10 SRF operations

10.1 Introduction

This International Standard specifies operations on SRF coordinates and, in the case of 3D object-spaces, on SRF spatial directions. Underlying these operations is the similarity transformation associated with two ORMs. Similarity transformations are treated first in 10.3. Then the general case of changing the representation of a position as a coordinate in one SRF to its representation as a coordinate in another SRF is specified in 10.4, followed by important special cases. The specification of a spatial direction in the context of an SRF is defined, and the general case of changing the representation of a spatial direction in one SRF to its representation in another SRF is specified (10.5).

Euclidean distance in 2D and 3D object-space is specified in 10.6. Distance and azimuth on the surface of an oblate ellipsoid (or sphere) is specified in 10.7. Vertical offset is defined in 9.3.

10.2 Notation and terminology

An important category of spatial operations is changing spatial information represented in one SRF to spatial information represented in a second SRF. For this category of operations, the adjective “source” shall be used to refer to the first SRF, and the adjective “target” shall be used to refer to the second SRF.

The notation in Table 10.1 is used throughout this clause.

Table 10.1 — Notation

Notation	Description
$\mathrm{ORM_S}$	Source ORM
$\mathrm{ORM_T}$	Target ORM
$\mathrm{ORM_R}$	Reference ORM for a given spatial object
$\boldsymbol{H}_{\mathrm{SR}}$	Reference transformation from $\mathrm{ORM_S}$ to the reference $\mathrm{ORM_R}$
$\boldsymbol{H}_{\mathrm{TR}}$	Reference transformation from $\mathrm{ORM_T}$ to the reference $\mathrm{ORM_R}$
$\boldsymbol{H}_{\mathrm{ST}}$	Similarity transformation from the embedding of $\mathrm{ORM_S}$ to $\mathrm{ORM_T}$
$\mathrm{SRF_S}$	Source SRF based on $\mathrm{ORM_S}$
$\mathrm{SRF_T}$	Target SRF based on $\mathrm{ORM_T}$
$\mathrm{SRF_L}$	The local tangent frame SRF at a coordinate (See 10.5.2)
$\mathrm{CS_S}$	CS of $\mathrm{SRF_S}$
$\mathrm{CS_T}$	CS of $\mathrm{SRF_T}$
$\boldsymbol{G}_{\mathrm{S}}$	Generating function of $\mathrm{CS_S}$
$\boldsymbol{G}_{\mathrm{T}}^{-1}$	Inverse generating function of $\mathrm{CS_T}$
$\boldsymbol{c}_{\mathrm{S}}$	Coordinate of a spatial position in $\mathrm{SRF_S}$
$\boldsymbol{c}_{\mathrm{T}}$	Coordinate of a spatial position in $\mathrm{SRF_T}$
$\boldsymbol{n}_{\mathrm{S}}$	Direction vector in $\mathrm{SRF_S}$ (See 10.5.2)
$\boldsymbol{n}_{\mathrm{T}}$	Direction vector in $\mathrm{SRF_T}$ (See 10.5.2)

10.3 Operations on ORMs

10.3.1 Introduction

The similarity transformations $\boldsymbol{H}_{\mathrm{ST}}$ between source/target pairs $\mathrm{ORM}_{\mathrm{S}}$ and $\mathrm{ORM}_{\mathrm{T}}$ underlie the coordinate operations in 10.4. Given a set of n ORMs there are $n(n\text{-}1)$ such source and target ORM pairs. Instead of specifying the full set of similarity transformations, this International Standard reduces the requirement to specifying the reference transformation $\boldsymbol{H}_{\mathrm{SR}}$ from each object-fixed source $\mathrm{ORM}_{\mathrm{S}}$ to the reference $\mathrm{ORM}_{\mathrm{R}}$ for a given object. This subclause specifies the methods of expressing a similarity transformation $\boldsymbol{H}_{\mathrm{ST}}$ in terms of the reference transformations for the source and target ORMs. The cases of ORMs for a single object are treated in 10.3.2. The more general cases in which $\mathrm{ORM}_{\mathrm{S}}$ and $\mathrm{ORM}_{\mathrm{T}}$ are ORMs for different objects are treated in 10.3.3.

10.3.2 ORMs for a single object

If $\mathrm{ORM}_{\mathrm{S}}$ is an object-fixed ORM, its reference transformation $\boldsymbol{H}_{\mathrm{SR}}$ may be specified as a seven-parameter transformation in the 3D case (see 7.3.2) and a by four-parameter transformation in the 2D case (see 7.3.3). The general form of $\boldsymbol{H}_{\mathrm{SR}}$ in the 3D case is given by Equation (7). The form in the 2D case is similar. As vector operations, they are in the form of a scaled invertible matrix multiplication followed by a vector addition. This form of vector operation is an invertible affine transformation. In the 3D case using the notation of Equation (5):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{R}} = \boldsymbol{H}_{\mathrm{SR}}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}\right) \equiv \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{SR}} + \left(1+\Delta s_{\mathrm{SR}}\right)\boldsymbol{T}_{\mathrm{SR}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} \tag{7}$$

NOTE The processes by which ORMs for the Earth are established are based on physical measurements. These measurements are subject to error and therefore introduce various types of relative distortions between ORMs. Equation (7) is based on the assumption that positions in object-space are error free and the equation includes no compensation for these distortions.

The reference transformation $\boldsymbol{H}_{\mathrm{TR}}$ from $\mathrm{ORM}_{\mathrm{T}}$ to $\mathrm{ORM}_{\mathrm{R}}$ is similarly specified. An important operation is the similarity transformation $\boldsymbol{H}_{\mathrm{ST}}$ from $\mathrm{ORM}_{\mathrm{S}}$ to $\mathrm{ORM}_{\mathrm{T}}$, when neither the source nor the target is necessarily the reference ORM. The $\boldsymbol{H}_{\mathrm{ST}}$ transformation may be expressed as the composition of $\boldsymbol{H}_{\mathrm{SR}}$ with $\boldsymbol{H}_{\mathrm{TR}}^{-1}$ (the inverse of $\boldsymbol{H}_{\mathrm{TR}}$) as in Equation (8) (see Figure 10.1):

$$\boldsymbol{H}_{\mathrm{ST}} = \boldsymbol{H}_{\mathrm{TR}}^{-1} \circ \boldsymbol{H}_{\mathrm{SR}} \tag{8}$$

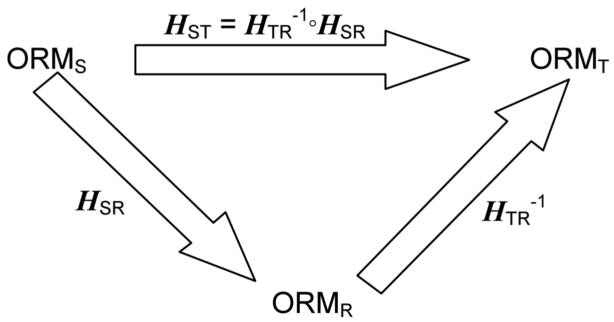

Figure 10.1 — Composed transformations

The inverse operation $\boldsymbol{H}_{\mathrm{TR}}^{-1}$ is also an affine transformation:

$$\boldsymbol{H}_{\mathrm{TR}}^{-1}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{R}}\right) = \frac{1}{(1+\Delta s_{\mathrm{TR}})}\boldsymbol{T}_{\mathrm{TR}}^{-1}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{R}} - \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{ST}}\right)$$

$$= \left(\frac{-1}{(1+\Delta s_{\mathrm{TR}})}\boldsymbol{T}_{\mathrm{TR}}^{-1}\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{TR}}\right) + \frac{1}{(1+\Delta s_{\mathrm{TR}})}\boldsymbol{T}_{\mathrm{TR}}^{-1}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{R}}$$

$$= \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{RT}} + \frac{1}{(1+\Delta s_{\mathrm{TR}})}\boldsymbol{T}_{\mathrm{TR}}^{-1}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{R}}$$

Because the matrix $\boldsymbol{T}_{\mathrm{TR}}$ is a rotation matrix, its transpose $\boldsymbol{T}_{\mathrm{TR}}^{T}$ is also its inverse $\boldsymbol{T}_{\mathrm{TR}}^{-1}$. Its inverse is also the matrix $\boldsymbol{T}_{\mathrm{RT}}$ corresponding to the reverse rotations of $\mathrm{ORM}_{\mathrm{T}}$ with respect to $\mathrm{ORM}_{\mathrm{R}}$. In particular:

$$\boldsymbol{T}_{\mathrm{RT}} = \boldsymbol{T}_{\mathrm{TR}}^{-1} = \boldsymbol{T}_{\mathrm{TR}}^{T}$$

and

$$\boldsymbol{H}_{\mathrm{TR}}^{-1}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{R}}\right) = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{RT}} + \frac{1}{(1+\Delta s_{\mathrm{TR}})}\boldsymbol{T}_{\mathrm{RT}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{R}} .$$

The composite operation $\boldsymbol{H}_{\mathrm{ST}} = \boldsymbol{H}_{\mathrm{TR}}^{-1} \circ \boldsymbol{H}_{\mathrm{SR}}$ reduces to:

$$\boldsymbol{H}_{\mathrm{ST}}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}\right) = \boldsymbol{H}_{\mathrm{TR}}^{-1} \circ \boldsymbol{H}_{\mathrm{SR}}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}\right) \tag{9}$$

$$= \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{ST}} + \frac{(1+\Delta s_{\mathrm{SR}})}{(1+\Delta s_{\mathrm{TR}})}\boldsymbol{T}_{\mathrm{ST}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}$$

where:

$\boldsymbol{T}_{\mathrm{ST}} = \boldsymbol{T}_{\mathrm{RT}}\boldsymbol{T}_{\mathrm{SR}}$, and

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{ST}} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{RT}} + \frac{1}{(1+\Delta s_{\mathrm{TR}})}\boldsymbol{T}_{\mathrm{RT}}\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{SR}} .$$

If the rotation parameters are equal, then $\boldsymbol{T}_{\mathrm{ST}}$ is the identity matrix, and if $\Delta s_{\mathrm{R}} = \Delta s_{\mathrm{T}}$, $\boldsymbol{H}_{\mathrm{ST}}$ simplifies to a translation of the origin:

$$\boldsymbol{H}_{\mathrm{ST}}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}}\right) = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}_{\mathrm{ST}} + \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\mathrm{S}} .$$

Equation (8) and Figure 10.1 also apply to the 2D case.

If the source $\mathrm{ORM}_{\mathrm{S}}$ is a time-dependent ORM for a spatial object, $\mathrm{ORM}_{\mathrm{S}}(t)$ shall denote the $\mathrm{ORM}_{\mathrm{S}}$ at time t, and $\boldsymbol{H}_{\mathrm{SR}}(t)$ shall denote the similarity transformation from the embedding of $\mathrm{ORM}_{\mathrm{S}}(t)$ to the embedding of the object-fixed reference $\mathrm{ORM}_{\mathrm{R}}$. If the similarity transformation $\boldsymbol{H}_{\mathrm{SR}}(t)$ can be determined, it is a time-dependent affine transformation. For a fixed value of time t_0, Equation (8) and Figure 10.1 generalize to

$\boldsymbol{H}_{\mathrm{ST}}(t_0) = \boldsymbol{H}_{\mathrm{TR}}^{-1} \circ \boldsymbol{H}_{\mathrm{SR}}(t_0)$. The generalizations to a time-dependent target $\mathrm{ORM_T}(t)$ are $\boldsymbol{H}_{\mathrm{ST}}(t_0) = \boldsymbol{H}_{\mathrm{TR}}^{-1}(t_0) \circ \boldsymbol{H}_{\mathrm{SR}}$ and $\boldsymbol{H}_{\mathrm{ST}}(t_0) = \boldsymbol{H}_{\mathrm{TR}}^{-1}(t_0) \circ \boldsymbol{H}_{\mathrm{SR}}(t_0)$ for the $\mathrm{ORM_S}$ static and time-dependent cases, respectively.

EXAMPLE $\mathrm{ORM_S}(t)$ is the ORM EARTH_INERTIAL_J2000r0 at time t. $\mathrm{ORM_R}$ is the Earth reference ORM WGS_1984. Because $\mathrm{ORM_S}(t)$ and $\mathrm{ORM_R}$ share the same embedding origin, the $\boldsymbol{H}_{\mathrm{SR}}(t)$ transformation is a (rotation) matrix multiplication operation (without vector addition). The matrix coefficients for selected values of t account for polar motion, Earth rotation, nutation, and precession. Predicted values for these coefficients are computed and updated weekly by the International Earth Rotation Service (IERS) [IERS] (see 7.5.2). See Annex B for other examples of dynamic ORM reference transformations.

10.3.3 Relating ORMs for different objects

If a spatial object ***S*** exists in the space of another spatial object ***T***, and if $\mathrm{ORM_R}$ is the reference ORM for object ***T***, and if the two objects are fixed with respect to each other, then $\boldsymbol{H}_{\mathrm{SR}}$ shall denote a similarity transformation from the embedding of $\mathrm{ORM_S}$ to the embedding of $\mathrm{ORM_R}$. $\boldsymbol{H}_{\mathrm{SR}}$ is an affine transformation. If $\mathrm{ORM_T}$ is an object-fixed ORM for the object ***T*** then $\boldsymbol{H}_{\mathrm{ST}}$ is given by Equation (8). The time dependent generalizations of Equation (8), defined in 10.3.2, are also applicable to this case.

EXAMPLE $\mathrm{ORM_S}$ is an ORM for the space shuttle (as a spatial object). $\mathrm{ORM_R}$ is the Earth reference ORM WGS_1984. When in orbit at time t, $\boldsymbol{H}_{\mathrm{SR}}(t)$ transforms positions with respect to $\mathrm{ORM_S}$ to positions with respect to ORM WGS_1984.

If the object-space of ***S*** and the object-space of ***T*** do not share locations or are otherwise unrelated, a similarity transformation between ORMs for the respective object-spaces is not defined. An abstract object ***S*** and a physical object ***T*** is an important instance of this case (see 10.4.6). However, if $\boldsymbol{H}_{\mathrm{SR}}$ is an invertible affine transformation between $\mathrm{ORM_S}$ and the reference ORM for ***T***, then, given an object-fixed ORM for object ***T,*** $\mathrm{ORM_T}$, Equation (8) may be used to define an invertible affine transformation $\boldsymbol{H}_{\mathrm{ST}}$, from $\mathrm{ORM_S}$ to $\mathrm{ORM_T}$.

10.4 Operations to change spatial coordinates between SRFs

10.4.1 Introduction

Given a coordinate c_{S} in a source SRF, $\mathrm{SRF_S}$, and a target SRF, $\mathrm{SRF_T}$, the change coordinate SRF operation[22] computes the corresponding coordinate c_{T} in $\mathrm{SRF_T}$. The general case of changing the spatial coordinate of a location from $\mathrm{SRF_S}$ to $\mathrm{SRF_T}$ is presented in formulations in 10.4.2 for time-independent (static) and time-dependent ORM relationships. The general case assumes that the source coordinate corresponds to a location that exists in both the source and target object spaces.

In the general case, $\mathrm{ORM_S}$ and $\mathrm{ORM_T}$ may differ, and the coordinate systems, $\mathrm{CS_S}$ and $\mathrm{CS_T}$, may differ. The formulation simplifies in the special case[23] for which $\mathrm{ORM_S} = \mathrm{ORM_T}$ or, more generally, in the case for which the associated normal embeddings match. This case is presented in 10.4.3. In a further specialization of the $\mathrm{ORM_S} = \mathrm{ORM_T}$ case, it is assumed that $\mathrm{CS_S}$ and $\mathrm{CS_T}$ are geodetic and/or map projection CSs. These assumptions produce further simplifications (see 10.4.4).

The case for which $\mathrm{CS_S} = \mathrm{CS_T}$ and $\mathrm{ORM_S}$ and $\mathrm{ORM_T}$ differ[24] does not generally produce a computational simplification of the general case. However, when both the source and target SRFs are based on the CS LOCOCENTRIC_EUCLIDEAN_3D, a simplification is produced and is presented in 10.4.5. This case is important for operations on directions (10.5.4).

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[22] ISO 19111 defines this case as a coordinate operation.

[23] ISO 19111 defines this case as a coordinate conversion.

[24] ISO 19111 defines this case as a coordinate transformation.

An extension of the change SRF operation to the case of unrelated source and target object-spaces is presented in 10.4.6 for linear SRFs. In that case, the ORM transformation is only restricted to an invertible affine transformation.

10.4.2 Change coordinate SRF operation

SRF_S and SRF_T are two object-fixed SRFs for a spatial object and $\boldsymbol{p}$ is a point in object-space that is in the coordinate system domains for both SRFs. $\boldsymbol{c}_\text{S}$ denotes the coordinate of $\boldsymbol{p}$ in SRF_S, and $\boldsymbol{c}_\text{T}$ denotes the coordinate of $\boldsymbol{p}$ in SRF_T. The determination of $\boldsymbol{c}_\text{T}$ as a function of $\boldsymbol{c}_\text{S}$ is an operation on the SRF pair (SRF_S, SRF_T). The most general form of the operation is:

$$\boldsymbol{c}_\text{T} = \boldsymbol{G}_\text{T}^{-1} \circ \boldsymbol{H}_\text{ST} \circ \boldsymbol{G}_\text{S}\left(\boldsymbol{c}_\text{S}\right) \tag{10}$$

where:

- $\boldsymbol{G}_\text{S}$ is the CS generating function of SRF_S,
- $\boldsymbol{H}_\text{ST}$ is the embedding transformation from ORM_S to ORM_T, and
- $\boldsymbol{G}_\text{T}$ is the CS generating function of SRF_T.

See Figure 10.2. CS generating and inverse generation functions are specified in Clause 5.

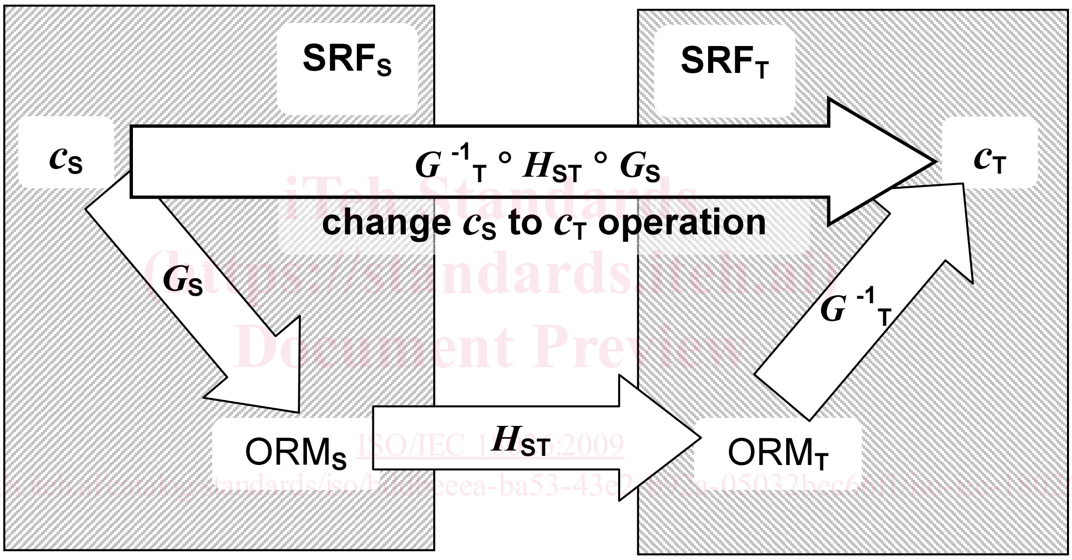

Figure 10.2 — Change coordinate SRF operation

Equation (10) is known as the *Helmert transformation* when $\boldsymbol{H}_\text{ST}$ is approximated with the Bursa-Wolfe equation (see Annex B).

In the time-dependent case, Equation (10) may be generalized to:

$$\boldsymbol{c}_\text{T}(t) = \boldsymbol{G}_\text{T}^{-1} \circ \boldsymbol{H}_\text{ST}(t) \circ \boldsymbol{G}_\text{S}\left(\boldsymbol{c}_\text{S}\right).$$

EXAMPLE 1 If SRF_S and SRF_T are two celestiodetic SRFs for the same spatial object with different ellipsoid RDs, Equation (10) transforms the coordinate $\boldsymbol{c}_\text{S} = \left(\lambda_\text{S}, \varphi_\text{S}, h_\text{S}\right)$ with respect to one oblate ellipsoid to $\boldsymbol{c}_\text{T} = \left(\lambda_\text{T}, \varphi_\text{T}, h_\text{T}\right)$ with respect to the other oblate ellipsoid.

NOTE A transformation between two celestiodetic SRFs for the spatial object Earth is known as a *horizontal datum shift*. A number of numerical approximations developed to implement this operation have been published. Under the assumption of zero rotations and no scale differences ($\omega_1 = \omega_2 = \omega_3 = 0$ and $\Delta s = 0$), a widely used approximation[25] to directly transform $\boldsymbol{c}_S = (\lambda_S, \varphi_S, h_S)$ to $\boldsymbol{c}_T = (\lambda_T, \varphi_T, h_T)$, is the *standard Molodensky transformation* formula [83502T] as follows:

$$\begin{pmatrix} \lambda \\ \varphi \\ h \end{pmatrix}_T = \begin{pmatrix} \lambda \\ \varphi \\ h \end{pmatrix}_S + \begin{pmatrix} \Delta\lambda \\ \Delta\varphi \\ \Delta h \end{pmatrix}$$

where:

$$\Delta\lambda = \frac{-\Delta x \sin\lambda + \Delta y \cos\lambda}{\left(R_N(\varphi) + h\right)\cos\varphi}$$

$$\Delta\varphi = \left(\frac{1}{R_N(\varphi) + h}\right)\left\{\begin{array}{l} -\Delta x \sin\varphi\cos\lambda - \Delta y \sin\varphi\sin\lambda + \Delta z\cos\varphi \\ \quad + \Delta a\left(R_N(\varphi)\varepsilon^2 \sin\varphi\cos\varphi\right)/a \\ \qquad + \Delta f\left(R_N(\varphi)(a/b)R_M(\varphi)(b/a)\right)\sin\varphi\cos\varphi \end{array}\right\}$$

$$\Delta h = \left\{\begin{array}{l} \Delta x\cos\varphi\cos\lambda + \Delta y\cos\varphi\sin\lambda + \Delta z\sin\varphi \\ \quad - \Delta a\left(a/R_N(\varphi)\right) + \Delta f(b/a)R_N(\varphi)\sin^2\varphi \end{array}\right\}$$

Δa = difference in ellipsoid major semi-axis from source to target

Δf = difference in ellipsoid flattening from source to target

The quantities a, b, ε^2, $R_N(\varphi)$, and $R_M(\varphi)$ are defined in Table 5.6.

Equation (10) is only defined for a value of $\boldsymbol{c}_S$ in the CS_S domain if its corresponding position belongs to the CS_T range. If D_S^{-1} is the domain of the inverse generating function $\boldsymbol{G}_S^{-1}$ and D_T^{-1} is the domain of the inverse generating function $\boldsymbol{G}_T^{-1}$, Equation (10) is only defined for $\boldsymbol{c}_S$ in the set:

$$\boldsymbol{G}_S^{-1}\left(D_S^{-1} \cap \boldsymbol{H}_{ST}^{-1}\left(D_T^{-1}\right)\right) \equiv \left\{\boldsymbol{c}_S \text{ in } D_S \mid \boldsymbol{H}_{ST}\left(\boldsymbol{G}_S(\boldsymbol{c}_S)\right) \text{ in } D_T^{-1}\right\} \tag{11}$$

EXAMPLE 2 SRF_S is SRF GEOCENTRIC_WGS_1984 and SRF_T is an instance of SRFT MERCATOR, with ORM WGS_1984. Equation (10) is not defined for any $\boldsymbol{c}_S$ that is on the z-axis of SRF_S, because the z-axis is not contained in the set in Equation (11).

SRF_T may optionally specify a valid-region V_T and may optionally specify an extended-valid region E_T (see 8.3.2.4). If D_T is the domain of the generating function $\boldsymbol{G}_T$, then $V_T \subseteq E_T \subseteq D_T$. If Equation (10) is defined for $\boldsymbol{c}_S$, $\boldsymbol{c}_T$ may be valid ($\boldsymbol{c}_T$ is in V_T), or extended valid ($\boldsymbol{c}_T$ is in $E_T \setminus V_T$) or neither. The set of $\boldsymbol{c}_S$ coordinates for which $\boldsymbol{c}_T$ is valid is:

$$\boldsymbol{G}_S^{-1}\left(D_S^{-1} \cap \boldsymbol{H}_{ST}^{-1}\left(\boldsymbol{G}_T\left(E_T\right)\right)\right) \equiv \left\{\boldsymbol{c}_S \text{ in } D_S \mid \boldsymbol{H}_{ST}\left(\boldsymbol{G}_S(\boldsymbol{c}_S)\right) \text{ in } \boldsymbol{G}_T\left(E_T\right)\right\}$$

where:

$$\boldsymbol{G}_T\left(E_T\right) \equiv \left\{\boldsymbol{p} \text{ in } D_T^{-1} \mid \boldsymbol{G}_T^{-1}(\boldsymbol{p}) \text{ in } E_T\right\}.$$

In applications that functionally conform to an SRM profile, the domain of an SRF operation is restricted to the accuracy domain of the SRF as specified by that profile (see Clause 12).

25 Historically it was thought that these approximations would require less computation than direct conversion. The perceived computational advantage may have been overcome by technology advances. New efficient algorithms for converting celestiocentric coordinates to celestiodetic coordinates have been developed that result in appreciably faster transformations without the attendant loss of accuracy.

