

7 Reference datums, embeddings, and object reference models

7.1. Introduction

This International Standard specifies reference datums as geometric primitives in position-space that are used to model aspects of object-space through a process termed reference datum binding. A reference datum binding is an identification of a reference datum in position-space with a corresponding constructed entity in object-space (see 7.2). Reference datums for celestial bodies of interest are specified in Annex D.

A normal embedding is a distance-preserving function from position-space to object-space. A normal embedding establishes a position-space model of object-space. The image of a bound reference datum under a normal embedding may or may not coincide with the constructed entity of the reference datum binding. If they coincide, the reference datum binding and the normal embedding are said to be compatible (see 7.3).

A set of bound reference datums can be selected so as to be compatible with only one normal embedding. In this way, a set of bound reference datums with properly constrained relationships can specify a unique normal embedding. Such a constrained set of bound reference datums is called an object reference model. Object reference models that use the same set of reference datum primitives and similar binding constraints are abstracted in the notion of an object reference model template. Object reference model templates provide a uniform method of object reference model specification (see 7.4).

Object reference models for celestial objects of interest are specified in Annex E. For these celestial objects, one object reference model is designated as the reference model for the object. The transformation from each object reference model to the reference model for the object is termed the reference transformation. Time-independent reference transformations are also specified.

Object-specific rules to bind reference datums in a way that is compatible with the binding constraints of an object reference model template are defined in 7.5. These object-specific binding rules are used to provide a uniform method of specifying object reference models for specific dynamically-related celestial bodies.

7.2. Reference datums

7.2.1 Introduction

A *reference datum* (RD) is a geometric primitive in position-space that is used to model an aspect of object-space through a process termed RD binding. In this International Standard, the reference datum concept is defined for 1D, 2D, and 3D position-spaces. In the 2D and 3D cases, this International Standard specifies a small set of reference datums for use in its own specifications. This set is not intended to be exhaustive. Users of this International Standard may specify additional reference datums by registration in accordance with Clause 13.

7.2.2 Reference datums

In this International Standard, an RD geometric primitive is expressed in terms of analytic geometry in position-space. RDs are designed to correspond to constructed entities of similar geometric type in an object-space through a process called RD binding (see 7.2.5). These geometric types are limited to a point, a directed curve, or an oriented surface. The analytic form of the position-space representation and its corresponding object-space geometric representation are described by category and position-space dimension in Table 7.1. An RD of a given category is specified by the parameters and/or the analytic expression of its position-space representation.

Table 7.1 — RD categories

RD category	Position-space representation			Object-space representation
	1D	2D	3D	
Point	(a) real a	(a, b) real a, b	(a, b, c) real a, b, c	a point in the object-space
Directed curve		$p = F(t)$, F is smooth and \mathbf{R}^2 valued. Direction at $p_0 = F(t_0)$ is $n = \frac{dF}{dt}(t_0)$.	$p = F(t)$, F is smooth and \mathbf{R}^3 valued. Direction at $p_0 = F(t_0)$ is $n = \frac{dF}{dt}(t_0)$.	a curve in the object-space with a designation of direction along the curve
Oriented surface			Implicit definition $f(p) = 0$. Positive side of surface (orientation): $f(p) > 0$	a surface in the object-space with a designation of one side as positive

This International Standard specifies 2D and 3D RDs by RD category in [Table 7.4](#) through [Table 7.8](#). The specification elements of those tables are defined in [Table 7.2](#). 3D RDs based on ellipsoids are described in [7.2.3](#) and [7.2.4](#) and specified in [Annex D](#) with specification elements defined in [Table 7.9](#). [Table 7.3](#) is a directory of RD specification tables or, in the case of 3D RDs based on ellipsoids, RD specification directories.

Table 7.2 — RD specification elements

Element	Definition
RD label	The label for the RD (see 13.2.2).
RD code	The code for the RD (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Description	A description of the RD including any common name for the concept.
Position-space representation	The analytic formulation of the RD in position-space

Table 7.3 — RD specification directory

Position-space dimension	RD category	Table number
2D	point	Table 7.4
3D	point	Table 7.5
2D	directed curve	Table 7.6
3D	directed curve	Table 7.7
3D	oriented surface	Table 7.8 and Table 7.10

Table 7.4 — 2D RDs of category point

RD label	RD code	Description	Position-space representation
ORIGIN_2D	1	Origin in 2D	(0,0)
X_UNIT_POINT_2D	2	<i>x</i> -axis unit point in 2D	(1,0)
Y_UNIT_POINT_2D	3	<i>y</i> -axis unit point in 2D	(0,1)

Table 7.5 — 3D RDs of category point

RD label	RD code	Description	Position-space representation
ORIGIN_3D	4	Origin in 3D	(0,0,0)
X_UNIT_POINT_3D	5	<i>x</i> -axis unit point in 3D	(1,0,0)
Y_UNIT_POINT_3D	6	<i>y</i> -axis unit point in 3D	(0,1,0)
Z_UNIT_POINT_3D	7	<i>z</i> -axis unit point in 3D	(0,0,1)

Table 7.6 — 2D RDs of category directed curve

RD label	RD code	Description	Position-space representation
X_AXIS_2D	8	<i>x</i> -axis in 2D	$F(t) \equiv (t, 0)$
Y_AXIS_2D	9	<i>y</i> -axis in 2D	$F(t) \equiv (0, t)$

Table 7.7 — 3D RDs of category directed curve

RD label	RD code	Description	Position-space representation
X_AXIS_3D	10	<i>x</i> -axis in 3D	$F(t) \equiv (t, 0, 0)$
Y_AXIS_3D	11	<i>y</i> -axis in 3D	$F(t) \equiv (0, t, 0)$
Z_AXIS_3D	12	<i>z</i> -axis in 3D	$F(t) \equiv (0, 0, t)$

Table 7.8 — 3D RDs of category oriented surface

RD label	RD code	Description	Position-space representation
XY_PLANE_3D	13	<i>xy</i> -plane	$f(x, y, z) \equiv z = 0$
XZ_PLANE_3D	14	<i>xz</i> -plane	$f(x, y, z) \equiv y = 0$
YZ_PLANE_3D	15	<i>yz</i> -plane	$f(x, y, z) \equiv x = 0$

7.2.3 Ellipsoidal RDs

The RDs specified in this International Standard include RDs based on oblate ellipsoids, prolate ellipsoids, and tri-axial ellipsoids. These RDs are 3D and of category oriented surface. These RDs are specified based upon certain geometrically-defined parameters. The position-space representations of oblate and prolate ellipsoid RDs are expressed in the form:

$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1 = 0. \quad (1)$$

When $a \geq b$, an RD of this form is an *oblate ellipsoid RD* with major semi-axis a and minor semi-axis b as illustrated in [Figure 7.1](#).

Spheres shall be considered as a special case of oblate ellipsoids. If $a = b$, an oblate ellipsoid RD may be called a *sphere RD*. In this case, the value $r = a = b$ is the radius of the sphere RD.

NOTE In general usage, spheres are a limiting case of oblate, prolate, and tri-axial ellipsoids. To remove ambiguity, in this International Standard spheres are a special case of oblate ellipsoids only.

When $a < b$, an RD of this form is a *prolate ellipsoid RD* with major semi-axis b and minor semi-axis a , as illustrated in [Figure 7.1](#).

Instead of specifying the parameters of an oblate ellipsoid RD as the major semi-axis a and the minor semi-axis b , it is both equivalent and sometimes convenient to use the major semi-axis a and the flattening f as defined in [Equation \(2\)](#). The minor semi-axis b may be expressed in terms of the major semi-axis a and the flattening f as in [Equation \(3\)](#). The flattening of a sphere RD is zero ($f = 0$).

flattening definition: $f \equiv \frac{a-b}{a}$ (2)

minor semi-axis relationship: $b = a - af$ (3)

The position-space representation of a tri-axial ellipsoid RD is expressed in the form:

$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0. \quad (4)$$

The semi-axes a , b , and c shall be positive non-zero and $a \neq b \neq c \neq a$.

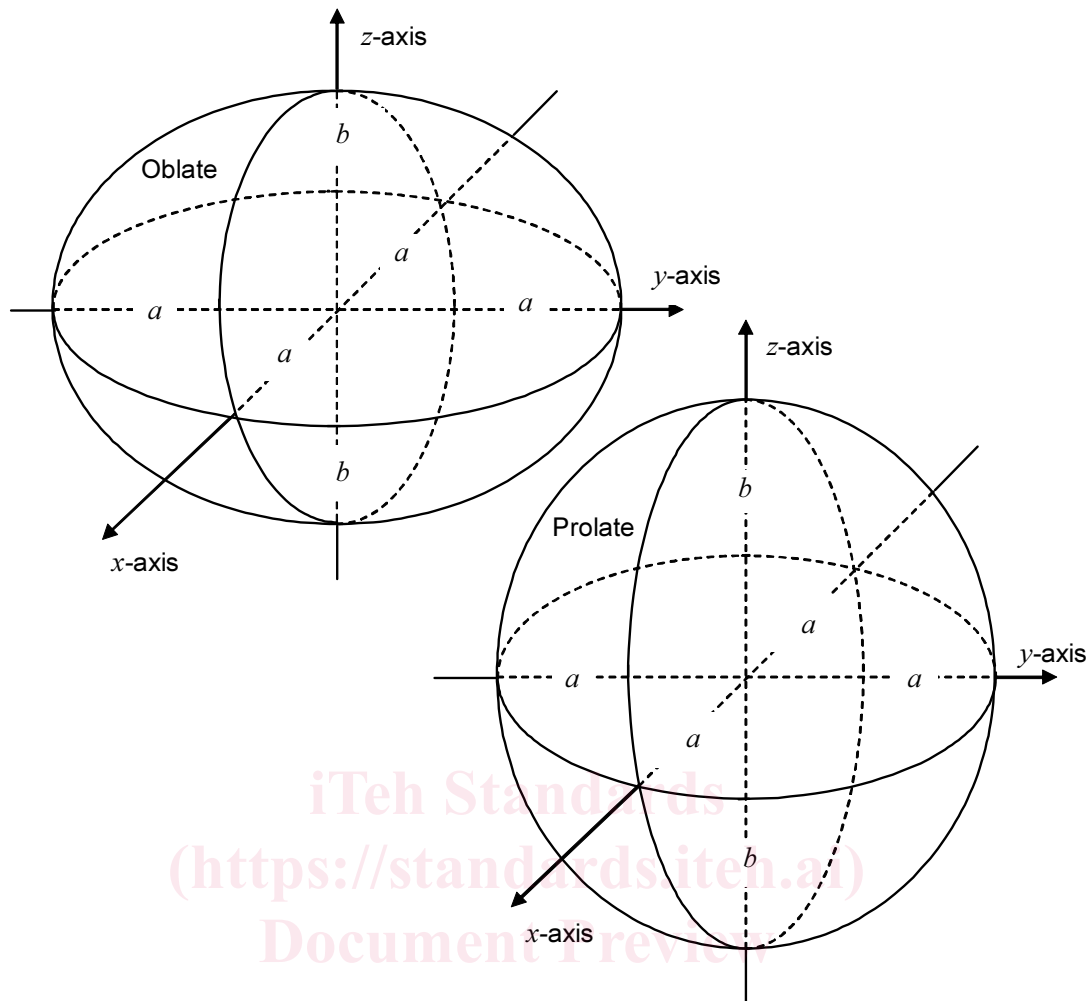


Figure 7.1 — Oblate and prolate ellipsoids

7.2.4 RDs associated with physical objects

In the case of ellipsoid RDs intended for modelling physical objects of interest, published parameter values for these RDs are used. The specification of these RDs includes the published ellipsoid parameters and the identification of the associated physical object. The specification elements for physical object RDs are defined in [Table 7.9](#).

Table 7.9 — Physical object RD specification elements

Element	Specification
RD label	The label for the RD (see 13.2.2).
RD code	The code for the RD (see 13.2.3).
Description	The description including the name as published or as commonly known.
Physical object	The name of the physical object.

Element	Specification	
Parameters	Oblate ellipsoid case (including the sphere case)	Major semi-axis, a Flattening, f
	Prolate ellipsoid case	Minor semi-axis, a Major semi-axis, b
	Tri-axial ellipsoid case	x -semi-axis, a y -semi-axis, b z -semi-axis, c
	<p>RD parameters shall be specified by value or by reference (see 13.2.5).</p> <p>If by value, the value(s) shall be followed by an error estimate expressed in one of the following forms:</p> <ul style="list-style-type: none"> a) error estimate: unknown b) error estimate: assumed precise c) error estimate (1σ): <parameter name>:<error value> d) error interval: <parameter name> \pm <error value> <p>EXAMPLE error estimate (1σ): $a : 1\,250$, $f^{-1} : 0,25$.</p> <p>If by reference, this specification element shall express the value(s) and error estimate(s) using the terminology found in the reference. These terms shall be enclosed in brackets ($\{ \}$). Any parameter value that is not specified in the citation(s) shall be specified as in the “by value” case. An error estimate for b or for f^{-1} may be substituted in place of an error estimate for f.</p>	
Date	The date the RD parameters were specified or published.	
References	The references (see 13.2.5).	

<https://standards.iteh.ai/catalog/standards/iso/bddbceea-ba53-43e2-b92a-05032bec66f1/iso-iec-18026-2009>

The RDs associated with physical objects are specified in [Annex D](#). [Table 7.10](#) is a directory of these RDs organized by type of ellipsoid. The semi-axis and radius parameters are unitless in position-space, but are bound to metre lengths when the RD is identified with the corresponding physical object-space constructed entity.

Table 7.10 — Physical RD specification table locations

Type of ellipsoid	RD table
Oblate ellipsoid	Table D.2
Sphere	Table D.3
Prolate ellipsoid	Table D.4
Tri-axial ellipsoid	Table D.5

Additional RDs associated with physical objects may be specified by registration in accordance with [Clause 13](#).

7.2.5 RD binding

An RD is *bound* when the RD in position-space is identified with a corresponding constructed entity in object-space. In this context, a "constructed entity" is defined to mean an intrinsic, artificial, measured, or conceptual entity in object-space that is uniquely identifiable within the user's application domain. The term "corresponding" in this context means that each RD is bound to a constructed entity of the same geometric object type. That is, position-space points are bound to identified points in object-space, position-space directed lines to constructed lines or line segments in object-space, position-space directed curves to constructed curves or curve segments in object-space, position-space oriented planes to constructed planes or partial planes in object-space, and position-space oriented surfaces to constructed surfaces or partial surfaces in object-space.

When a curve or surface RD is bound, the radii of curvature on the corresponding constructed entity in object-space shall correspond to the radii of curvature in position-space. In this International Standard, in the case of physical objects, one unit in position-space corresponds to one metre in object-space. In the case of abstract objects, one unit in position-space corresponds to the designated length scale unit in the abstract object-space. In particular, the semi-axes of an ellipsoid RD shall correspond to the semi-axes of the constructed ellipsoid to which it is bound.

If the constructed entity of an RD binding is fixed in position with respect to object-space, then the RD binding shall be called an *object-fixed RD binding*. This definition assumes that the position of the constructed entity does not change in time by an amount significant for the accuracy and time scale of an application.

EXAMPLE 1 For points on the surface of the Earth, tectonic plate movements are insignificant for many applications.

EXAMPLE 2 An RD X AXIS 3D is bound to the line segment from the centre of the Earth to the centre of the Sun. This RD binding is not an object-fixed RD binding with respect to the spatial object Earth.

Figure 7.2 illustrates two distinct bindings of a point RD. On the left, it is bound to a specific point in the abstract object-space of a CAD/CAM model. On the right, it is bound to a point in physical object-space that is on an object that has been manufactured from that CAD model.

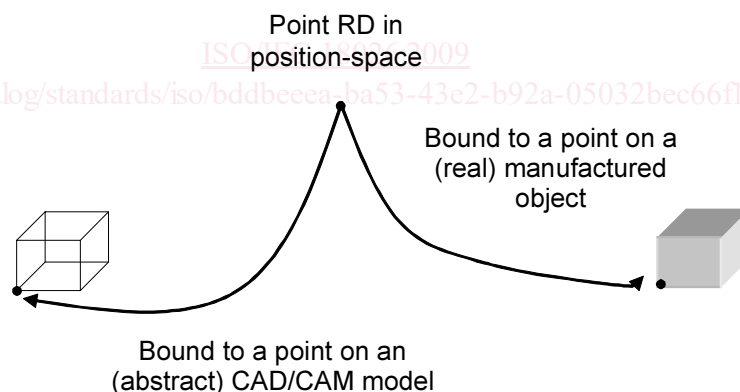


Figure 7.2 — An RD bound to an abstract object and to a real object

7.3. Normal embeddings of position-space into object-space

7.3.1 Normal embeddings

An embedding is a position-space model of object-space formed by a one-to-one function of positions in position-space to points in object-space. A *normal embedding* is an embedding that satisfies the following distance-preserving property:

A function E from position-space to object-space is *distance-preserving* if for any two positions p and q in position-space, the measured distance in object-space from $E(p)$ to $E(q)$ in metres is equal to the [Euclidean distance](#) $d(p, q)$.

NOTE As a consequence of the normal distance-preserving property, a normal embedding is also a continuous function, that preserves angles, area, and other geometric properties.

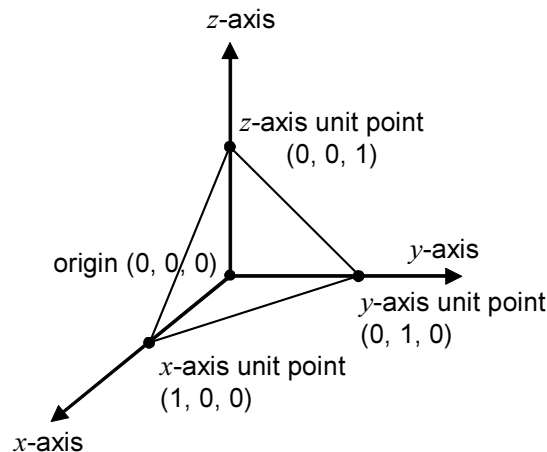


Figure 7.3 — A right-handed normal embedding¹⁹

In object-space, the point $E(0)$ is called the *origin of the normal embedding* E , and the point $E(e_1)$ is the *x-axis unit point* of the normal embedding E . If the dimension of position-space is 2D or 3D, the point $E(e_2)$ is the *y-axis unit point* of the normal embedding E . If the dimension of position-space is 3D, the point $E(e_3)$ is the *z-axis unit point* of the normal embedding E .

A normal embedding of a 3D position-space is *right-handed* if the directed triangle formed by the three points, x-axis unit point, y-axis unit point, and z-axis unit point, in that sequence, has a clockwise orientation when viewed from the origin of the embedding. Otherwise, the embedding is *left-handed*. A right-handed normal embedding is illustrated in [Figure 7.3](#). All 3D normal embeddings in this International Standard shall be right-handed.

7.3.2 Specification of 3D similarity transformations

A 3D object-space may have many normal embeddings of 3D position-space. Given two 3D normal embeddings E_1 and E_2 into the same object-space, one embedding can be expressed in terms of the other normal embedding. Given a position $(x, y, z)_{E_2}$ in position-space, the normal embedding E_2 associates to it a unique point p in object-space. The normal embedding E_1 uniquely associates some position $(x, y, z)_{E_1}$ to the same point p . This association of $(x, y, z)_{E_2}$ to $(x, y, z)_{E_1}$ may be expressed as a similarity transformation from E_2 to E_1 (see [Figure 4.2](#)). A *similarity transformation* is defined as a transformation on position-space that performs a translation, rotation, and/or scaling operation.

In general, $E_2(0)$ may be displaced with respect to $E_1(0)$ and the axes of the E_2 normal embedding may also be rotated and/or differently scaled with respect to the axes of the E_1 normal embedding (see [Figure 7.4](#)). If E_1

¹⁹ The y-axis and z-axis are in the plane of the presentation, and the x-axis is directed generally towards the observer.

associates the position $(\Delta x, \Delta y, \Delta z)_{E_1}$ to $E_2(\mathbf{0})$, the similarity transformation from E_2 to E_1 may be specified in the form of the *seven-parameter transformation*:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{E_1} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{E_1} + (1 + \Delta s) T_3 T_2 T_1 \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{E_2} \quad (5)$$

where:

$$T_3 = \begin{pmatrix} \cos(\omega_3) & -\sin(\omega_3) & 0 \\ \sin(\omega_3) & \cos(\omega_3) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_2 = \begin{pmatrix} \cos(\omega_2) & 0 & \sin(\omega_2) \\ 0 & 1 & 0 \\ -\sin(\omega_2) & 0 & \cos(\omega_2) \end{pmatrix}$$

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega_1) & -\sin(\omega_1) \\ 0 & \sin(\omega_1) & \cos(\omega_1) \end{pmatrix}$$

and where:

ω_1 = the rotation of the x -axis_{E2} with respect to E_1 ,

ω_2 = the rotation of the y -axis_{E2} with respect to E_1 ,

ω_3 = the rotation of the z -axis_{E2} with respect to E_1 , and

Δs = the scale adjustment.

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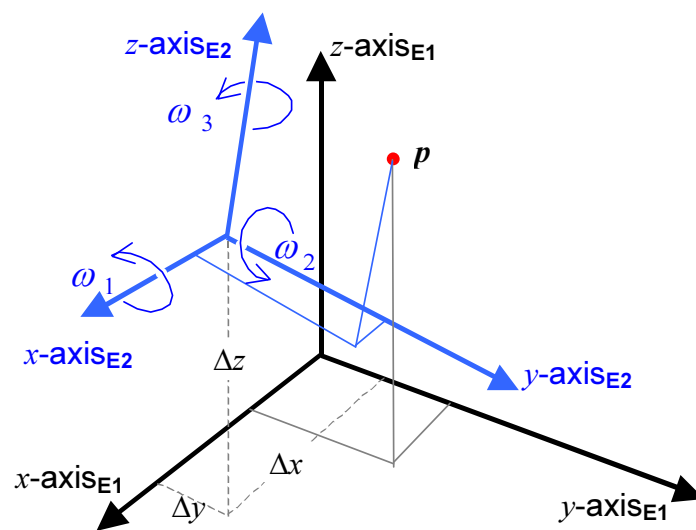


Figure 7.4 — 3D normal embedding relationships

The scale adjustment is needed to account for differing length scales in abstract object-space. In the case of physical object-space, small non-zero values of Δs may be required to adjust for spatial distortions in empirically estimated data. This is addressed in 7.4.5.

The convention of viewing the rotations with respect to E_1 is the *position vector rotation* convention. The *coordinate frame rotation* convention views rotations with respect to E_2 instead of E_1 . The rotations ω_1 , ω_2 , and ω_3 in Equation (5) are position vector rotations. If coordinate frame rotations are used, the rotations reverse sign (see Figure 7.5).

NOTE 1 Sign reversal does not affect cosine terms in the equation. Only the sine terms reverse sign.

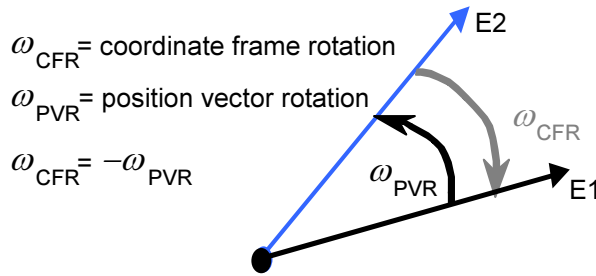


Figure 7.5 — Rotation between E_1 and E_2 in two conventions

NOTE 2 A small rotation approximation of the seven-parameter transformation is described in Annex B.

The *seven-parameter embedding specification* of E_2 with respect to E_1 is defined by the seven-parameter values Δx , Δy , Δz , ω_1 , ω_2 , ω_3 , and Δs in the position vector rotation convention (as in Equation (5)).

NOTE 3 In the cases that $\omega_1 = \omega_2 = \omega_3 = \Delta s = 0$, the formula for the transformation from $(x, y, z)_{E_2}$ to $(x, y, z)_{E_1}$ reduces to a translation operation $(x, y, z)_{E_1} = (x + \Delta x, y + \Delta y, z + \Delta z)_{E_2}$.

7.3.3 Specification of 2D similarity transformations

Given two 2D normal embeddings E_1 and E_2 into the same 2D object-space, one embedding can be expressed in terms of the other normal embedding. Given a position $(x, y)_{E_2}$ in position-space, the normal embedding E_2 associates to it a unique point p in object-space. As in the 3D case, this association may be expressed as a similarity transformation.

If E_1 associates $(\Delta x, \Delta y)_{E_1}$ to $E_2(0)$, $(1 + \Delta s)$ is the scale factor, and ω is the relative rotation, then the 2D similarity transformation from E_2 to E_1 may be specified in the form of the *four parameter transformation*:

$$\begin{pmatrix} x \\ y \end{pmatrix}_{E_1} = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}_{E_1} + (1 + \Delta s) \begin{pmatrix} \cos(\omega) & \sin(\omega) \\ -\sin(\omega) & \cos(\omega) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}_{E_2} \quad (6)$$

with parameters: Δx , Δy , Δs , and ω .

7.4. Object reference model

7.4.1 Introduction

A set of bound RDs can be selected so as to be compatible with only one normal embedding. In this way, a set of bound RDs with properly constrained relationships can specify a unique normal embedding. Such a constrained set of bound RDs is called an object reference model. Some object reference models use a set of

RDs that model application-specific geometric aspects of the object-space. Of particular interest are object reference models that include an oriented surface RD that models a surface significant to the object (see [7.4.2](#)).

A relationship between two or more bound RDs needed to ensure compatibility with a normal embedding is termed a binding constraint (see [7.4.3](#)). Object reference models that use the same set of RD primitives and the same binding constraints are abstracted in the notion of an object reference model template. Object reference model templates provide a uniform method of object reference model specification. If the bound RDs of an object reference model are compliant with the RD set and binding constraints of a particular object reference model template, then the object reference model is said to *realize* that template (see [7.4.4](#)).

A set of standardized object reference model templates are defined in this International Standard (see [7.4.5](#)). Realizations of these templates are specified in [Annex E](#).

7.4.2 ORM

A normal embedding and an RD binding are *compatible* if the normal embedding image of the RD primitive is coincident with the points (and direction or orientation, as applicable) of the constructed entity of the RD binding.

EXAMPLE 1 The constructed point in object-space to which RD [ORIGIN 3D](#) is bound is the origin of a normal embedding if, and only if, that normal embedding is compatible with the RD binding.

EXAMPLE 2 The directed line constructed in object-space to which RD [X AXIS 3D](#) is bound is the locus of the *x*-axis image under a compatible normal embedding, and similarly for other axis RDs.

An *object reference model* (ORM) for a spatial object is a set of bound RDs for which there exists exactly one normal embedding that is compatible with each bound RD in the set. In the 3D case, this unique embedding shall also be right-handed.

An ORM is *object-fixed* if each of its RD bindings are object-fixed, otherwise it is called *object-dynamic*. The object-fixed definition assumes that the object itself is not changing in time by an amount significant for the accuracy and time scale of an application. The normal embedding determined by an ORM is, correspondingly, either an *object-fixed embedding* or an *object-dynamic embedding*.

EXAMPLE 3 The Sun and the gas giants Jupiter, Saturn, Uranus, and Neptune are not rigid. The ORM specified for the Sun uses RD bindings defined in part by ephemeris and is thus object-dynamic. In the case of the ORMs specified for the gas giants, the object for binding is the magnetic field of the planet, thus these ORMs are object-fixed.

An ORM is often selected to contain an RD of category oriented surface that corresponds to a physical or conceptual surface significant to the modelled spatial object. An RD is chosen and its position with respect to the object is bound so that the RD instance is a “best fit” to the object in some application-specific sense. In particular, if the RD surface is “fitted” to a specific part of the object surface, the ORM is called a *local model*. If the RD is selected to best fit the entire surface, the ORM is called a *global model*.

An ORM may also contain an RD for the purpose of providing a CS binding parameter (see [8.3.2.2](#)). In particular the radius of a sphere RD or the semi-axis values of an oblate ellipsoid RD may be used for this purpose.

An *Earth reference model* (ERM) is an ORM for which the spatial object is the Earth.

EXAMPLE 4 If the object is a planet, an ORM containing an oblate ellipsoid RD is usually selected to model all or part of the general shape of the planet.