, 3.1.23. (The notes are not included here.)

3.4
centre point

vector of factor level settings of the form (a_1, a_2, ..., a_k), where all a_i equal 0, as notation for the coded levels of the factors

NOTE Adapted from ISO 3534-3:—, 3.1.38. (The note and example are not included here.)

3.5
design matrix

matrix with rows representing individual treatments (possibly transformed according to the assumed model) which can be extended by deduced levels of other functions of factor levels (interactions, quadratic terms, etc.) but are dependent upon the assumed model

NOTE Adapted from ISO 3534-3:—, definition 3.2.24. (The notes are not included here.)

3.6
full factorial experiment
factorial experiment

designed experiment consisting of all possible treatments formed from two or more factors, each being studied at two or more levels

NOTE Adapted from ISO 3534-3:—, 3.2.1. (The notes are not included here.)

3.7
interaction

combination of two or more factors

NOTE Adapted from ISO 3534-3:—, 3.1.15. (The notes are not included here.)

3.8
factor level

setting, value or assignment of a factor in accordance with the design region

NOTE Adapted from ISO 3534-3:—, definition 3.1.10. (The notes and examples are not included here.)

3.9
normal distribution
Gaussian distribution

continuous distribution having the probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where $-\infty < x < \infty$ and with parameters $-\infty < \mu < \infty$ and $\sigma > 0$

NOTE Adapted from ISO 3534-1:2006, definition 2.50. (The notes are not included here.)

3.10
predictor variable
factor
variable that can contribute to the explanation of the outcome of an experiment

NOTE 1 The extent to which a given predictor variable can be controlled dictates its potential role in a designed experiment. Predictor variables can be controllable (fixed), modifiable (controllable only for short duration or at considerable expense) or uncontrollable (random).

NOTE 2 A predictor variable can include a random element in it or it can, for example, be from a set of qualitative classes which can be observed or assigned without random error.

NOTE 3 The term predictor variable is typically used in contexts involving the mathematical relationship among the response variable and predictor variable(s) or functions of predictor variables. The term “factor” tends to be used operationally as a means to assess the response variable as particular factors vary.

NOTE 4 A factor may be associated with the creation of blocks.

NOTE 5 “Independent variable” is not recommended as a synonym due to potential confusion with “independence” (see ISO 3534-1:2006, 2.4). Other terms sometimes substituted for predictor variable include “input variable”, “descriptor variable” and “explanatory variable”.

[ISO 3534-3:—, definition 3.1.4]

3.11
randomization
strategy in which each experimental unit has an equal chance of being assigned a particular treatment

NOTE Adapted from ISO 3534-3:—, definition 3.1.26. (The notes are not included here.)

3.12
replication
⟨experiment⟩ multiple occurrences of a given treatment combination or setting of predictor variables

NOTE Adapted from ISO 3534-3:—, definition 3.1.35. (The notes are not included here.)

3.13
repetition
performance of an experiment more than once for a given set of predictor variables not necessarily with a complete set-up of the predictor variables

3.14
split-plot design
experimental design in which the group of experimental units (“plot”) to which the same factor level assigned to the principal factor is subdivided (“split”) so as to study one or more additional principal factors within each level of that factor

NOTE Adapted from ISO 3534-3:—, definition 3.2.16. (The example and notes have not been included here.)

3.15
design resolution
⟨design of experiments; fractional factorials⟩ length of the shortest word in the defining relation

NOTE 1 Design resolution indicates the extent of aliasing among main effects and two-way and higher-order interactions.

NOTE 2 The design resolution describes the aliasing in a particular experimental design. The numerical length is generally given by upper case Roman numerals. The three most common practical situations are resolutions III, IV and V.

- For a **resolution III** design, main effects are not aliased with other main effects. This observation can be made by examining the expressions in the defining relation. For example, $I = ABD = BCE = ACDE$ includes the expressions I, ABD, BCE and $ACDE$ and counting the number of letters for each term (1, 3, 3 and 4, respectively for this example). The shortest length of these aside from 1 (corresponding to I) is 3, which is known as the length of the shortest “character string”. At least one main effect is aliased with a two-way interaction. For example, for $I = ABD$ it would be the case that A is confounded with BD, B is confounded with AD and D is confounded with AB.

- For a **resolution IV** design, main effects are not aliased with other main effects or with any two-way interactions. At least one two-way interaction is aliased with another two-way interaction. For example, for the defining relation $I = ABCE = BCDF = ADEF$ includes the expressions I, $ABCE$, $BCDF$ and $ADEF$ and counting the number of letters for each term (1, 4, 4 and 4, respectively). The smallest value aside from 1 (corresponding to I) is 4, which is known as the length of the shortest “character string” for this design. For example, for $I = ABCE$, it would be the case that AB is confounded with CE, AC with BE, AE with BC, while A is confounded with BCE, B is confounded with ACE, C is confounded with ABE and E is confounded with ABC.

- For a **resolution V** design, main effects and two-way interactions are not aliased with any other main effects or with any other two-way interactions. For example, with $I = ABCDE$, it is clear that each main effect is confounded with a four-way interaction (A is confounded with $BCDE$) and each two-way interaction is confounded with a three-way interaction (AB is confounded with CDE).

NOTE 3 The higher the resolution, the more effects (main or interactions) can be estimated unambiguously, provided the higher-order interactions are negligible. Given a choice of two potential designs involving the same number of factors and experimental units, the design with the higher resolution should be selected. Fortunately, for most cases of k and p of practical interest, the most appropriate defining relations are recorded.

NOTE 4 Full factorial designs have no confounding. For most practical purposes, a resolution V is excellent and a resolution IV design may be adequate. Resolution III designs are useful as economical screening designs.

[ISO 3534-3:—, definition 3.2.6]

4 Symbols and abbreviated terms

The main symbols and abbreviated terms used in this Technical Report are given below.

Y	Response variable
A, B, C, D	Factors
AB, AC, AD, BC, BD, CD	Two-way interactions
ABC, ACD, BCD	Three-way interactions
$ABCD$	Four-way interactions
+1, −1	High and low settings
2^4	Four factors each at two levels
σ	Standard deviation

5 Generic description of fractional factorial designs

5.1 Overview of the structure of the examples in Annexes A to F

This Technical Report provides general guidelines on the design, conduct and analyses of two-level fractional factorial designs and illustrates the steps with six distinct applications given in the annexes. Each of the six examples in Annexes A to F follows the basic structure as given in Table 1.

The steps given in Table 1 apply to design and analysis of screening experiments in general. Each of the seven steps is explained below.

Table 1 — Basic steps in experimental design

Step 1	State the overall objective(s) of the experiment
Step 2	Describe the response variable(s)
Step 3	List the factors that might affect the response(s)
Step 4	Select a fractional factorial design
Step 5	Analyse the results – numerical summaries and graphical displays
Step 6	Present the findings
Step 7	Perform a confirmation run

5.2 Overall objective(s) of the experiment (Step 1)

Experiments are conducted for a variety of reasons. The primary motivation for the experiment should be clearly stated and agreed to by all parties involved in the design, conduct, analysis and implications of the experimental effort. There may be secondary objectives that could be addressed with this experiment but, in general, the focus is on identifying a subset of the factors that impact the response variable.

The ultimate outcome of the experiment might be to take immediate action on factor levels or to obtain a predictive model, both of which dictate some elements of the analyses. Further, the experiment could be recognized as the first step in an ongoing sequence of experiments to determine improvements to the product or process.

5.3 Response variable(s) (Step 2)

Associated with the objective of an experiment is a measurable outcome or performance measure. A response of interest could involve maximization (larger is better), minimization (smaller is better) or meet a target value (be close to a specified value). The response variable (denoted here by the variable Y) should be intimately (if not directly) related to the objective of the experiment. For some situations, there may be multiple characteristics of interest to be considered, although there is typically a primary response variable associated with the experiment. In other cases, multiple responses must be considered.

5.4 Factors affecting the response(s) (Step 3)

The response variable very likely depends in some unknown way on a variety of conditions. These could be set in the course of generating a response variable outcome such as a temperature or a time setting. These conditions are presumed to relate to controllable factors that may be continuous (temperature, concentration) or discrete (two assembly lines A or B, two vendors, two packaging styles, and so forth). For the fractional factorial experiments considered in this Technical Report, the experimental design process is simplified by selecting two levels for each factor to be varied in the experiment.

— For discrete factors, with only two possible settings, the levels are just these two settings.

— For continuous factors, discretion can be used in choosing the two specific values. In some cases, one setting could be the historical value and the other a proposed value. In other cases, the two settings could be nominal adjustments from the historical setting. In any event, the settings should be sufficiently far removed to have an opportunity to reveal an impact subject to the inherent uncertainty, while not being so disparate that the settings are unreasonable from a practical, safety or sensibility standpoint. The setting of levels of continuous factors benefits from an expert collaborating in the experiment.

There may be additional factors that could impact that response variable but may be deemed much less important than the chosen factors or are too difficult or expensive to control. Finally, it is the case that the factors are to be set independently from each other. It could, however, be discovered that the factors interact (the setting of one factor impacts how a second factor affects the response).

5.5 Select a fractional factorial design (Step 4)

It is usual that the number of factors thought to be important to experiment with is large, e.g. 10 or more. Designing an experiment to investigate all possible factors and their combinations would lead to excessively large experiments. For example, in studying only eight factors at two levels each, the number of experimental runs is 256. For three levels, this would be 6 561 runs! For most organizations, this would be a very extravagant use of resources. Thus, the need is there to design experiments using fewer runs but still giving the experimenter all of the important experimental results. Although this gives full information about the effects of the factors, it also delivers information about higher-order factor combinations that are of minimal practical utility. Consequently, a balance is sought to reduce the experimental effort without foregoing the information about main factors of interest by designing a fractional experiment.

The examples provided in the annexes use 6 to 15 factors with a maximum of only 40 runs to identify the critical factors.

A classical design consists of 16 runs obtained by considering all combinations of four factors with two possible levels. See ISO/TR 29901 for a description. Table 3 provides the basic layout in a standard order for ease of understanding. Each row of the table represents one set of experimental conditions that when run will produce a value of the response variable Y. The four factors are designated as A, B, C and D. For an individual factor, the level "–1" is the "low" setting, or one of the two levels if the factor is categorical. The level "+1" is the "high" setting, or the other level of the categorical factor. The column "Y" is a placeholder for the response value once a run has occurred.

This will provide unambiguous information about the following factors and their interactions, namely:

A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD and $ABCD$

For most practical applications, the higher-order interactions, e.g. ABC, are often taken as unimportant and can therefore, in a design, be discounted. This, therefore, becomes the way in which larger numbers of factors can be incorporated into designs that are intended for full factor combinations, albeit for a smaller number of factors.

It is imperative that the experimenter be aware of the exact pattern of main effects and the associated two-way interactions created by this discounting of higher-order interactions. A careless selection may inhibit the clear identification of the effects (be they main or two-way interactions). These aspects can be seen in the six different examples contained in the annexes.

Table 2 provides a useful summary of the shortcomings of certain designs. The table describes the three most common design resolutions. The examples in the annexes (see Table 4) refer to these resolutions.

Table 2 — Selected design resolutions

Resolution	Description
III	No main effects are aliased with any other main effect, but at least one main effect is aliased with two-factor interactions.
IV	No main effects are aliased with any other main effect or two-factor interactions, but at least one two-factor interaction is aliased with other two-factor interactions and at least one main effect is aliased with three-factor interactions.
V	No main effects or two-factor interactions are aliased with any other main effect or two-factor interactions, but at least one two-factor interaction is aliased with three-factor interactions and at least one main effect is aliased with four-factor interactions.

The design choice ultimately rests with the aforementioned considerations as well as resource constraints. Guidance of a general nature can be gleaned from the reference texts in the Bibliography. Increasingly, software packages contain useful guides to design selection and algorithms for optimal design where the problem is well defined.

Table 3 — Layout of a generic 2^4 full factorial design

Row number	A	B	C	D	Y	Run order
1	−1	−1	−1	−1	y_1	6
2	+1	−1	−1	−1	y_2	14
3	−1	+1	−1	−1	y_3	4
4	+1	+1	−1	−1	y_4	11
5	−1	−1	+1	−1	y_5	9
6	+1	−1	+1	−1	y_6	2
7	−1	+1	+1	−1	y_7	3
8	+1	+1	+1	−1	y_8	1
9	−1	−1	−1	+1	y_9	8
10	+1	−1	−1	+1	y_{10}	13
11	−1	+1	−1	+1	y_{11}	7
12	+1	+1	−1	+1	y_{12}	10
13	−1	−1	+1	+1	y_{13}	15
14	+1	−1	+1	+1	y_{14}	16
15	−1	+1	+1	+1	y_{15}	5
16	+1	+1	+1	+1	y_{16}	12

5.6 Analyse the results — Numerical summaries and graphical displays (Step 5)

At the completion of the conduct of the experiment, the y_i values would be replaced by the actual observed responses. Many statistical software packages exist to produce output to aid in the understanding of the results of the experiment. Of immediate concern is the determination of the impact of the factors individually on the response variable. By the nature of fractional factorial designs, it is hoped, if not presumed, that the main effects that are estimated are indeed attributable to the main factor although it is recognized that aliasing could be taking place.

Much of the analysis that is conducted for 2^4 full factorials (see ISO/TR 29901) is applicable here with the proviso that aliasing patterns must be recognized and properly evaluated. In particular, main effects plots, interaction plots, Pareto diagrams, normal and/or half-normal probability plots are relevant instruments of analysis. Although the emphasis of the analysis is likely to be graphical in its nature, additional supporting documentation, e.g. effects table, should be given.

5.7 Present the findings (Step 6)

Almost certainly, the conclusions from a screening experiment will constitute a description of the next phase of experimentation. Design of experiments is inherently sequential in nature and a screening experiment is frequently the first phase of investigation and is often followed by subsequent experiments with fewer factors and, sometimes, more levels. In some cases, the screening experiment supports a predictive model that can suggest alternative settings for the important factors. Although the process has not been fully optimized, superior settings may, in the meantime, have been identified.

Further, the experiment may reveal important interactions which may be, unfortunately, aliased with others. An example of this can be found in Annex C.

5.8 Perform confirmation runs (Step 7)

Confirming experiments are considered to be good “practice” in that they should provide the experimenter assurance that

a) the original experiment was correctly conducted,

b) the findings still hold true at a later point in time, and

c) there were no apparent “lurking” variables that were ignored in the original experimental design.

This being said, there are circumstances that preclude a confirmation run, e.g. time pressures due to commercial issues or matters of cost (resources) or management acceptance of the risk of a faulty screening experiment.

6 Description of Annexes A to F

6.1 Comparing and contrasting the examples

Six distinct examples of fractional factorial designs are illustrated in Annexes A to F. Each example follows the general template given in Table 1.

6.2 Experiment summaries

Table 4 summarizes the six examples detailed in the annexes and indicates aspects of the analyses that were unique to that experiment.

Table 4 — Experiment summaries found in annexes

Annex	Experiment	Business area	Resolution	Problem-specific aspects
A	Direct mail campaign	Marketing	IV	Proportion response variable, standard errors based on binomial distribution.
B	Polymer emulsion optimization	Chemical engineering	IV	Blocking, response transformation, two responses.
C	PVC foam formulation	Chemical engineering	III	Analysis of three responses with different levels of success, due to severe confounding of effects.
D	Insulin process validation	Pharmaceuticals	IV	Eight responses, blocking, factor levels expressed as a range, interest in no effect as a result.
E	Washing machine robustness	Mechanical engineering	Not applicable	Taguchi orthogonal arrays, inner/outer arrays.
F	Aggregated shipworm bacterium	Bioengineering	III, IV	Plackett-Burman design.