9 Designated spatial surfaces and vertical offsets

9.1 Introduction

Many spatial applications require the specification of object-space surfaces that are more complex than a surface represented by an RD. An RD surface generating function is restricted by definition to be a multi-variate polynomial of degree 2 or less. Surfaces of interest are often more complex than this restriction allows. These surfaces are termed designated spatial surfaces. These surfaces often represent some conceptual or physical aspect of object-space such as a gravity equipotential surface. Some designated spatial surfaces can be analytically represented by means of a smooth surface in position-space and a normal embedding. Such a model is termed a designated spatial surface model.

For SRFs that have a vertical coordinate-component, certain designated spatial surfaces may be used to define vertical offset values. Many real-world measurement systems used in geodesy define the value of the vertical coordinate-component of an SRF to be zero at a designated spatial surface. If the point of intersection between each vertical coordinate-component curve and the designated spatial surface is unique, it specifies a vertical offset value, and the designated spatial surface is termed a vertical offset surface for the given SRF.

In this International Standard, the vertical coordinate-component is always zero at an RD surface. For a given point, the difference in values of the vertical coordinate-component between these two vertical measurement systems is called the vertical offset. If the designated spatial surface has a designated spatial surface model, then the vertical offset may be computed. In the case of SRFs which designate ellipsoidal height as the vertical coordinate-component, the API (Clause 11) provides a method for the vertical offset computation.

9.2 Designated spatial surface

A *designated spatial surface* (DSS) is a surface in object-space. A DSS may be used to represent an application-specific aspect of the object-space.

Two important cases of DSSs are:

a) equipotential surfaces including geoids, and

b) models of mean sea level surfaces based on sounding and tidal data.

A DSS *model* is comprised of a smooth surface in position-space and a normal embedding such that the normal embedding image of the position-space surface either coincides with the DSS or approximates it in an application-specific sense.

EXAMPLE The International Great Lakes Datum 1955 is associated with a DSS that conceptually represents the mean water level of certain bodies of water and extensions of the surface to inland areas. It is empirically represented by a physical network of locations with assigned values for height above the conceptual surface. Various levelling techniques are applied to extrapolate these height values to other locations. There is currently no mathematically defined surface in position-space to model the International Great Lakes Datum 1955 DSS.

An *equipotential surface* is an implicitly defined surface given by $P(x,y,z)-c=0$, where P is a potential function defined in (a portion of) position-space and c is a value in the range of P.

If P is a smooth function, the equipotential surface is a smooth surface. If the smooth surface is embedded into object-space with a normal embedding, it is a DSS model for the corresponding DSS in object-space.

An important special case of an equipotential surface is a mathematical model of the gravity potential of a celestial body. The *geoid* is a specific equipotential surface of the Earth's gravity field that best fits the global mean sea surface in a minimum variance sense. Global, regional, and local approximations of the geoid are developed from empirical measurements in association with specific ERMs. Gravity equipotential surfaces have also been modelled for other planets.

NOTE The geoid cannot be measured directly. Current models of the Earth's gravity potential are usually realized as truncated power series in spherical harmonics.

9.3 Vertical offset surface

A DSS is a *vertical offset surface* (VOS) with respect to an SRF in a region of object-space if, for each point in the region, the DSS intersects the vertical coordinate-component curve containing the point exactly once. The VOS concept is restricted to SRFs that have a designated vertical coordinate-component and that are based on an object-fixed ORM. The vertical coordinate-component designation for an SRF is defined in 8.4.

The vertical coordinate-component zero surface is the set of points for which the vertical coordinate-component value is zero (see 5.5.2). Given a point $\boldsymbol{p}$ on the vertical coordinate-component zero surface that is in the region of a VOS, the *vertical offset* at $\boldsymbol{p}$ is the value of the vertical coordinate-component at the intersection of the VOS with the vertical coordinate-component curve that contains $\boldsymbol{p}$. The vertical offset at $\boldsymbol{p}$ is denoted $v(\boldsymbol{p})$. If $\boldsymbol{p}$ is not in the region of a VOS or if a VOS has not been specified, the vertical offset at $\boldsymbol{p}$ shall be defined to be zero.

NOTE All points on the same vertical coordinate-component curve have the same vertical offset value.

For a VOS with respect to an SRF based on an oblate ellipsoid (or sphere) ORM, the vertical offset at a point $\boldsymbol{p}$ on the oblate ellipsoid (or sphere) with surface geodetic coordinate (λ,φ) is denoted by $v(\lambda,\varphi)$.

In many cases, the values $v(\lambda,\varphi)$ are not known or the values are approximately known at specific locations. When a DSS has a DSS model, the $v(\lambda,\varphi)$ values may be computed. If a DSS is a VOS for two SRFs, SRF_S and SRF_T, and if the vertical offset function for SRF_S $v_\text{S}(\lambda,\varphi)$ is known, then the vertical offset function for SRF_T $v_\text{T}(\lambda,\varphi)$ may be computed from v_S as follows:

Each SRF_S coordinate of the form $c_\text{S} = (\lambda,\varphi,v_\text{S}(\lambda,\varphi))$ lies on the VOS. If $c_\text{T} = (\lambda',\varphi',h')$ is the corresponding coordinate representation in SRF_T, then $v_\text{T}(\lambda',\varphi') = h'$.

The API (Clause 11) provides a vertical offset computation for DSS models that are a VOS with respect to a given SRF with ellipsoidal height as the vertical coordinate-component.

EXAMPLE 1 If an SRF is derived from SRFT CELESTIODETIC or from a map projection SRFT, the ellipsoidal height coordinate-component is the designated vertical coordinate-component. Given a VOS with respect to the SRF, $v(\lambda,\varphi)$ is the distance from the ellipsoid to the VOS along the ellipsoidal height curve at (λ,φ) in the region of the VOS (see Figure 9.1).

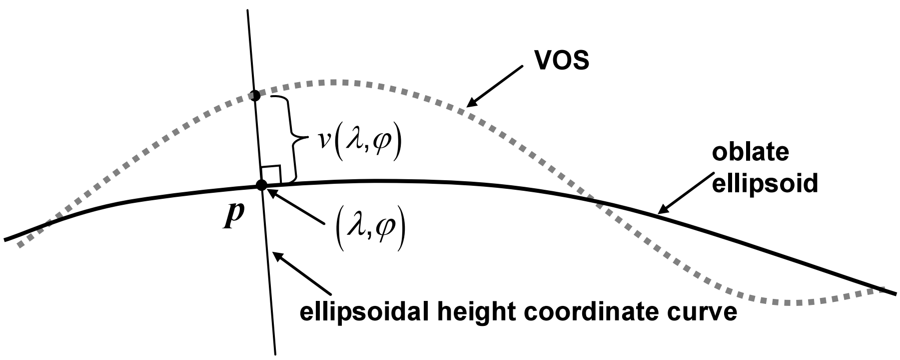

Figure 9.1 — Vertical offset surface for ellipsoidal height

EXAMPLE 2 If an SRF is derived from SRFT LOCAL_TANGENT_SPACE_EUCLIDEAN or SRFT LOCAL_TANGENT_SPACE_CYLINDRICAL, the designated vertical coordinate-component is height and the vertical coordinate-component zero surface is a plane. Given a VOS with respect to the SRF, The vertical offset at a point $\boldsymbol{p}$ in the plane is the distance from $\boldsymbol{p}$ to the VOS along a line normal to the plane (see Figure 9.2).

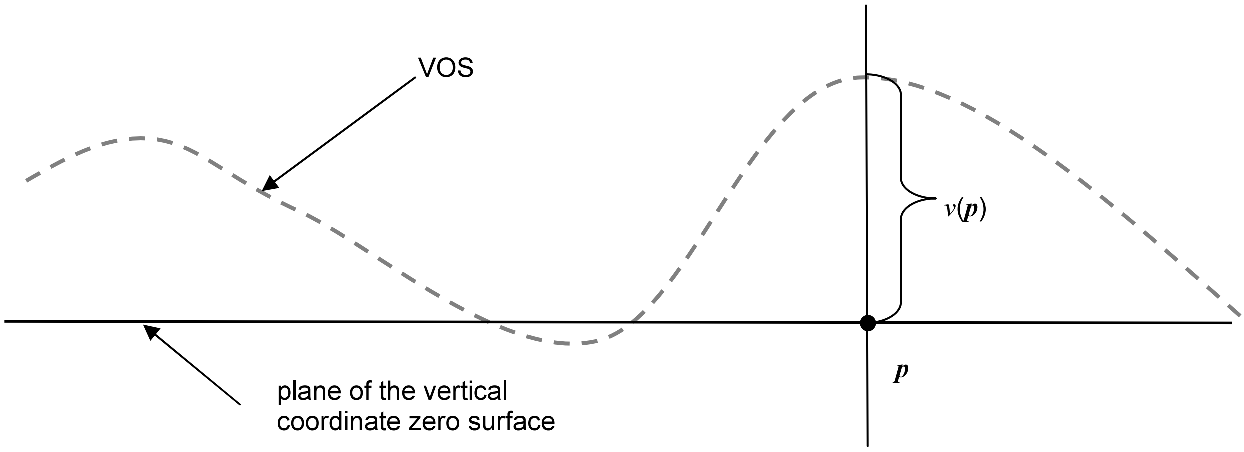

Figure 9.2 — Vertical offset surface tangent plane

9.4 Geoidal separation

If the VOS is a geoid, $v(\lambda,\varphi)$ is called the *geoidal separation* at (λ,φ) (see Figure 9.3). The specification of the geoidal separation is equivalent to the specification of the geoid surface because the geoid DSS can be constructed from the set of geoidal separation values.

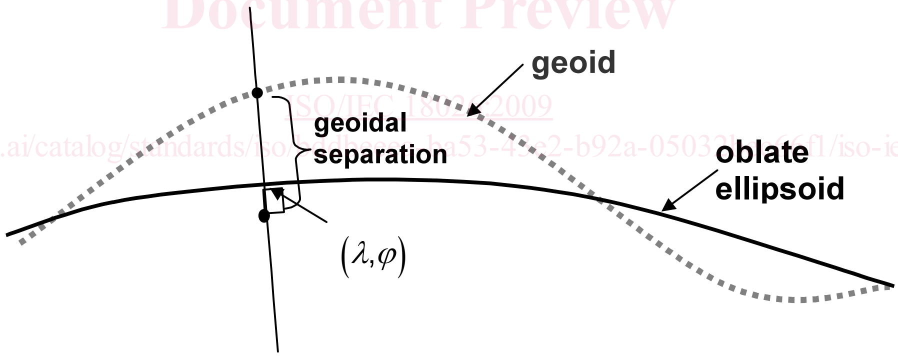

Figure 9.3 — Geoidal separation

NOTE The geoidal separation is often published as a table of values of $v(\lambda,\varphi)$.

9.5 Vertical offset height and elevation

Given a VOS with respect to an SRF with vertical coordinate-component h, the *vertical offset height* h_e at point $\boldsymbol{p}$ is defined as $h_e = h - v(\boldsymbol{p})$ (see Figure 9.4).

EXAMPLE If the SRF is derived from SRFT CELESTIODETIC and If (λ, φ, h) is the coordinate of $\boldsymbol{p}$, then $h_e = h - v(\lambda, \varphi)$.

If the VOS is a geoid, then h_e is called the *elevation* of $\boldsymbol{p}$ with respect to the geoid. Note that in the geoid case, (ellipsoidal height) - (elevation) = $v(\lambda, \varphi)$.

NOTE 1 h_e is an approximation of the distance from $\boldsymbol{p}$ to the VOS. In general, the vertical coordinate-component curve intersection with the VOS is not perpendicular to the VOS. When the intersection is not perpendicular, h_e does not equal the distance.

NOTE 2 VOS is similar in concept to vertical datum as defined in ISO 19111. ISO 19111 uses the term vertical datum to define a vertical coordinate reference system as part of a (3D) compound coordinate reference system.

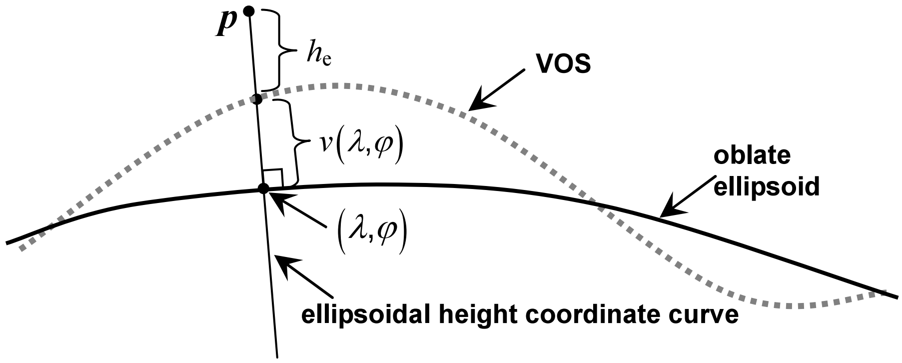

Figure 9.4 — Vertical coordinate-component with respect to a vertical offset surface

9.6 Use of vertical offset height in spatial referencing

If a DSS is a VOS for a 3D SRF, and $\boldsymbol{c} = (c_1, c_2)$ is a surface coordinate in the induced surface SRF, then $\boldsymbol{c}$ together with vertical offset height h_e represent a unique location in object-space. If $v(\boldsymbol{c})$ is known, then the SRF 3D coordinate of that location is $(c_1, c_2, h_e + v(\boldsymbol{c}))$. In this case, the 3D coordinate may be changed to other SRF coordinate representations in accordance with the operations specified in Clause 10.

In general, the value of $v(\boldsymbol{c})$ is not known. In that case, the 3D SRF coordinate of the location cannot be computed nor can it be changed to a different SRF. An important exception is the case of two 3D SRFs, SRF_S and SRF_T, that:

a) use the same ORM, and

b) use the same vertical coordinate-component.

In this case, if $\boldsymbol{c}_S = (c_{1S}, c_{2S})$ is a coordinate in the induced surface SRF of SRF_S and if $\boldsymbol{c}_T = (c_{1T}, c_{2T})$ is the coordinate in the induced surface SRF of SRF_T for the same surface position, then $\boldsymbol{c}_S$ with vertical height h_e in SRF_S and $\boldsymbol{c}_T$ with vertical height h_e in SRF_T represent the same location in object-space.

EXAMPLE SRF_S is derived from SRFT LAMBERT_CONFORMAL_CONIC with ORM WGS_1984 and SRF_T is derived from SRFT MERCATOR with the same ORM and $\boldsymbol{p}$ is on the ORM ellipsoid RD and is in the valid-region of both SRFs. If $\boldsymbol{c}_S = (c_{1S}, c_{2S})$ and $\boldsymbol{c}_T = (c_{1T}, c_{2T})$ are the surface coordinates of $\boldsymbol{p}$ in the respective SRFs, then $\boldsymbol{c}_S$ with vertical height h_e in SRF_S and $\boldsymbol{c}_T$ with vertical height h_e in SRF_T represents the same location in object-space.

9.7 Other vertical measurements

In addition to vertical offset height (and elevation), different fields of application define other vertical measurements. These include:

a) Orthometric height h_o depends on a gravity model that specifies a potential for each position in position-space. The gradient operator on the geoidal equipotential surface specifies a vector field in position-space. A *plumbline* is defined to be a curve that follows the gradient vector field (*i.e.*, the

