INTERNATIONAL
STANDARD

ISO
22514-6

First edition
2013-02-15

Statistical methods in process management — Capability and performance —

Part 6:
Process capability statistics for characteristics following a multivariate normal distribution

Méthodes statistiques dans la gestion des processus — Capabilité et performance —

Partie 6: Statistiques de capabilité pour un processus caractérisé par une distribution normale multivariée

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Reference number
ISO 22514-6:2013(E)

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ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.org
Web www.iso.org

Published in Switzerland

Contents

Page

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ISO 22514-6:2013
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Foreword

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International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 22514-6 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

ISO 22514 consists of the following parts, under the general title *Statistical methods in process management — Capability and performance*:

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- — *Part 1: General principles and concepts*
- — *Part 2: Process capability and performance of time-dependent process models*
- — *Part 3: Machine performance studies for measured data on discrete parts*
- — *Part 4: Process capability estimates and performance measures* [Technical Report]
- — *Part 5: Process capability statistics for attribute characteristics*
- — *Part 6: Process capability statistics for characteristics following a multivariate normal distribution*
- — *Part 7: Capability of measurement processes*
- — *Part 8: Machine performance of a multi-state production process*

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Introduction

Due to the increased complexity of the production methods and the increasing quality requirements for products and processes, a process analysis based on univariate quantities is in many cases not sufficient.

Instead, it may be necessary to analyse the process on the basis of multivariate product quantities. This can, for instance, be in such cases where geometric tolerances, dynamic magnitudes such as imbalance, correlated quantities of materials or other procedural products are observed.

By analogy with ISO 22514-2, ISO 22514-6 provides calculation formulae for process performance and process capability indices, which take into consideration process dispersion as well as process location as an extension to the corresponding indices for univariate quantities. The indices proposed are indeed based on the classical C_p and C_{pk} indices for the one-dimensional case. The motivation for the extension to the multivariate case is explained in Annex A.

Examples of possible applications are two-dimensional or three-dimensional positions, imbalance or several correlated quantities of chemical products.

The dispersion of the measuring results comprises the dispersion of the product realization process and the precision of the measuring process. It is assumed that the capability of the used measuring system was demonstrated prior to the determination of the capability of the product realization process.

The calculation method described here should be used to support an unambiguous decision, especially if

- limiting values for process capability indices for multivariate, continuous product quantities are specified as part of a contract between customers and suppliers, or
- the capabilities of different constructions, production methods or suppliers are to be compared, or
- production processes are to be approved, or
- problems are to be analysed and decisions made in complaint cases or damage events.

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NOTE Product realization processes include e.g. manufacturing processes, service processes, product assembly processes.

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Statistical methods in process management — Capability and performance —

Part 6:
Process capability statistics for characteristics following a multivariate normal distribution

1 Scope

This part of ISO 22514 provides methods for calculating performance and capability statistics for process or product quantities where it is necessary or beneficial to consider a family of singular quantities in relation to each other. The methods provided here mostly are designed to describe quantities that follow a bivariate normal distribution.

NOTE In principle, this part of ISO 22514 can be used for multivariate cases.

This part of ISO 22514 does not offer an evaluation of the different provided methods with respect to different situations of possible application of each method. For the current state, the selection of one preferable method might be done following the users preferences.

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The purpose is to give definitions for different approaches of index calculation for performance and capability in the case of a multiple process or product quantity description.

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2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 22514-1, *Statistical methods in process management — Capability and performance — Part 1: General principles and concepts*

ISO 22514-2, *Statistical methods in process management — Capability and performance — Part 2: Process capability and performance of time-dependent process models*

3 Terms and definitions

For the purpose of this document, the terms and definitions given in ISO 22514-1 and ISO 22514-2 and the following apply.

3.1
quantity
property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference

[ISO/IEC Guide 99:2007, 1.1]

3.2
multivariate quantity
set of distinguishing features

Note 1 to entry: The set can be expressed by a *d*-tuple, i.e. an ordered set consisting of *d* elements.

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Note 2 to entry: If the single quantities in the set are denoted by x_i where $i = 1, 2…d$, the multivariate quantity is expressed as the vector $\underline{\mathbf{x}} = (x_1, x_2, ... x_d)^\mathrm{T}$. Thus, a multivariate quantity can be considered as a feature vector of a product. The value of the multivariate quantity is represented by a point in the *d*-dimensional feature space.

Note 3 to entry: The selection of the quantities in a vector is made for specific technical reason.

Note 4 to entry: All single quantities combined in the vector of a multivariate must be measurable in the same product or object.

Note 5 to entry: If the multivariate quantity is to be described by means of statistics, the vector is to be considered as a random vector following a *d*-dimensional multivariate distribution.

EXAMPLE 1 A number of $d = 3$ quantities like x_1 = colour, x_2 = mass and x_3 = number of defects are combined in order to use only one statistic for process assessment. The dimension of vector $\mathbf{x}$ is $d = 3$.

EXAMPLE 2 In order to evaluate a boring process, the position of the borehole axis is measured in an *x*-coordinate and *y*-coordinate. The coordinates are combined to the two-dimensional multivariate quantity $\mathbf{x}$ where the component x_1 is the *x*-coordinate and x_2 is the *y*-coordinate.

EXAMPLE 3 Imbalance of a wheel.

3.3
tolerance region

region in the feature space that contains all permitted values of the *multivariate quantity* (3.2)

Note 1 to entry: The region is limited by lines, surfaces or hyper-surfaces in the *d*-dimensional space and not necessarily closed. The form and extension of the region are specified by one or more parameters.

Note 2 to entry: Typical shapes of tolerance regions are rectangles, ellipses (or circles) in the two-dimensional case, cuboids or hyper-cuboids, ellipsoids or hyper-ellipsoids or composite prismatic shapes. Figure 1 shows examples of tolerance regions in the two-dimensional space.

Note 3 to entry: The tolerance region is specified based on the required function of the product. Products showing values outside the region are assumed to not fulfil functional requirements. Those products are considered to be nonconforming parts.

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Note 4 to entry: In order to assess a product with respect to the limits of the tolerance region, the order of the single quantity in the multivariate quantity and the number *d* of dimension must be equal to that of the tolerance region description.

EXAMPLE A tolerance zone as it is defined in ISO 1101 for geometrical product features can be considered as a tolerance region. In that case, limiting geometrically perfect lines or surfaces correspond to the boundary and the tolerance correspond to the parameter of the tolerance region.

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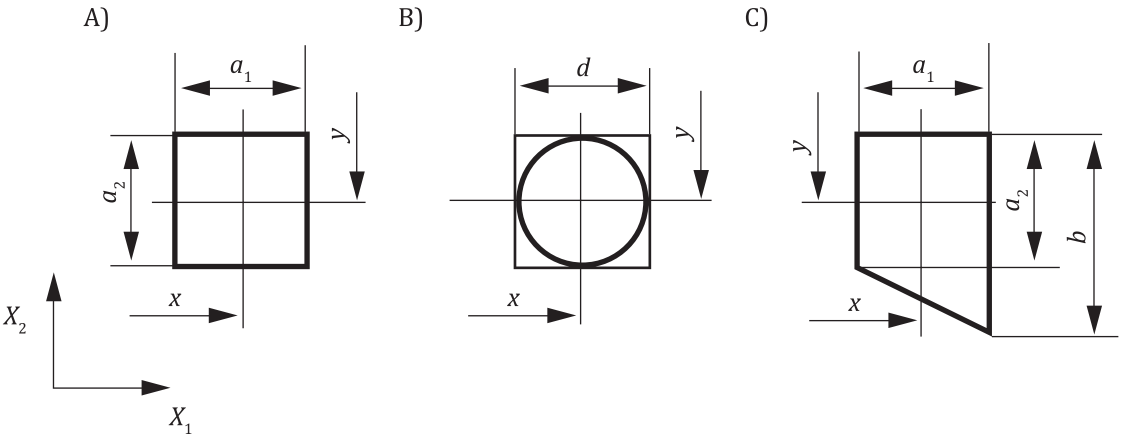

Key

A	rectangular tolerance region with parameters a_1, a_2, x and y
B	circular tolerance region with parameters d, x and y
C	triangularly extended rectangular region with parameters a_1, a_2, b, x and y

Figure 1 — Examples of tolerance regions in the two-dimensional space of the bivariate quantity $(x_1, x_2)^T$

3.4

process capability

distribution of measured *quantity* (3.1) values from a process that has been demonstrated to be in statistical control and which describes the ability of a process to produce quantity values that will fulfil the requirements for that quantity

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Note 1 to entry: The process capability index provides the ability to meet requirements of the measured quantity.

Note 2 to entry: The abbreviation for process capability index is PCI.

3.5

estimated process capability

statistical description of a *process capability* (3.4)

3.6

process performance

distribution of measured *quantity* (3.1) values from a process

Note 1 to entry: The process may not have been demonstrated to be in statistical control.

3.7

estimated process performance

statistical description of a *process performance* (3.6)

4 Abbreviated terms

MMC	maximum material condition
PCI	process capability index

5 Process analysis

The purpose of process analysis is to obtain sound knowledge of a process. This knowledge is necessary for controlling the process efficiently, so that the products realized by the process fulfil the quality requirement.

A process analysis is always an analysis of one or more quantities that are considered to be important to the process.

Product quantities can often be analysed instead of process quantities because product quantities not only characterize the products, but due to their correlation with process quantities they also characterize the process creating these products.

The values of the quantities under consideration are typically determined on the basis of samples taken from the process flow. The sample size and frequency should be chosen depending on the type of process and the type of product so that all important changes are detected in time. The samples should be representative for the multivariate quantities under consideration. (Univariate quantity values are considered in ISO 22514-2.) This part of ISO 22514 describes multivariate capability statistics.

To estimate the PCI, the sample size should preferably be at least 125.

6 Use of multivariate process capability and performance assessment

The purpose of a process capability index is to reflect how well or how badly a process generates qualified products. The use of PCI for multivariate quantities should reflect this process behaviour better than PCI for single quantities would. Since a variety of multivariate PCI definitions exists, the selection of a specific definition to be used will remain in the user's accountability. However, the following guidance is given as to when a multivariate PCI should be preferred at all.

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A multivariate assessment of process capability and performance is suitable if at least one of the following circumstances is applicable.

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- It is found to be advantageous to describe process capability and performance with only one comprehensive statistic instead of a high number of single statistics for each product quality quantity.
- The boundary of the tolerance region cannot be expressed independently for all quantities, i.e. at least one tolerance limit for one quantity is a function of another quantity. This is the case if the tolerance region is not of rectangular or cuboid shape.
- The single quantities that could be combined to a multivariate one appear to be correlated among each other.

EXAMPLE In the case of a two-dimensional position tolerance for a borehole axis, the tolerance region is a circle with defined distances in an x- and y-coordinate direction from the references, see 8.1. The result of the hole axis measurement will be a value for the x- and y-coordinate. The tolerance limit for the x-coordinate cannot be expressed independently from the y-coordinate. Thus, a bivariate assessment is to be applied.

7 Calculation of process capability and process performance

7.1 Description of Types I and II

In the multivariate domain, different approaches exist for measuring process capability and process performance. This part of ISO 22514 describes examples of two different types of indices: Type I and Type II. The distinction between the types is based on whether the index is defined based on probability or defined geometrically by relating the area or volume of a tolerance or process region.

The following description of the types applies:

- Type I Based on the probability of conforming or non-conforming products P, the index is calculated using the relationship between the index and the said probability in a univariate normal case.

- Type II The index is calculated as the ratio of the area or volume of the tolerance region to the area or volume of the region covered by the process variation.

For practical reasons, the multivariate normal distribution mode has been chosen for the calculation of the statistics which are described in this clause. However, the choice of normal distribution does not exclude that in special cases other model distributions will describe the reality better. Also, for practical reasons, in this part of ISO 22514, the process variation region has been chosen to be of ellipsoid shape.

The most important properties of the multivariate normal distribution are explained in Annex A.

Because of that choice, additional transformations should be applied to make the shape of the process variation intervals comparable to the shape of the tolerance region. Thus, three further principles are to be distinguished. These are the principles of transforming the shape of the

a) tolerance region into the shape of the process variation interval,

b) process variation interval into the shape of the tolerance region, and

c) tolerance region and/or the process variation into a new function-oriented dimension.

Both the above-mentioned types and the principles can be combined to define a multivariate PCI. Each combination, however, may not be useful. There is, for instance, no known definition of a type Ib PCI.

The term “capability” can only be used for processes that have been demonstrated to be in statistical control using control charts. In the multivariate case, the distinction between special and common causes is usually more difficult than in the univariate case. If the process has not been demonstrated to be in statistical control, the term “performance” is used in this part of ISO 22514.

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ISO 22514-6:2013
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7.2 Designation and symbols of the indices

7.2.1 General

Different symbols are currently used for multivariate index definitions in industry and science. Currently used symbols try to distinguish between the types of calculation or to specify their use. This part of ISO 22514 uses the designation C_p and/or C_pk for basic definitions of calculation. Furthermore, it will be distinguished between process capability and process performance in applying the indices by using capitals “C” for capability and “P” for performance.

7.2.2 Process capability index

Consider a d-dimensional normal distribution $N_d(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with the mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. If the tolerance region is not of elliptic shape (circle, ellipse if $d = 2$ or sphere, ellipsoid if $d = 3$ or hyper-sphere, hyper-ellipsoid if $d > 3$), it is to be transformed into a modified tolerance region that is of elliptic shape. This is to be done by determining the largest ellipse (or ellipsoid, hyper-ellipsoid) that is centred at the target and completely fits into the original tolerance region.

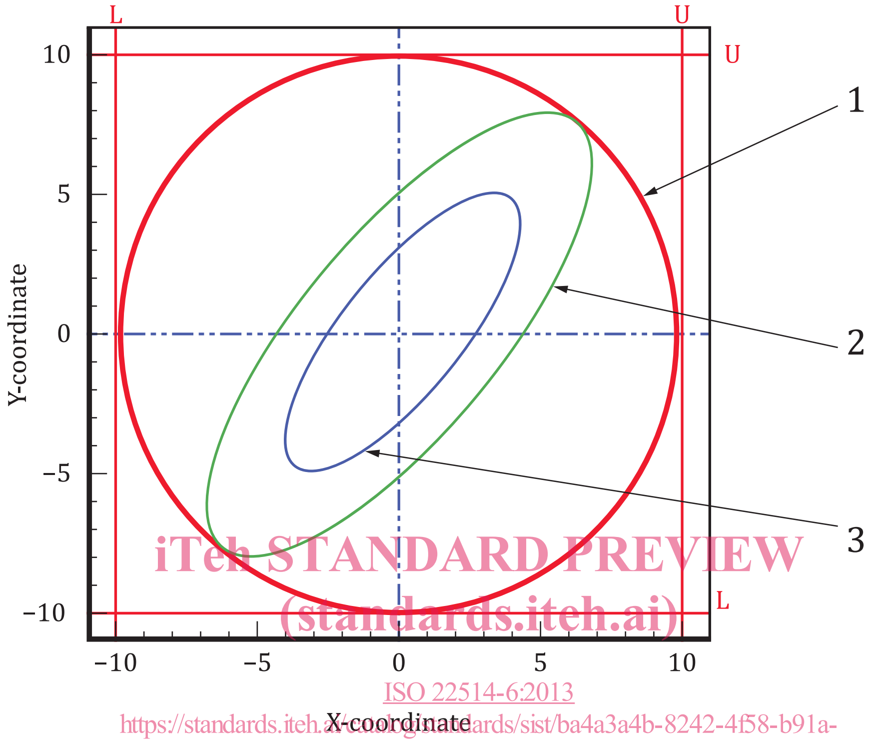
In order to calculate the multivariate C_p index, the normal distribution shall be centred to have the mean at the centre of the elliptic tolerance region. For that normal distribution, determine the largest contour ellipsoid that is completely contained in the elliptic tolerance region and calculate the probability of the volume bounded by that contour ellipse under the d-dimensional normal distribution with covariance matrix $\boldsymbol{\Sigma}$ and mean at the centre of the elliptic tolerance region. Denote that probability by P. Then, the multivariate C_p index is

$$C_\mathrm{p} = \frac{1}{3}\Phi^{-1}\left(\frac{P+1}{2}\right)$$

The calculation of P, the probability for observations of $\underline{x}$ within the determined contour ellipse (ellipsoid/hyper-ellipsoid) for any d can be done by using the relation to the F-distribution. The explanation is given in Clause A.1.

In order to estimate a C_p index from d-dimensional data, start by estimating the covariance matrix of the multivariate normal distribution from the data. Denote the estimate by $\hat{\Sigma}$ and use that covariance matrix to determine the contour ellipsoid and its probability $\hat{P}$. Finally, the estimated multivariate C_p index is

$$\hat{C}_\text{p} = \frac{1}{3}\Phi^{-1}\left(\frac{\hat{P}+1}{2}\right)$$



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ISO 22514-6:2013
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Key

1 elliptic tolerance zone

2 contour ellipse used for the calculation of the capability index

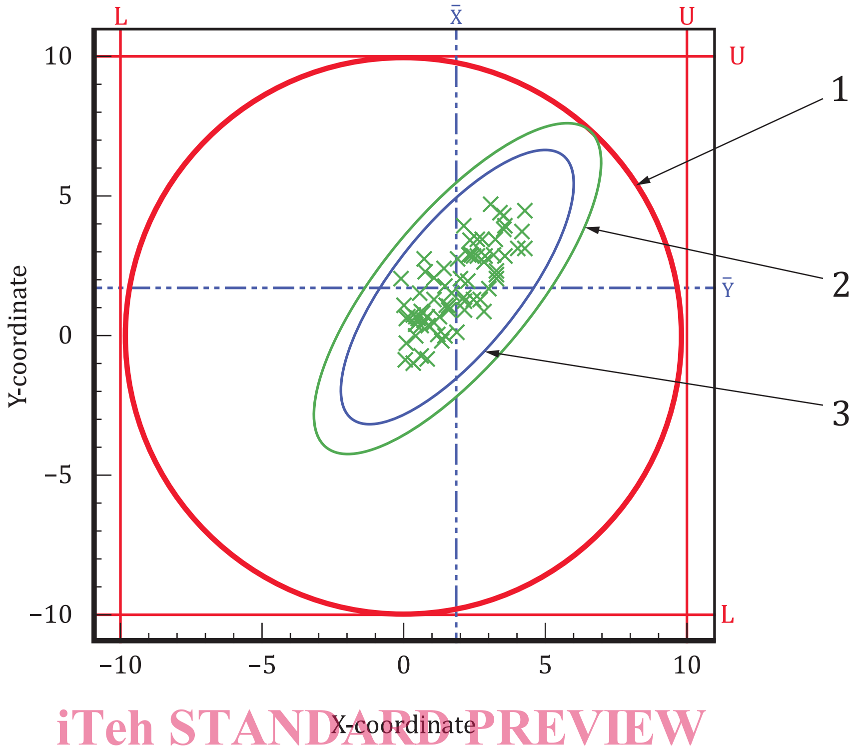
3 contour ellipse corresponding to the probability zone 99,73 %

Figure 2 — Contour ellipse and tolerance zone used to calculate the capability index for $d = 2$

In Figure 1, the contour ellipse with probability 99,73 % is completely contained in the contour ellipse used for the calculation of the index. When this is the case, the index will be larger than 1.

We use the symbol C_p for this index as for the classical capability index for the univariate normal distribution. The reason is that this calculation method in the one-dimensional case gives the classical C_p index. This is explained in Clause A.1.

7.2.3 Minimum process capability index



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(standards.iteh.ai)
ISO 22514-6:2013
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Key

1 elliptic tolerance zone
2 contour ellipse used for the calculation of the C_{pk} index
3 contour ellipse corresponding to the probability 99,73 %

Figure 3 — Tolerance zone and contour ellipse used to calculate the capability index for $d = 2$

Calculation of the C_{pk} index involves both the mean and the variance of the distribution, so consider again a d-dimensional normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. For the $N_d(\boldsymbol{\mu},\boldsymbol{\Sigma})$ distribution, calculate

— the largest contour ellipse (ellipsoid, hyper-ellipsoid) that is completely contained in the elliptic tolerance region, if $\boldsymbol{\mu}$ is contained in the tolerance region, or

— the largest contour ellipse (ellipsoid, hyper-ellipsoid) that is not contained in the tolerance region, if $\boldsymbol{\mu}$ is not contained in the tolerance zone.

Now, the probability, P, of the area (volume) contained in the contour ellipse (ellipsoid, hyper-ellipsoid) under the $N_d(\boldsymbol{\mu},\boldsymbol{\Sigma})$ distribution is calculated. Finally, the C_{pk} index is calculated as

$$C_{\mathrm{pk}} = \frac{1}{3}\Phi^{-1}\left(\frac{P+1}{2}\right)$$

if $\boldsymbol{\mu}$ is in the tolerance region and as

$$C_{\mathrm{pk}} = \frac{1}{3}\Phi^{-1}\left(\frac{1-P}{2}\right)$$

if $\boldsymbol{\mu}$ is not in the tolerance region.

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