# 4 Concepts

## 4.1 Introduction

The SRM provides an integrated framework and precise terminology for describing spatial concepts and operations on spatial information (including positions, directions, and distances). The SRM includes the following features:

a) precise and uniform definitions of commonly used spatial coordinate systems, including those based on map projections,

b) spatial referencing support for physical spatial objects, including artificial objects and non-terrestrial celestial bodies, as well as abstract spatial objects,

c) spatial operations on positions and directions, including coordinate conversions and transformations, and calculations of distances and other geometric quantities, along with an application program interface for performing these spatial operations,

d) codes and labels to support the encoding and exchange of spatial data,

e) an extensible framework that supports the registration of additional instances of SRM concepts, and

f) profiles to allow subsets of the SRM to be defined to conform to the specific requirements of an application or an application domain.

This International Standard is based on the following conceptual approach:



a) Spatial positions are identified by coordinates in a spatial coordinate system. The collection of spatial positions associated with a spatial object of interest, such as the Earth, is called its object-space (see 4.2).

b) A spatial reference frame specifies a spatial coordinate system by combining an abstract coordinate system with an object reference model. An abstract coordinate system may be combined with many different object reference models. Thus, a geodetic coordinate tied to the Earth object reference model WGS_1984 does not identify the same place as when tied to the Earth object reference model EUROPE_1950, or when tied to an object reference model for Mars (see 4.6.3 and Clause 8).

c) An abstract coordinate system associates coordinates with positions in an abstract Euclidean space, which is called its position-space. Abstract coordinate systems are defined independently of any object-space. Thus, there are many spatial spherical coordinate systems for a given object-space, but there is only one abstract spherical coordinate system for a position-space (see 4.6.1 and Clause 5).

d) An object reference model determines a precise relationship between position-space and the object-space for a spatial object of interest. Different object reference models for the Earth relate position-space to the object-space of the Earth in different ways. Thus, the object reference model WGS_1984 relates the position-space $x$-axis to the direction from the Earth's center of mass towards the intersection of the Greenwich meridian with the equator, while the object reference model EARTH_INERTIAL_J2000r0 relates the position-space $x$-axis to the direction from the Earth's center of mass towards the first point of Aries (see 4.5 and Clause 7).

e) The position-space to object-space relationship determined by an object reference model is expressed mathematically by a length-preserving embedding function called a normal embedding (see 4.3 and Clause 7).

f) A reference datum is a geometric primitive that relates measurements and/or geometric characteristics of object-space to position-space. Object reference models use reference datums to specify the position-space to object-space relationship. An object reference model may also use reference datums to model a geometric aspect of a spatial object. Thus, an oblate ellipsoid reference datum may be used to model the figure of the Earth or other celestial bodies (see 4.4 and Clause 7).

g) Temporal coordinate systems are introduced to describe the time-varying characteristics of spatial reference frames (see 4.6 and Clause 6).

h) Vertical offset surfaces are introduced to define heights with respect to equipotential or other complex surfaces. In particular, the vertical offset surface EGM96_GEOID represents the geopotential surface defined by the WGS 84 EGM 96 Earth Gravitational Model that is closely associated with the mean ocean surface (see 4.8 and Clause 9).

The relationships among some of these concepts are depicted in Figure 4.1. An abstract coordinate system is based on the underlying Euclidean structure of position-space. The reference datums that comprise the object reference model determine how position-space relates to object-space. That relationship is mathematically expressed by a normal embedding. A spatial reference frame combines the abstract coordinate system with the object reference model to specify a spatial coordinate system. This allows positions to be expressed relative to a spatial object of interest, such as the Earth.

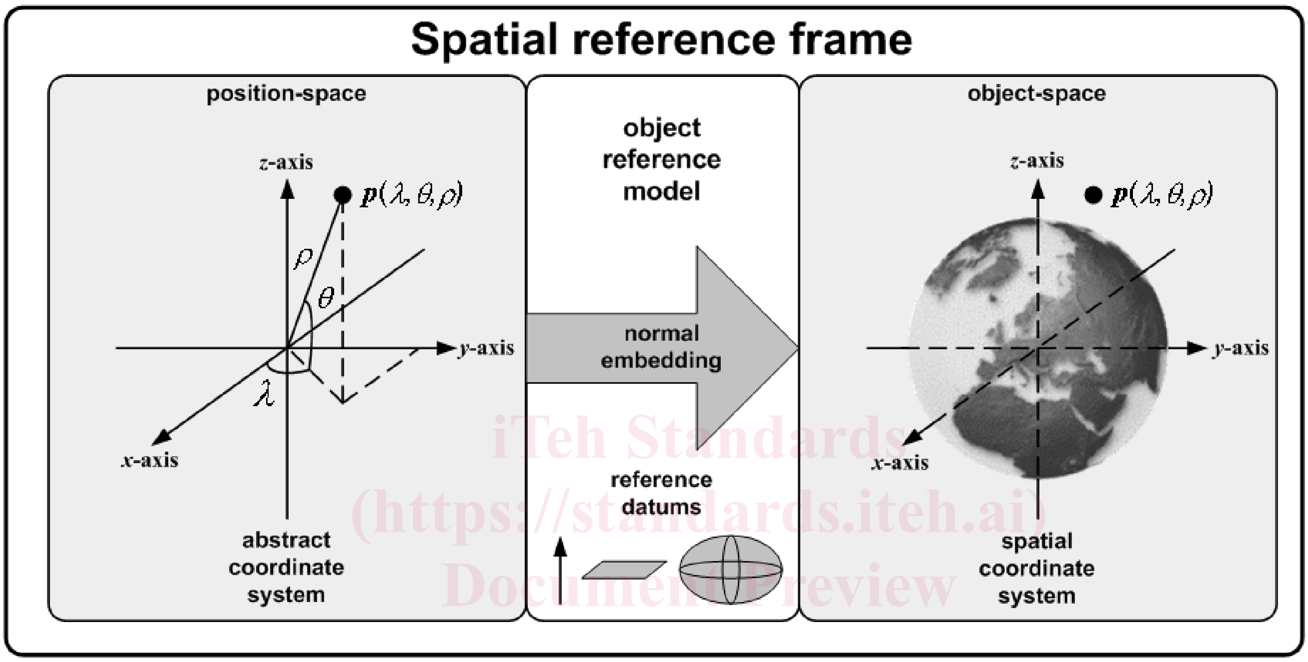




**Figure 4.1 — SRM conceptual relationships**

The concepts outlined in this sub-clause are discussed in greater detail in the remainder of this clause. This International Standard takes a functional approach to the definition of these concepts. Basic geometric concepts, including the concepts of point, line, and plane, are assumed. Annex A provides a concise summary of important specialized mathematical concepts and notational conventions used in this International Standard.

## 4.2 Spatial objects and object-space

*Object-space* is an abstract universe[1] or a real universe that is associated with a designated spatial object of interest. Object-space is the context in which all spatial locations are represented and spatial operations are performed. A rigid spatial object is assumed to be fixed in its object-space (see Example 1).

The spatial objects of concern in this International Standard may be divided into two types: physical objects and abstract objects.

[1] The set of all continuations of a spatial object is called the universe of the object. In physics, this is called "the space of the object". [EINS]

 

*Physical objects* are real world objects. The length of one metre has intrinsic meaning in the object-space of a physical object.

*Abstract objects* are conceptual objects including virtual, engineering, and/or mathematical models. A length of one metre does not have intrinsic meaning in the object-space of abstract objects. For the purpose of specifying relationships among abstract object-spaces, a designated length scale shall be associated with each abstract object-space.

A point in object-space may have a fixed location with respect to the spatial object. If points and objects have a time-dependent relationship, locations shall be qualified by a time value in a temporal coordinate system. Thus, at a specified time, the points and objects have a spatially-fixed relationship.

EXAMPLE 1 The Sun and the Earth are both physical objects, each with its own object-space. In the object-space of the Sun, the Sun is fixed and the Earth moves. In the object-space of the Earth, the Earth is fixed and the Sun moves.

EXAMPLE 2 At any given instant the International Space Station (ISS) has a location in the object-space of the Earth.

EXAMPLE 3 Each component of the ISS has a location in the object-space of the ISS.

EXAMPLE 4 A solar collector component of the ISS was manufactured in compliance with an engineering design. The CAD/CAM model that specifies the design for that component is defined in an abstract object-space.

## 4.3 Position-space and normal embeddings

*Position-space* is an $n$-dimensional Euclidean space, for $n$ = 1, 2, or 3. Position-space serves as a vector space abstraction of object-space so that the methods of linear algebra and multivariate calculus may be applied to spatial concepts.



A normal embedding is a distance-preserving function from positions in position-space to points in object-space. The distance-preserving property requires that, whenever a pair of points in object-space corresponds via the normal embedding function to a pair of positions in position-space, the Euclidean distance between the pair of positions in position-space equals the distance in metres between the pair of object-space points. The distance-preserving property implies that a normal embedding is one-to-one and continuous. Normal embeddings also preserve other important geometric properties. Position-space together with a normal embedding is a specific algebraic model of object-space. In particular, normal embeddings are used to relate abstract coordinate systems in Euclidean space to spatial coordinate systems in object-space (see 4.6.3).

The origin of a normal embedding is the point in object-space associated by a normal embedding with the position-space origin. A normal embedding of a 3D position-space is right-handed if the triangle formed by the three points associated to the $x$-axis unit point, the $y$-axis unit point, and the $z$-axis unit point, in that order, has a clockwise orientation when viewed from the origin of the normal embedding. Otherwise, the normal embedding is left-handed. In this International Standard, all 3D normal embeddings shall be right-handed.

Algebraic models of an object-space are not unique. There are infinitely many normal embeddings of an $n$-dimensional position-space. Figure 4.2 illustrates two distinct normal embeddings. These normal embeddings provide two distinct algebraic models of the same object-space. Some dynamic applications require a continuum of normal embeddings parameterized by time.

 

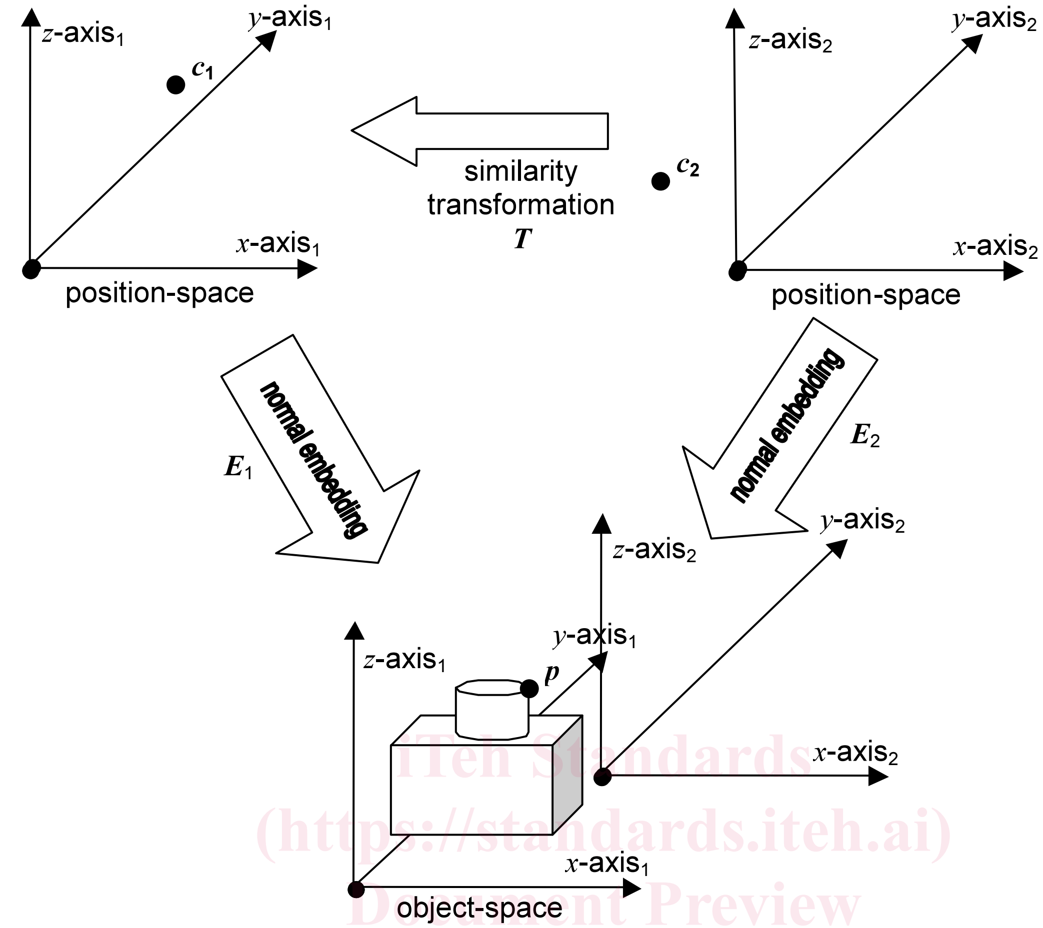




**Figure 4.2 — Two position-space normal embeddings**[2]



Any two normal embeddings for the same object-space can be inter-converted with a transformation consisting of a translation, a rotation and a scale factor. Such a transformation is called a similarity transformation. If $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are two normal embeddings, there is a similarity transformation $\boldsymbol{T}$ such that $\boldsymbol{E}_2$ is the composition of $\boldsymbol{E}_1$ with $\boldsymbol{T}$, $\boldsymbol{E}_2 = \boldsymbol{E}_1 \circ \boldsymbol{T}$. This is depicted in Figure 4.2 where $\boldsymbol{p} = \boldsymbol{E}_1(c_1)$ and $\boldsymbol{p} = \boldsymbol{E}_2(c_2)$. The similarity transformation is such that $c_1 = \boldsymbol{T}(c_2)$ so that $\boldsymbol{E}_2(c_2) = \boldsymbol{E}_1(\boldsymbol{T}(c_2))$.

The method of specifying a normal embedding varies across disciplines and application domains. In the case of an abstract spatial object, the object-space is itself a Euclidean space and the identity function is an implicit normal embedding. In some application domains, a normal embedding is implicitly defined by the specification of the origin point and axis directions. In the case of geodesy, an origin point at the centre of the Earth cannot be directly specified. Instead, its location is implied by specifying other geometric entities from physical measurements. Other disciplines use a variety of techniques to either implicitly or explicitly define a normal embedding. This International Standard abstracts and encapsulates these techniques within the concepts of reference datum and object reference model.

[2] Each $y$-axis points away from the viewer.

 

## 4.4 Reference datums

A reference datum is a geometric primitive in position-space. Reference datums are points or directed curves in 2D position-space or points, directed curves or oriented surfaces in 3D position-space.

A reference datum is bound when the reference datum in position-space is identified with a corresponding constructed entity (*i.e.*, a measured or conceptual geometric aspect of a spatial object) in object-space. The term "corresponding" in this context means that each position-space reference datum is bound to a constructed geometric entity of the same geometric object type. That is, position-space points are bound to object-space points, position-space lines to object-space lines, position-space curves to object-space curves, position-space planes to object-space planes, and position-space surfaces to object-space surfaces.

EXAMPLE 1 An ellipsoid reference datum with major semi-axis $a$ and minor semi-axis $b$ is the surface implicitly defined by:

$$f(x,y,z)=\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}-1=0$$

and is illustrated in Figure 4.3.

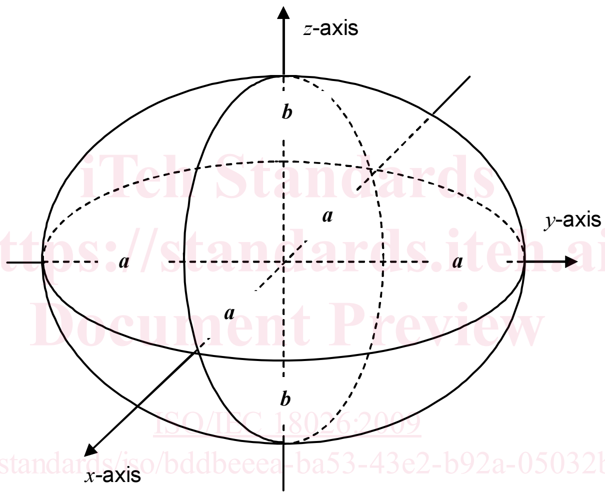



**Figure 4.3 — An ellipsoid reference datum**

Figure 4.4 illustrates two distinct bindings of a point reference datum. On the left, the point reference datum is bound to a specific point in the abstract object-space of a CAD/CAM model. On the right, the point reference datum is bound to a corresponding point in the physical object-space of an object that has been manufactured in accordance to that CAD/CAM model.

In some application domains, bound reference datums are used to model a significant aspect of the problem domain. In particular, in geodesy, oblate ellipsoids are used to model the figure of the Earth or a subset thereof.

 

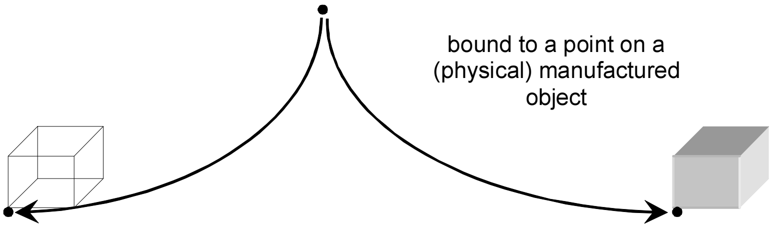


**Figure 4.4 — A reference datum bound to abstract and physical object-spaces**

EXAMPLE 2 Semi-axis values $a$ and $b$, $a \ge b$, are selected to specify an oblate ellipsoid reference datum. The following steps (see Figure 4.5) illustrate one way to bind an ellipsoid reference datum specified by semi-axis values $a$ and $b$ to a conceptual ellipsoid that represents the figure of the Earth in a region as approximated by a geodetic survey control network:

a) A point on the surface of the reference datum is specified. This point has a computable geodetic latitude $\varphi$.

b) The specified position-space point is identified with a specific point in object-space.

c) The direction of the oblate ellipsoid rotational axis is constructed in object-space.

d) The direction of the outward surface normal at the point is constructed in object-space so that the angle it makes with respect to the oblate ellipsoid rotational axis direction is $(\pi/2-\varphi)$.

This binding requires the specification of a point and two directions in object-space.



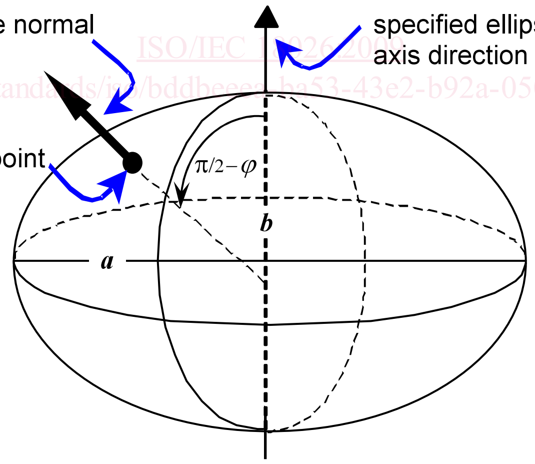


**Figure 4.5 — A reference datum binding**

This International Standard specifies a set of reference datums for use in subsequent specifications, including points, axis lines, planes, and ellipsoids. Additional reference datums may be defined by registration, as described in 4.13.

A reference datum may be associated with a corresponding constructed entity in object-space by means of a reference datum binding. The same reference datum may also be associated with a set of points in object-

 