



Designation: D 1535 – 07

## Standard Practice for Specifying Color by the Munsell System<sup>1</sup>

This standard is issued under the fixed designation D 1535; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last approval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

*This standard has been approved for use by agencies of the Department of Defense.*

### 1. Scope

1.1 This practice provides a means of specifying the colors of objects in terms of the Munsell color order system, a system based on the color-perception attributes hue, lightness, and chroma. The practice is limited to opaque objects, such as painted surfaces viewed in daylight by an observer having normal color vision. This practice provides a simple visual method as an alternative to the more precise and more complex method based on spectrophotometry and the CIE system (see Practices E 308 and E 1164). Provision is made for conversion of CIE data to Munsell notation.

1.2 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

### 2. Referenced Documents

#### 2.1 ASTM Standards:<sup>2</sup>

- D 1729 Practice for Visual Appraisal of Colors and Color Differences of Diffusely-Illuminated Opaque Materials
- D 3134 Practice for Establishing Color and Gloss Tolerances
- E 284 Terminology of Appearance
- E 308 Practice for Computing the Colors of Objects by Using the CIE System
- E 1164 Practice for Obtaining Spectrometric Data for Object-Color Evaluation

### 3. Terminology

3.1 Terms and definitions in Terminology E 284 are applicable to this practice.

### 3.2 Definitions:

3.2.1 *Munsell notation,  $n$* —(1) the Munsell hue, value, and chroma assigned to the color of a specimen by visually comparing the specimen to the chips in the *Munsell Book of Color*;<sup>3</sup> (2) a notation in the Munsell color system, derived from luminous reflectance factor  $Y$  and chromaticity coordinates  $x$  and  $y$ , in the CIE system for standard illuminant  $C$ , by the use of scales defined by the Optical Society of America Subcommittee on the Spacing of the Munsell Colors (1).<sup>4</sup>

3.2.1.1 *Discussion*—The Munsell notation is written as a combination of letters and numbers by which the color of an opaque object may be specified with respect to Munsell hue  $H$ , Munsell value  $V$ , and Munsell chroma  $C$ , written in the form  $H V/C$ .

3.2.2 *hue,  $n$* —the attribute of color perception by means of which a color is judged to be red, orange, yellow, green, blue, purple, or intermediate between adjacent pairs of these, considered in a closed ring (red and purple being an adjacent pair).

3.2.3 *Munsell hue,  $n$* —an attribute of color used in the Munsell color system to indicate the hue of a specimen viewed in daylight.

3.2.3.1 *Discussion*—Two systems of designating Munsell hue are shown in Fig. 1, a letter-number system and an all-number system. The two systems are equivalent, but the letter-number system is preferred, because it requires no prior knowledge or memory of the correspondence of numbers to hues. The hue circle is graduated in steps judged visually to be approximately equal.

3.2.4 *lightness,  $n$* —the attribute of color perception by which a non-self-luminous body is judged to reflect more or less light.

3.2.5 *Munsell value,  $n$* —an attribute of color used in the Munsell color system to indicate the lightness of a specimen viewed in daylight, on a scale extending from 0 for ideal black to 10 for ideal white, in steps that are visually approximately equal in magnitude.

<sup>1</sup> This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.07 on Color Order Systems.

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<sup>2</sup> For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

<sup>3</sup> Available from GretagMacbeth, 617 Little Britain Road, New Windsor, NY 12553-6148.

<sup>4</sup> The boldface numbers in parentheses refer to a list of references at the end of this standard.

3.2.5.1 *Discussion*—Achromatic or neutral colors are designated *N* followed by the value notation, thus: *N* 5.61/.

3.2.6 *chroma, n*—the attribute of color used to indicate the degree of departure of the color from a neutral color of the same lightness.

3.2.7 *Munsell chroma, n*—an attribute of color used in the Munsell color system to indicate the degree of departure of a color from a gray of the same Munsell value, in steps that are visually approximately equal in magnitude.

### 3.3 *Definitions of Terms Specific to This Standard:*

3.3.1 *Munsell surface-color perception solid, n*—a spatial representation of colors in the form of a cylindrical coordinate system based on the three perceptual attributes: hue, lightness and chroma.

3.3.1.1 *Discussion*—(1) This solid (see Fig. 2 (2)) forms the basis of the Munsell notation in which Munsell hue corresponds to hue, Munsell value corresponds to lightness, and Munsell chroma corresponds to chroma. The central, vertical axis dimension represents neutral colors, ranging from black at the bottom, through a gradation of grays, to white at the top. The lightness of a color perceived as chromatic (not gray) is represented by the distance above the base plane. Hue is represented by the angular position about this axis (see Discussion (2)). Chroma is represented by the perpendicular distance from the central axis. If the observer has normal color vision, is adapted to daylight, and views the specimen illuminated by CIE source *C* or *D65*, against a medium gray to white background, the Munsell value of the specimen correlates well with the observer's perception of the lightness of the color. Under the same conditions, the Munsell hue correlates well with the observer's perception of hue and the Munsell chroma with the perception of chroma.

3.3.1.2 *Discussion*—(2) Although the original system proposed by Munsell was a left-handed coordinate system, the system is often represented as a right-handed system because it facilitates comparison to the CIE chromaticity diagram, taken to be right-handed.

3.3.2 *Munsell hue circle, n*—a spatial representation of the Munsell hue sectors on a circle, where the angular spacing represents a uniform scaling of hue; see Fig. 2.

## 4. Significance and Use

4.1 This practice is used by artists, designers, scientists, engineers, and government regulators, to specify an existing or desired color. It is used in the natural sciences to record the colors of specimens, or identify specimens, such as human complexion, flowers, foliage, soils, and minerals. It is used to specify colors for commerce and for control of color-production processes, when instrumental color measurement is not economical. The Munsell system is widely used for color tolerancing, even when instrumentation is employed (see Practice D 3134). It is common practice to have color chips made to illustrate an aim color and the just tolerable deviations from that color in hue, value, and chroma, such a set of chips being called a *Color Tolerance Set*. A color tolerance set exhibits the aim color and color tolerances so that everyone involved in the selection, production, and acceptance of the color can directly perceive the intent of the specification, before bidding to supply the color or starting production. A

color tolerance set may be measured to establish instrumental tolerances. Without extensive experience, it may be impossible to visualize the meaning of numbers resulting from color measurement, but by this practice, the numbers can be translated to the Munsell color-order system, which is exemplified by colored chips for visual examination. This color-order system is the basis of the ISCC-NBS Method of Designating Colors and a Dictionary of Color Names, as well as the Universal Color Language, which associates color names, in the English language, with Munsell notations (3).

## 5. Apparatus

5.1 *Munsell Book of Color*, matte or glossy edition.<sup>3</sup>

5.2 *Gray Masks*, with rectangular openings the size of the chips in the *Munsell Book of Color*.

5.3 *Daylight Illuminating Equipment*, as described in Practice D 1729.

## 6. Preparation of Test Specimens

6.1 This practice does not cover the preparation of test specimens. If preparation is necessary, see other ASTM standards covering the appropriate materials or agree among interested parties on what the procedure shall be.

## 7. Munsell Notation by Visual Means

7.1 *Lighting and Viewing Conditions:*

7.1.1 Specimens must be examined by an observer with normal color vision.

7.1.2 For critical applications, use daylight illuminating equipment as described in Practice D 1729.

7.1.3 If the lighting equipment described in Practice D 1729 is not available, natural daylight can be used to obtain notations having accuracy adequate for many purposes.

7.2 *Procedure:*

7.2.1 When using daylight illuminating equipment, follow the lighting and viewing recommendations of Practice D 1729.

7.2.2 When determining the Munsell notation with natural daylight, select a window through which the sun is not shining. A north window is usually used in the northern hemisphere, and a south window is usually used in the southern hemisphere. Place a working surface at the window so the light reaches the surface from the observer's side, chiefly from the sky, and at angles centering on 45° above the horizontal. Place a canopy of black cloth above the working surface to prevent errors caused by the ceiling or other objects being reflected from the surface of the specimens, or by light other than daylight falling on the work surface. Place the specimen on a neutral medium gray to white background, where it is uniformly illuminated by daylight. View the specimen along a direction just far enough from the normal to avoid reflection of your forehead. Although 45° illumination and perpendicular viewing are recommended by the CIE, converse conditions are equivalent if a black matte surface is placed opposite the observer to minimize the amount of light reflected from the specimen surface.

7.2.3 If both matte and glossy editions of the *Munsell Book of Color* are available, use the one having gloss most like the specimen. Select the two adjacent Munsell constant-hue charts or chips between which the hue of the specimen lies. Place one

on each side of the specimen. Cover the specimen and charts with the gray masks so the specimen and one chip from each chart can be seen. Move the masks from chip to chip to find the chips most like the specimen. The glossy chips are removable. Remove them and place immediately adjacent to the specimen. Estimate, in the following order, the value, the chroma, and the hue, by interpolation or extrapolation of the notations on the chips, as described in 7.2.3.1 to 7.2.3.3. Interchange the positions of the charts, repeat the estimations, and average the results.

**7.2.3.1 Value**—Find the chips between which the value of the specimen lies. Estimate the value of the specimen to the nearest tenth of the one-value-step interval between adjacent value levels and record it, for example, 4.2.

**7.2.3.2 Chroma**—Move the masks to present successive colors of the same chroma and, by interpolation or extrapolation, determine the Munsell chroma. Pay chief attention to the Munsell chips having values nearest that of the specimen and secondary attention to those next nearest. Although all Munsell chips of the same Munsell chroma are intended to appear to have the same perceptual chroma, a slightly different estimate of chroma may be obtained by comparison with the chips of the next value. In such cases, average the estimated Munsell chromas. Note that there are usually two chroma steps between adjacent columns of a chart. Estimate chroma to the nearest fifth of the 2-chroma interval and record it, for example, 6.4.

**7.2.3.3 Hue**—Estimate the hue of the specimen by interpolation between the chips of the nearest Munsell value and chroma in the selected hue charts. Estimate to the nearest fifth of the 2.5-hue steps between adjacent hue charts and record it, for example, 4.5 R. (The tenth step of one hue sector is the zero of the next. The 10 is used; the zero is not.) If the value and chroma of the specimen do not correspond closely to those of any chip, repeat the interpolation of hue with the next closest pair of chips and record the average or estimate the hue as being closer to that of one or the other of the selected pairs of chips.

**7.2.3.4** The Munsell notation for the hue *H*, the value *V*, and the chroma *C*, is written in the form *HV/C*. Using the examples given, the Munsell notation would be written 4.5 R 4.2/6.4.

## 8. Munsell Color Notation from CIE Measurement

**NOTE 1**—The CIE results for the specimen must be based upon color measurements in which the specular component was excluded, and with calculations made using the 1931 2° standard observer and illuminant C.

**8.1 Procedure**—Convert the luminous reflectance, *Y*, and the chromaticity coordinates, *x*, *y*, of the specimen to Munsell color notation by use of Table 1 and Figs. 3-16.<sup>5</sup> Table 2 contains the numerical data from Ref (1) upon which Figs. 3-16 were based.

**NOTE 2**—For further information concerning Figs. 3-7, Fig. 9, Fig. 11, Fig. 13, Fig. 15 and Fig. 16 see Newhall, et al. (1). For further information concerning Fig. 8 and Fig. 10, see I. Nimeroff (2).

<sup>5</sup> Fig. 8, Fig. 10, Fig. 12, Fig. 14, and Fig. 16 are enlargements of the low-chroma areas of Fig. 7, Fig. 9, Fig. 11, Fig. 13, and Fig. 15. Large-scale diagrams of Figs. 3-16 are available from GretagMacbeth.

**NOTE 3**—The luminous reflectance in the original reference (1) was measured relative to Magnesium Oxide. The luminous reflectance values in Table 2 were changed so that it is relative to the perfect reflecting diffuser.

**8.2** In Table 1, find the value, *V*, equivalent to the luminous reflectance, *Y*. Use Figs. 3-16 to estimate hue and chroma for value levels above and below the value found and linearly interpolate the hues and chromas for the desired value level. If the required value level differs from the nearest level by 0.05 or less, simply use the hue and chroma for the nearest level.

**8.3 Munsell Notation of Dark Colors**—If the Munsell value is less than 1.0, use the extension of the Munsell system to very dark colors (4). Table 3 contains the numerical data from Ref (4) for 40 hues at values 0.8/, 0.6/, 0.4/, and 0.2/ and chromas up to the theoretical pigment limits.

**NOTE 4**—The luminous reflectance in the original reference (4) was measured relative to Magnesium Oxide. The luminous reflectance in Table 3 was changed so that it is relative to the perfect reflecting diffuser.

**8.4** Table 1 was derived from the following relationships (5):

$$\text{For } Y \leq 0.9: V = UY^W \quad (1)$$

$$\text{For } Y \geq 0.9: V = AY^{1/3} - B - C/[(DY - E)^2 + F]$$

$$+ G/Y^H + J \sin(KY^{1/3} + 1)$$

$$+ (M/Y) \sin[N(Y - 2)]$$

$$- (P/QY) \sin[S(Y - T)]$$

where:

$$A = 2.49268$$

$$B = 1.5614$$

$$C = 0.985$$

$$D = 0.1073$$

$$E = 3.084$$

$$F = 7.54$$

$$G = 0.0133 - aaf7-23da1f2163e8/astm-d1535-07$$

$$H = 2.3$$

$$J = 0.0084$$

$$K = 4.1$$

$$M = 0.0221$$

$$N = 0.39$$

$$P = 0.0037$$

$$Q = 0.44$$

$$S = 1.28$$

$$T = 0.53$$

$$U = 0.87445$$

$$W = 0.9967$$

**8.5 Computer Conversion of CIE Measurement Data**—Computer programs that convert CIE data to Munsell color notations are available commercially from various manufacturers of color control instruments or software, or both. The accuracy of a computer program can be determined by comparing the results obtained with that program to those obtained using the graphical method described in this practice. Before using a computer conversion program, the user should ascertain that the program's accuracy is sufficient for the proposed usage. Table 4 contains graphical conversions that may be used to verify the accuracy of data obtained by computer conversions.

**NOTE 5**—Many of the original computer programs used Magnesium

Oxide as the reference white for determining luminous reflectance, Y, and Munsell Value, V. The reference white was changed to the perfect reflecting diffuser, and the user should ascertain that the computer conversion program uses the correct reference white.

NOTE 6—Although the chromaticity coordinates were not affected by the change of the reference white to the perfect reflecting diffuser, CIE X and Z tristimulus values calculated from them will change. The changes in X, Y, and Z will also affect color coordinates determined by transforming those tristimulus values.

8.6 In the interest of completeness, and because the reference white has changed since the year 1943 when it was proposed and published (1) (See Notes 3-6), the Munsell fifth-order equation relating Munsell value V to CIE luminance Y is presented here with the perfect reflecting diffuser as the white reference:

$$Y = 1.1914 V - 0.22533 V^2 + 0.23352 V^3 - 0.020484 V^4 + 0.00081939 V^5 \quad (2)$$

The coefficients of this equation are obtained from the 1943 equation by multiplying each coefficient by 0.975, the reflectance factor of magnesium oxide with respect to the perfect reflecting diffuser, and rounding to five digits of precision.

Presumably these five digits consist of four that are significant and a guard digit. Results obtained from this equation should thus be rounded to no more than four significant digits.

**9. Report**

9.1 Report the notation in the Munsell system, specifying whether the notation was obtained visually, using the matte or glossy *Munsell Book of Color*, or by conversion of CIE colorimetric data.

9.1.1 If obtained visually, note the source of illumination (artificial daylight or natural daylight).

9.1.2 If obtained from colorimetric data, note the instrument used.

**10. Precision**

10.1 The estimated precision within which a color notation can be determined by visual interpolation is 0.5 hue step, 0.1 value step, and 0.4 chroma step.

**11. Keywords**

11.1 color; Munsell; Munsell color order system; Munsell notation

**APPENDIXES**

(Nonmandatory Information)

**X1. EXAMPLE OF CONVERTING OF MUNSELL NOTATION**

X1.1 Given the CIE data  $Y = 46.02$ ,  $x = 0.500$  and  $y = 0.454$ , find the Munsell notation.

X1.1.1 In Table 1,  $Y = 46.02$  corresponds to Munsell value 7.28.

X1.1.2 The value lies between 7 and 8, so the hue and chroma will be found by interpolating these quantities between those found in Fig. 11 and Fig. 13. On Fig. 11,  $x = 0.500$  and  $y = 0.454$  corresponds to a hue of 10YR and a chroma of 13.1. On Fig. 13, the same  $x$  and  $y$  correspond to a hue just a small amount redder than 10YR, an amount less than 0.25 hue step, so the hue is read as 10YR. The chroma is 14.6.

X1.1.3 The value is 7.28, which is 0.28 of the way from 7 to 8, so the interpolated hue is that for value 7 plus 0.28 times the difference between the hues found at those two value levels. Since the difference was zero, the interpolated hue is simply the hue found for value 7. The interpolated chroma is found in the same way. The difference in chroma for the two value levels is  $14.6 - 13.1 = 1.5$ . The difference is multiplied by the interpolation factor:  $1.5 \times 0.28 = 0.42$ , which may be rounded to 0.4. This amount is added to the chroma for value level 7:  $0.4 + 13.1 = 13.5$ .

X1.1.4 The Munsell notation is 10YR 7.2/13.5.

**X2. METHOD FOR CALCULATING TRISTIMULUS VALUES OF MUNSELL NOTATIONS IN ILLUMINANT-OBSERVER COMBINATIONS OTHER THAN ILLUMINANT C-2° OBSERVER**

X2.1 *Starting From Munsell Notations*—Look-up the tristimulus values of the Munsell notation of interest in Table 2 or Table 3. Follow the procedures of 8.1-8.3 in the opposite sense to that given there. 8.1-8.3 give instructions for table look-up from CIE C-2° tristimulus values to Munsell notation. You will be looking up the CIE C-2° tristimulus values from Munsell notation. The table will yield CIE tristimulus values in Y, x, y format. Calculate the tristimulus values from:

$$\begin{aligned} X &= x Y/y \\ Y &= Y \end{aligned} \quad (X2.1)$$

and

$$Z = z Y/y$$

$$z = 1 - x - y$$

X2.2 *Starting From Tristimulus Values in Any Illuminant-Observer Combination*—Calculate, by the following matrix equation, three values C using data supplied in Table X2.1 of this practice appropriate to the Illuminant-Observer combination of the tristimulus values.

$$C = T^{-1} (Q - Q_{v0}) \quad (X2.2)$$

where  $C$  is a  $3 \times 1$  vector containing the three derived principal component coefficients,  $T^{-1}$  is a  $3 \times 3$  transformation matrix given in **Table X2.1**,  $Q$  is a  $3 \times 1$  vector of the three tristimulus values such that:

$$Q = [X \ Y \ Z]^T \quad (\text{X2.3})$$

and  $Q_{V_0}$  is a  $3 \times 1$  vector of mean reflectances given in **Table X2.1** in a column labeled  $V_0$  and the symbol  $T$  refers to the transpose of the matrix.

X2.2.1 The tristimulus values in another illuminant-observer combination may be calculated from the following:

$$Q = TC + V_0 \quad (\text{X2.4})$$

where  $T$  and  $V_0$  are chosen from **Table X2.1** and refer to the new illuminant-observer combination in whose view the tristimulus values are sought.

X2.2.2 It should be understood that measuring and integrating the reflectance factor curve of a specimen from the *Munsell Book of Color* may not give exactly the same results as this matrix transformation. This occurs because the specimen's notation specification is set only in terms of tristimulus values and only in CIE Illuminant C for the  $2^\circ$  Observer. Further there exists a color-difference tolerance from that specification in producing the atlas.

X2.2.3 Reference (6) covers the derivation of the principal components of reflectance used here. This reference also provides additional information about these transformations and provides methods for transforming among spectral data, tristimulus values, principal component coordinates and all combinations thereof.

X2.3 A worked example of the above calculations:

X2.3.1 Begin with Munsell notation 7.5G 5/10. From the **Table 2** look-up of **X2.1**, the  $Y$ ,  $x$ ,  $y$  values are:  $Y = 19.27$ ,  $x = 0.2200$ , and  $y = 0.4082$ . From Eq X2.1 the tristimulus values in Illuminant C-1931 Standard Colorimetric Observer are  $X = 10.39$ ,  $Y = 19.27$ , and  $Z = 17.55$ .

X2.3.2 For Eq X2.2 the  $T^{-1}$  matrix from **Table X2.1** that applies to the current illuminant-observer combination (Illuminant C-1931 Observer) is chosen with the values of  $V_0$  from the same table:

$$C = \begin{bmatrix} 6.264 & -1.516 & 0.695 \\ 7.811 & -5.680 & -2.358 \\ 4.048 & -5.450 & 1.440 \end{bmatrix} \begin{bmatrix} 10.39 - 31.67 \\ 19.27 - 31.96 \\ 17.55 - 31.07 \end{bmatrix} \quad (\text{X2.5})$$

thus:

$$\begin{aligned} C_1 &= -123.46 \\ C_2 &= -62.26 \\ C_3 &= -36.45. \end{aligned}$$

X2.3.3 For Eq X2.4 the  $T$  matrix that applies to the new illuminant-observer combination is chosen from **Table X2.1** with the values of  $V_0$  also for the new illuminant-observer combination:

$$Q = \begin{bmatrix} 0.1779 & 0.0071 & -0.0761 \\ 0.1800 & -0.0687 & -0.1582 \\ 0.1554 & -0.2214 & 0.1957 \end{bmatrix} \begin{bmatrix} -123.46 \\ -62.26 \\ -36.45 \end{bmatrix} + \begin{bmatrix} 30.53 \\ 31.64 \\ 28.04 \end{bmatrix} \quad (\text{X2.6})$$

and the tristimulus values for the new illuminant-observer combination (Illuminant D65-1964 Standard Colorimetric Observer) are  $X = 10.86$ ,  $Y = 19.46$ , and  $Z = 15.51$ .

**TABLE X2.1 The Values of the Matrix  $V_0$  and the  $3 \times 3$  Values of the Matrices  $T$  and  $T^{-1}$  for Several Important Illuminant-Observer Combinations**

T Matrices				
	$V_0$	$V_1$	$V_2$	$V_3$
Illuminant D65 – 1964 10° Observer				
X	30.53	0.1779	0.0071	-0.0761
Y	31.64	0.1800	-0.0687	-0.1582
Z	28.04	0.1554	-0.2214	0.1957
Illuminant A – 1964 10° Observer				
X	37.56	0.2206	0.0802	-0.1076
Y	32.60	0.1881	-0.0142	-0.1618
Z	9.32	0.0514	-0.0739	0.0580
Illuminant C – 1931 2° Observer				
X	31.67	0.1848	0.0141	-0.0661
Y	31.96	0.1827	-0.0545	-0.1775
Z	31.07	0.1720	-0.2460	0.2086
T <sup>-1</sup> Matrices				
		$V_1$	$V_2$	$V_3$
Illuminant D65 – 1964 10° Observer				
X		7.097	-2.262	0.929
Y		8.757	-6.828	-2.118
Z		4.272	-5.929	1.976
Illuminant A – 1964 10° Observer				
X		4.342	-1.122	4.929
Y		6.533	-6.227	-5.252
Z		4.477	-6.940	6.192
Illuminant C – 1931 2° Observer				
X		6.264	-1.516	0.695
Y		7.811	-5.680	-2.358
Z		4.048	-5.450	1.440

Document Preview

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REFERENCES

<https://standards.iteh.ai/catalog/standards/sist/7517a81d-9a64-4012-aaf7-23da1f2163e8/astm-d1535-07>

- (1) Newhall, S. M., Nickerson, D., and Judd, D. B., "Final Report of the OSA Subcommittee on the Spacing of the Munsell Colors," *Journal, Optical Society of America*, Vol 33, 1943, pp. 385–418.
- (2) Taken from Nimeroff, I., "Colorimetry," *Monograph 104*, National Bureau of Standards, NBS, January 1968.
- (3) Kelly, K. L., and Judd, D. B., *Color: Universal Language and Dictionary of Names*, National Technical Information Service (NTIS), Springfield, VA 22161, Order No. PB 265225.
- (4) Judd, D. B., and Wyszecki, G., "Extension of the Munsell Renotation System to Very Dark Colors," *Journal, Optical Society of America*, Vol 46, 1956, pp. 281–285
- (5) McCamy, C. S., "Munsell Value as Explicit Functions of CIE Luminance Factor," *Color Research and Application*, Vol 17, 1992, pp. 205–207.
- (6) Fairman, H. S. and Brill, M. H., "The Principal Components of Reflectance," *Color Research and Application*, Vol 29, 2004, pp. 104–110.

**TABLE 1 Munsell Value V for Given Luminous Reflectance Factor Y, in Percent, Relative to the Perfect Reflecting Diffuser**

Y	V	Y	V	Y	V	Y	V	Y	V
0.01	0.01	0.71	0.62	1.41	1.16	2.11	1.57	2.81	1.90
0.02	0.02	0.72	0.63	1.42	1.17	2.12	1.58	2.82	1.90
0.03	0.03	0.73	0.64	1.43	1.18	2.13	1.58	2.83	1.91
0.04	0.04	0.74	0.65	1.44	1.18	2.14	1.59	2.84	1.91
0.05	0.04	0.75	0.66	1.45	1.19	2.15	1.59	2.85	1.92
0.06	0.05	0.76	0.67	1.46	1.20	2.16	1.60	2.86	1.92
0.07	0.06	0.77	0.67	1.47	1.20	2.17	1.60	2.87	1.92
0.08	0.07	0.78	0.68	1.48	1.21	2.18	1.61	2.88	1.93
0.09	0.08	0.79	0.69	1.49	1.22	2.19	1.61	2.89	1.93
0.10	0.09	0.80	0.70	1.50	1.22	2.20	1.62	2.90	1.94
0.11	0.10	0.81	0.71	1.51	1.23	2.21	1.62	2.91	1.94
0.12	0.11	0.82	0.72	1.52	1.24	2.22	1.63	2.92	1.94
0.13	0.11	0.83	0.73	1.53	1.24	2.23	1.63	2.93	1.95
0.14	0.12	0.84	0.73	1.54	1.25	2.24	1.64	2.94	1.95
0.15	0.13	0.85	0.74	1.55	1.25	2.25	1.64	2.95	1.96
0.16	0.14	0.86	0.75	1.56	1.26	2.26	1.65	2.96	1.96
0.17	0.15	0.87	0.76	1.57	1.27	2.27	1.65	2.97	1.97
0.18	0.16	0.88	0.77	1.58	1.27	2.28	1.66	2.98	1.97
0.19	0.17	0.89	0.78	1.59	1.28	2.29	1.66	2.99	1.97
0.20	0.18	0.90	0.79	1.60	1.29	2.30	1.67	3.00	1.98
0.21	0.18	0.91	0.79	1.61	1.29	2.31	1.67	3.01	1.98
0.22	0.19	0.92	0.80	1.62	1.30	2.32	1.68	3.02	1.99
0.23	0.20	0.93	0.81	1.63	1.30	2.33	1.68	3.03	1.99
0.24	0.21	0.94	0.81	1.64	1.31	2.34	1.69	3.04	1.99
0.25	0.22	0.95	0.82	1.65	1.32	2.35	1.69	3.05	2.00
0.26	0.23	0.96	0.83	1.66	1.32	2.36	1.70	3.06	2.00
0.27	0.24	0.97	0.84	1.67	1.33	2.37	1.70	3.07	2.01
0.28	0.25	0.98	0.85	1.68	1.33	2.38	1.71	3.08	2.01
0.29	0.25	0.99	0.86	1.69	1.34	2.39	1.71	3.09	2.01
0.30	0.26	1.00	0.86	1.70	1.35	2.40	1.72	3.10	2.02
0.31	0.27	1.01	0.87	1.71	1.35	2.41	1.72	3.11	2.02
0.32	0.28	1.02	0.88	1.72	1.36	2.42	1.72	3.12	2.03
0.33	0.29	1.03	0.89	1.73	1.36	2.43	1.73	3.13	2.03
0.34	0.30	1.04	0.90	1.74	1.37	2.44	1.73	3.14	2.03
0.35	0.31	1.05	0.90	1.75	1.38	2.45	1.74	3.15	2.04
0.36	0.32	1.06	0.91	1.76	1.38	2.46	1.74	3.16	2.04
0.37	0.32	1.07	0.92	1.77	1.39	2.47	1.75	3.17	2.05
0.38	0.33	1.08	0.93	1.78	1.39	2.48	1.75	3.18	2.05
0.39	0.34	1.09	0.94	1.79	1.40	2.49	1.76	3.19	2.05
0.40	0.35	1.10	0.94	1.80	1.40	2.50	1.76	3.20	2.06
0.41	0.36	1.11	0.95	1.81	1.41	2.51	1.77	3.21	2.06
0.42	0.37	1.12	0.96	1.82	1.42	2.52	1.77	3.22	2.06
0.43	0.38	1.13	0.97	1.83	1.42	2.53	1.78	3.23	2.07
0.44	0.39	1.14	0.97	1.84	1.43	2.54	1.78	3.24	2.07
0.45	0.39	1.15	0.98	1.85	1.43	2.55	1.78	3.25	2.08
0.46	0.40	1.16	0.99	1.86	1.44	2.56	1.79	3.26	2.08
0.47	0.41	1.17	1.00	1.87	1.44	2.57	1.79	3.27	2.08
0.48	0.42	1.18	1.00	1.88	1.45	2.58	1.80	3.28	2.09
0.49	0.43	1.19	1.01	1.89	1.45	2.59	1.80	3.29	2.09
0.50	0.44	1.20	1.02	1.90	1.46	2.60	1.81	3.30	2.10
0.51	0.45	1.21	1.03	1.91	1.47	2.61	1.81	3.31	2.10
0.52	0.46	1.22	1.03	1.92	1.47	2.62	1.82	3.32	2.10
0.53	0.46	1.23	1.04	1.93	1.48	2.63	1.82	3.33	2.11
0.54	0.47	1.24	1.05	1.94	1.48	2.64	1.82	3.34	2.11
0.55	0.48	1.25	1.05	1.95	1.49	2.65	1.83	3.35	2.11
0.56	0.49	1.26	1.06	1.96	1.49	2.66	1.83	3.36	2.12
0.57	0.50	1.27	1.07	1.97	1.50	2.67	1.84	3.37	2.12
0.58	0.51	1.28	1.08	1.98	1.50	2.68	1.84	3.38	2.13
0.59	0.52	1.29	1.08	1.99	1.51	2.69	1.85	3.39	2.13
0.60	0.53	1.30	1.09	2.00	1.51	2.70	1.85	3.40	2.13
0.61	0.53	1.31	1.10	2.01	1.52	2.71	1.86	3.41	2.14
0.62	0.54	1.32	1.10	2.02	1.53	2.72	1.86	3.42	2.14
0.63	0.55	1.33	1.11	2.03	1.53	2.73	1.86	3.43	2.14
0.64	0.56	1.34	1.12	2.04	1.54	2.74	1.87	3.44	2.15
0.65	0.57	1.35	1.12	2.05	1.54	2.75	1.87	3.45	2.15
0.66	0.58	1.36	1.13	2.06	1.55	2.76	1.88	3.46	2.15
0.67	0.59	1.37	1.14	2.07	1.55	2.77	1.88	3.47	2.16
0.68	0.60	1.38	1.14	2.08	1.56	2.78	1.89	3.48	2.16
0.69	0.60	1.39	1.15	2.09	1.56	2.79	1.89	3.49	2.17
0.70	0.61	1.40	1.16	2.10	1.57	2.80	1.89	3.50	2.17

**TABLE 1** *Continued*

Y	V	Y	V	Y	V	Y	V	Y	V
3.51	2.17	4.21	2.41	4.91	2.62	5.61	2.81	6.31	2.98
3.52	2.18	4.22	2.41	4.92	2.62	5.62	2.81	6.32	2.98
3.53	2.18	4.23	2.42	4.93	2.62	5.63	2.81	6.33	2.98
3.54	2.18	4.24	2.42	4.94	2.63	5.64	2.81	6.34	2.99
3.55	2.19	4.25	2.42	4.95	2.63	5.65	2.82	6.35	2.99
3.56	2.19	4.26	2.43	4.96	2.63	5.66	2.82	6.36	2.99
3.57	2.19	4.27	2.43	4.97	2.64	5.67	2.82	6.37	2.99
3.58	2.20	4.28	2.43	4.98	2.64	5.68	2.83	6.38	3.00
3.59	2.20	4.29	2.44	4.99	2.64	5.69	2.83	6.39	3.00
3.60	2.21	4.30	2.44	5.00	2.64	5.70	2.83	6.40	3.00
3.61	2.21	4.31	2.44	5.01	2.65	5.71	2.83	6.41	3.00
3.62	2.21	4.32	2.44	5.02	2.65	5.72	2.84	6.42	3.01
3.63	2.22	4.33	2.45	5.03	2.65	5.73	2.84	6.43	3.01
3.64	2.22	4.34	2.45	5.04	2.66	5.74	2.84	6.44	3.01
3.65	2.22	4.35	2.45	5.05	2.66	5.75	2.84	6.45	3.01
3.66	2.23	4.36	2.46	5.06	2.66	5.76	2.85	6.46	3.01
3.67	2.23	4.37	2.46	5.07	2.66	5.77	2.85	6.47	3.02
3.68	2.23	4.38	2.46	5.08	2.67	5.78	2.85	6.48	3.02
3.69	2.24	4.39	2.47	5.09	2.67	5.79	2.85	6.49	3.02
3.70	2.24	4.40	2.47	5.10	2.67	5.80	2.86	6.50	3.02
3.71	2.24	4.41	2.47	5.11	2.67	5.81	2.86	6.51	3.03
3.72	2.25	4.42	2.48	5.12	2.68	5.82	2.86	6.52	3.03
3.73	2.25	4.43	2.48	5.13	2.68	5.83	2.86	6.53	3.03
3.74	2.25	4.44	2.48	5.14	2.68	5.84	2.87	6.54	3.03
3.75	2.26	4.45	2.48	5.15	2.69	5.85	2.87	6.55	3.04
3.76	2.26	4.46	2.49	5.16	2.69	5.86	2.87	6.56	3.04
3.77	2.26	4.47	2.49	5.17	2.69	5.87	2.87	6.57	3.04
3.78	2.27	4.48	2.49	5.18	2.69	5.88	2.88	6.58	3.04
3.79	2.27	4.49	2.50	5.19	2.70	5.89	2.88	6.59	3.05
3.80	2.28	4.50	2.50	5.20	2.70	5.90	2.88	6.60	3.05
3.81	2.28	4.51	2.50	5.21	2.70	5.91	2.88	6.61	3.05
3.82	2.28	4.52	2.51	5.22	2.70	5.92	2.89	6.62	3.05
3.83	2.29	4.53	2.51	5.23	2.71	5.93	2.89	6.63	3.05
3.84	2.29	4.54	2.51	5.24	2.71	5.94	2.89	6.64	3.06
3.85	2.29	4.55	2.51	5.25	2.71	5.95	2.89	6.65	3.06
3.86	2.30	4.56	2.52	5.26	2.72	5.96	2.90	6.66	3.06
3.87	2.30	4.57	2.52	5.27	2.72	5.97	2.90	6.67	3.06
3.88	2.30	4.58	2.52	5.28	2.72	5.98	2.90	6.68	3.07
3.89	2.31	4.59	2.53	5.29	2.72	5.99	2.90	6.69	3.07
3.90	2.31	4.60	2.53	5.30	2.73	6.00	2.91	6.70	3.07
3.91	2.31	4.61	2.53	5.31	2.73	6.01	2.91	6.71	3.07
3.92	2.32	4.62	2.54	5.32	2.73	6.02	2.91	6.72	3.07
3.93	2.32	4.63	2.54	5.33	2.73	6.03	2.91	6.73	3.08
3.94	2.32	4.64	2.54	5.34	2.74	6.04	2.91	6.74	3.08
3.95	2.33	4.65	2.54	5.35	2.74	6.05	2.92	6.75	3.08
3.96	2.33	4.66	2.55	5.36	2.74	6.06	2.92	6.76	3.08
3.97	2.33	4.67	2.55	5.37	2.74	6.07	2.92	6.77	3.09
3.98	2.34	4.68	2.55	5.38	2.75	6.08	2.92	6.78	3.09
3.99	2.34	4.69	2.56	5.39	2.75	6.09	2.93	6.79	3.09
4.00	2.34	4.70	2.56	5.40	2.75	6.10	2.93	6.80	3.09
4.01	2.35	4.71	2.56	5.41	2.76	6.11	2.93	6.81	3.10
4.02	2.35	4.72	2.56	5.42	2.76	6.12	2.93	6.82	3.10
4.03	2.35	4.73	2.57	5.43	2.76	6.13	2.94	6.83	3.10
4.04	2.36	4.74	2.57	5.44	2.76	6.14	2.94	6.84	3.10
4.05	2.36	4.75	2.57	5.45	2.77	6.15	2.94	6.85	3.10
4.06	2.36	4.76	2.58	5.46	2.77	6.16	2.94	6.86	3.11
4.07	2.37	4.77	2.58	5.47	2.77	6.17	2.95	6.87	3.11
4.08	2.37	4.78	2.58	5.48	2.77	6.18	2.95	6.88	3.11
4.09	2.37	4.79	2.58	5.49	2.78	6.19	2.95	6.89	3.11
4.10	2.37	4.80	2.59	5.50	2.78	6.20	2.95	6.90	3.12
4.11	2.38	4.81	2.59	5.51	2.78	6.21	2.96	6.91	3.12
4.12	2.38	4.82	2.59	5.52	2.78	6.22	2.96	6.92	3.12
4.13	2.38	4.83	2.60	5.53	2.79	6.23	2.96	6.93	3.12
4.14	2.39	4.84	2.60	5.54	2.79	6.24	2.96	6.94	3.12
4.15	2.39	4.85	2.60	5.55	2.79	6.25	2.97	6.95	3.13
4.16	2.39	4.86	2.61	5.56	2.79	6.26	2.97	6.96	3.13
4.17	2.40	4.87	2.61	5.57	2.80	6.27	2.97	6.97	3.13
4.18	2.40	4.88	2.61	5.58	2.80	6.28	2.97	6.98	3.13
4.19	2.40	4.89	2.61	5.59	2.80	6.29	2.97	6.99	3.14
4.20	2.41	4.90	2.62	5.60	2.80	6.30	2.98	7.00	3.14



**TABLE 1** *Continued*

Y	V	Y	V	Y	V	Y	V	Y	V
7.01	3.14	7.71	3.29	8.41	3.43	9.11	3.56	9.81	3.69
7.02	3.14	7.72	3.29	8.42	3.43	9.12	3.56	9.82	3.69
7.03	3.14	7.73	3.29	8.43	3.43	9.13	3.57	9.83	3.69
7.04	3.15	7.74	3.30	8.44	3.44	9.14	3.57	9.84	3.69
7.05	3.15	7.75	3.30	8.45	3.44	9.15	3.57	9.85	3.70
7.06	3.15	7.76	3.30	8.46	3.44	9.16	3.57	9.86	3.70
7.07	3.15	7.77	3.30	8.47	3.44	9.17	3.57	9.87	3.70
7.08	3.16	7.78	3.30	8.48	3.44	9.18	3.58	9.88	3.70
7.09	3.16	7.79	3.31	8.49	3.45	9.19	3.58	9.89	3.70
7.10	3.16	7.80	3.31	8.50	3.45	9.20	3.58	9.90	3.70
7.11	3.16	7.81	3.31	8.51	3.45	9.21	3.58	9.91	3.71
7.12	3.16	7.82	3.31	8.52	3.45	9.22	3.58	9.92	3.71
7.13	3.17	7.83	3.31	8.53	3.45	9.23	3.59	9.93	3.71
7.14	3.17	7.84	3.32	8.54	3.46	9.24	3.59	9.94	3.71
7.15	3.17	7.85	3.32	8.55	3.46	9.25	3.59	9.95	3.71
7.16	3.17	7.86	3.32	8.56	3.46	9.26	3.59	9.96	3.71
7.17	3.18	7.87	3.32	8.57	3.46	9.27	3.59	9.97	3.72
7.18	3.18	7.88	3.32	8.58	3.46	9.28	3.59	9.98	3.72
7.19	3.18	7.89	3.33	8.59	3.47	9.29	3.60	9.99	3.72
7.20	3.18	7.90	3.33	8.60	3.47	9.30	3.60	10.00	3.72
7.21	3.18	7.91	3.33	8.61	3.47	9.31	3.60	10.01	3.72
7.22	3.19	7.92	3.33	8.62	3.47	9.32	3.60	10.02	3.72
7.23	3.19	7.93	3.34	8.63	3.47	9.33	3.60	10.03	3.73
7.24	3.19	7.94	3.34	8.64	3.48	9.34	3.60	10.04	3.73
7.25	3.19	7.95	3.34	8.65	3.48	9.35	3.61	10.05	3.73
7.26	3.19	7.96	3.34	8.66	3.48	9.36	3.61	10.06	3.73
7.27	3.20	7.97	3.34	8.67	3.48	9.37	3.61	10.07	3.73
7.28	3.20	7.98	3.35	8.68	3.48	9.38	3.61	10.08	3.73
7.29	3.20	7.99	3.35	8.69	3.48	9.39	3.61	10.09	3.74
7.30	3.20	8.00	3.35	8.70	3.49	9.40	3.62	10.10	3.74
7.31	3.21	8.01	3.35	8.71	3.49	9.41	3.62	10.11	3.74
7.32	3.21	8.02	3.35	8.72	3.49	9.42	3.62	10.12	3.74
7.33	3.21	8.03	3.36	8.73	3.49	9.43	3.62	10.13	3.74
7.34	3.21	8.04	3.36	8.74	3.49	9.44	3.62	10.14	3.74
7.35	3.21	8.05	3.36	8.75	3.50	9.45	3.62	10.15	3.75
7.36	3.22	8.06	3.36	8.76	3.50	9.46	3.63	10.16	3.75
7.37	3.22	8.07	3.36	8.77	3.50	9.47	3.63	10.17	3.75
7.38	3.22	8.08	3.37	8.78	3.50	9.48	3.63	10.18	3.75
7.39	3.22	8.09	3.37	8.79	3.50	9.49	3.63	10.19	3.75
7.40	3.22	8.10	3.37	8.80	3.51	9.50	3.63	10.20	3.76
7.41	3.23	8.11	3.37	8.81	3.51	9.51	3.64	10.21	3.76
7.42	3.23	8.12	3.37	8.82	3.51	9.52	3.64	10.22	3.76
7.43	3.23	8.13	3.38	8.83	3.51	9.53	3.64	10.23	3.76
7.44	3.23	8.14	3.38	8.84	3.51	9.54	3.64	10.24	3.76
7.45	3.24	8.15	3.38	8.85	3.51	9.55	3.64	10.25	3.76
7.46	3.24	8.16	3.38	8.86	3.52	9.56	3.64	10.26	3.77
7.47	3.24	8.17	3.38	8.87	3.52	9.57	3.65	10.27	3.77
7.48	3.24	8.18	3.39	8.88	3.52	9.58	3.65	10.28	3.77
7.49	3.24	8.19	3.39	8.89	3.52	8.59	3.65	10.29	3.77
7.50	3.25	8.20	3.39	8.90	3.52	9.60	3.65	10.30	3.77
7.51	3.25	8.21	3.39	8.91	3.53	9.61	3.65	10.31	3.77
7.52	3.25	8.22	3.39	8.92	3.53	9.62	3.65	10.32	3.78
7.53	3.25	8.23	3.40	8.93	3.53	9.63	3.66	10.33	3.78
7.54	3.25	8.24	3.40	8.94	3.53	9.64	3.66	10.34	3.78
7.55	3.26	8.25	3.40	8.95	3.53	9.65	3.66	10.35	3.78
7.56	3.26	8.26	3.40	8.96	3.54	9.66	3.66	10.36	3.78
7.57	3.26	8.27	3.40	8.97	3.54	9.67	3.66	10.37	3.78
7.58	3.26	8.28	3.41	8.98	3.54	9.68	3.67	10.38	3.79
7.59	3.26	8.29	3.41	8.99	3.54	9.69	3.67	10.39	3.79
7.60	3.27	8.30	3.41	9.00	3.54	9.70	3.67	10.40	3.79
7.61	3.27	8.31	3.41	9.01	3.54	9.71	3.67	10.41	3.79
7.62	3.27	8.32	3.41	9.02	3.55	9.72	3.67	10.42	3.79
7.63	3.27	8.33	3.41	9.03	3.55	9.73	3.67	10.43	3.79
7.64	3.28	8.34	3.42	9.04	3.55	9.74	3.68	10.44	3.80
7.65	3.28	8.35	3.42	9.05	3.55	9.75	3.68	10.45	3.80
7.66	3.28	8.36	3.42	9.06	3.55	9.76	3.68	10.46	3.80
7.67	3.28	8.37	3.42	9.07	3.56	9.77	3.68	10.47	3.80
7.68	3.28	8.38	3.42	9.08	3.56	9.78	3.68	10.48	3.80
7.69	3.29	8.39	3.43	9.09	3.56	9.79	3.68	10.49	3.80
7.70	3.29	8.40	3.43	9.10	3.56	9.80	3.69	10.50	3.81

**TABLE 1** *Continued*

Y	V	Y	V	Y	V	Y	V	Y	V
10.51	3.81	11.21	3.92	11.91	4.03	12.61	4.14	13.31	4.24
10.52	3.81	11.22	3.92	11.92	4.03	12.62	4.14	13.32	4.24
10.53	3.81	11.23	3.92	11.93	4.03	12.63	4.14	13.33	4.24
10.54	3.81	11.24	3.93	11.94	4.04	12.64	4.14	13.34	4.25
10.55	3.81	11.25	3.93	11.95	4.04	12.65	4.14	13.35	4.25
10.56	3.82	11.26	3.93	11.96	4.04	12.66	4.15	13.36	4.25
10.57	3.82	11.27	3.93	11.97	4.04	12.67	4.15	13.37	4.25
10.58	3.82	11.28	3.93	11.98	4.04	12.68	4.15	13.38	4.25
10.59	3.82	11.29	3.93	11.99	4.04	12.69	4.15	13.39	4.25
10.60	3.82	11.30	3.94	12.00	4.05	12.70	4.15	13.40	4.25
10.61	3.82	11.31	3.94	12.01	4.05	12.71	4.15	13.41	4.26
10.62	3.83	11.32	3.94	12.02	4.05	12.72	4.15	13.42	4.26
10.63	3.83	11.33	3.94	12.03	4.05	12.73	4.16	13.43	4.26
10.64	3.83	11.34	3.94	12.04	4.05	12.74	4.16	13.44	4.26
10.65	3.83	11.35	3.94	12.05	4.05	12.75	4.16	13.45	4.26
10.66	3.83	11.36	3.95	12.06	4.05	12.76	4.16	13.46	4.26
10.67	3.83	11.37	3.95	12.07	4.06	12.77	4.16	13.47	4.26
10.68	3.84	11.38	3.95	12.08	4.06	12.78	4.16	13.48	4.27
10.69	3.84	11.39	3.95	12.09	4.06	12.79	4.16	13.49	4.27
10.70	3.84	11.40	3.95	12.10	4.06	12.80	4.17	13.50	4.27
10.71	3.84	11.41	3.95	12.11	4.06	12.81	4.17	13.51	4.27
10.72	3.84	11.42	3.95	12.12	4.06	12.82	4.17	13.52	4.27
10.73	3.84	11.43	3.96	12.13	4.07	12.83	4.17	13.53	4.27
10.74	3.85	11.44	3.96	12.14	4.07	12.84	4.17	13.54	4.27
10.75	3.85	11.45	3.96	12.15	4.07	12.85	4.17	13.55	4.28
10.76	3.85	11.46	3.96	12.16	4.07	12.86	4.18	13.56	4.28
10.77	3.85	11.47	3.96	12.17	4.07	12.87	4.18	13.57	4.28
10.78	3.85	11.48	3.96	12.18	4.07	12.88	4.18	13.58	4.28
10.79	3.85	11.49	3.97	12.19	4.07	12.89	4.18	13.59	4.28
10.80	3.85	11.50	3.97	12.20	4.08	12.90	4.18	13.60	4.28
10.81	3.86	11.51	3.97	12.21	4.08	12.91	4.18	13.61	4.28
10.82	3.86	11.52	3.97	12.22	4.08	12.92	4.18	13.62	4.29
10.83	3.86	11.53	3.97	12.23	4.08	12.93	4.19	13.63	4.29
10.84	3.86	11.54	3.97	12.24	4.08	12.94	4.19	13.64	4.29
10.85	3.86	11.55	3.98	12.25	4.08	12.95	4.19	13.65	4.29
10.86	3.86	11.56	3.98	12.26	4.09	12.96	4.19	13.66	4.29
10.87	3.87	11.57	3.98	12.27	4.09	12.97	4.19	13.67	4.29
10.88	3.87	11.58	3.98	12.28	4.09	12.98	4.19	13.68	4.29
10.89	3.87	11.59	3.98	12.29	4.09	12.99	4.19	13.69	4.30
10.90	3.87	11.60	3.98	12.30	4.09	13.00	4.20	13.70	4.30
10.91	3.87	11.61	3.98	12.31	4.09	13.01	4.20	13.71	4.30
10.92	3.87	11.62	3.99	12.32	4.09	13.02	4.20	13.72	4.30
10.93	3.88	11.63	3.99	12.33	4.10	13.03	4.20	13.73	4.30
10.94	3.88	11.64	3.99	12.34	4.10	13.04	4.20	13.74	4.30
10.95	3.88	11.65	3.99	12.35	4.10	13.05	4.20	13.75	4.30
10.96	3.88	11.66	3.99	12.36	4.10	13.06	4.20	13.76	4.31
10.97	3.88	11.67	3.99	12.37	4.10	13.07	4.21	13.77	4.31
10.98	3.88	11.68	4.00	12.38	4.10	13.08	4.21	13.78	4.31
10.99	3.89	11.69	4.00	12.39	4.10	13.09	4.21	13.79	4.31
11.00	3.89	11.70	4.00	12.40	4.11	13.10	4.21	13.80	4.31
11.01	3.89	11.71	4.00	12.41	4.11	13.11	4.21	13.81	4.31
11.02	3.89	11.72	4.00	12.42	4.11	13.12	4.21	13.82	4.31
11.03	3.89	11.73	4.00	12.43	4.11	13.13	4.21	13.83	4.32
11.04	3.89	11.74	4.00	12.44	4.11	13.14	4.22	13.84	4.32
11.05	3.90	11.75	4.01	12.45	4.11	13.15	4.22	13.85	4.32
11.06	3.90	11.76	4.01	12.46	4.12	13.16	4.22	13.86	4.32
11.07	3.90	11.77	4.01	12.47	4.12	13.17	4.22	13.87	4.32
11.08	3.90	11.78	4.01	12.48	4.12	13.18	4.22	13.88	4.32
11.09	3.90	11.79	4.01	12.49	4.12	13.19	4.22	13.89	4.32
11.10	3.90	11.80	4.01	12.50	4.12	13.20	4.22	13.90	4.32
11.11	3.91	11.81	4.02	12.51	4.12	13.21	4.23	13.91	4.33
11.12	3.91	11.82	4.02	12.52	4.12	13.22	4.23	13.92	4.33
11.13	3.91	11.83	4.02	12.53	4.13	13.23	4.23	13.93	4.33
11.14	3.91	11.84	4.02	12.54	4.13	13.24	4.23	13.94	4.33
11.15	3.91	11.85	4.02	12.55	4.13	13.25	4.23	13.95	4.33
11.16	3.91	11.86	4.02	12.56	4.13	13.26	4.23	13.96	4.33
11.17	3.91	11.87	4.03	12.57	4.13	13.27	4.24	13.97	4.33
11.18	3.92	11.88	4.03	12.58	4.13	13.28	4.24	13.98	4.34
11.19	3.92	11.89	4.03	12.59	4.13	13.29	4.24	13.99	4.34
11.20	3.92	11.90	4.03	12.60	4.14	13.30	4.24	14.00	4.34

**TABLE 1** *Continued*

Y	V	Y	V	Y	V	Y	V	Y	V
14.01	4.34	14.71	4.44	15.41	4.53	16.11	4.62	16.81	4.71
14.02	4.34	14.72	4.44	15.42	4.53	16.12	4.62	16.82	4.71
14.03	4.34	14.73	4.44	15.43	4.53	16.13	4.62	16.83	4.71
14.04	4.34	14.74	4.44	15.44	4.53	16.14	4.62	16.84	4.71
14.05	4.35	14.75	4.44	15.45	4.53	16.15	4.62	16.85	4.71
14.06	4.35	14.76	4.44	15.46	4.54	16.16	4.63	16.86	4.71
14.07	4.35	14.77	4.44	15.47	4.54	16.17	4.63	16.87	4.72
14.08	4.35	14.78	4.45	15.48	4.54	16.18	4.63	16.88	4.72
14.09	4.35	14.79	4.45	15.49	4.54	16.19	4.63	16.89	4.72
14.10	4.35	14.80	4.45	15.50	4.54	16.20	4.63	16.90	4.72
14.11	4.35	14.81	4.45	15.51	4.54	16.21	4.63	16.91	4.72
14.12	4.36	14.82	4.45	15.52	4.54	16.22	4.63	16.92	4.72
14.13	4.36	14.83	4.45	15.53	4.54	16.23	4.64	16.93	4.72
14.14	4.36	14.84	4.45	15.54	4.55	16.24	4.64	16.94	4.72
14.15	4.36	14.85	4.46	15.55	4.55	16.25	4.64	16.95	4.73
14.16	4.36	14.86	4.46	15.56	4.55	16.26	4.64	16.96	4.73
14.17	4.36	14.87	4.46	15.57	4.55	16.27	4.64	16.97	4.73
14.18	4.36	14.88	4.46	15.58	4.55	16.28	4.64	16.98	4.73
14.19	4.37	14.89	4.46	15.59	4.55	16.29	4.64	16.99	4.73
14.20	4.37	14.90	4.46	15.60	4.55	16.30	4.64	17.00	4.73
14.21	4.37	14.91	4.46	15.61	4.56	16.31	4.65	17.01	4.73
14.22	4.37	14.92	4.46	15.62	4.56	16.32	4.65	17.02	4.73
14.23	4.37	14.93	4.47	15.63	4.56	16.33	4.65	17.03	4.74
14.24	4.37	14.94	4.47	15.64	4.56	16.34	4.65	17.04	4.74
14.25	4.37	14.95	4.47	15.65	4.56	16.35	4.65	17.05	4.74
14.26	4.37	14.96	4.47	15.66	4.56	16.36	4.65	17.06	4.74
14.27	4.38	14.97	4.47	15.67	4.56	16.37	4.65	17.07	4.74
14.28	4.38	14.98	4.47	15.68	4.56	16.38	4.65	17.08	4.74
14.29	4.38	14.99	4.47	15.69	4.57	16.39	4.66	17.09	4.74
14.30	4.38	15.00	4.48	15.70	4.57	16.40	4.66	17.10	4.74
14.31	4.38	15.01	4.48	15.71	4.57	16.41	4.66	17.11	4.75
14.32	4.38	15.02	4.48	15.72	4.57	16.42	4.66	17.12	4.75
14.33	4.38	15.03	4.48	15.73	4.57	16.43	4.66	17.13	4.75
14.34	4.39	15.04	4.48	15.74	4.57	16.44	4.66	17.14	4.75
14.35	4.39	15.05	4.48	15.75	4.57	16.45	4.66	17.15	4.75
14.36	4.39	15.06	4.48	15.76	4.57	16.46	4.66	17.16	4.75
14.37	4.39	15.07	4.48	15.77	4.58	16.47	4.67	17.17	4.75
14.38	4.39	15.08	4.49	15.78	4.58	16.48	4.67	17.18	4.75
14.39	4.39	15.09	4.49	15.79	4.58	16.49	4.67	17.19	4.76
14.40	4.39	15.10	4.49	15.80	4.58	16.50	4.67	17.20	4.76
14.41	4.40	15.11	4.49	15.81	4.58	16.51	4.67	17.21	4.76
14.42	4.40	15.12	4.49	15.82	4.58	16.52	4.67	17.22	4.76
14.43	4.40	15.13	4.49	15.83	4.58	16.53	4.67	17.23	4.76
14.44	4.40	15.14	4.49	15.84	4.59	16.54	4.67	17.24	4.76
14.45	4.40	15.15	4.50	15.85	4.59	16.55	4.68	17.25	4.76
14.46	4.40	15.16	4.50	15.86	4.59	16.56	4.68	17.26	4.76
14.47	4.40	15.17	4.50	15.87	4.59	16.57	4.68	17.27	4.77
14.48	4.41	15.18	4.50	15.88	4.59	16.58	4.68	17.28	4.77
14.49	4.41	15.19	4.50	15.89	4.59	16.59	4.68	17.29	4.77
14.50	4.41	15.20	4.50	15.90	4.59	16.60	4.68	17.30	4.77
14.51	4.41	15.21	4.50	15.91	4.59	16.61	4.68	17.31	4.77
14.52	4.41	15.22	4.50	15.92	4.60	16.62	4.68	17.32	4.77
14.53	4.41	15.23	4.51	15.93	4.60	16.63	4.69	17.33	4.77
14.54	4.41	15.24	4.51	15.94	4.60	16.64	4.69	17.34	4.77
14.55	4.41	15.25	4.51	15.95	4.60	16.65	4.69	17.35	4.78
14.56	4.42	15.26	4.51	15.96	4.60	16.66	4.69	17.36	4.78
14.57	4.42	15.27	4.51	15.97	4.60	16.67	4.69	17.37	4.78
14.58	4.42	15.28	4.51	15.98	4.60	16.68	4.69	17.38	4.78
14.59	4.42	15.29	4.51	15.99	4.60	16.69	4.69	17.39	4.78
14.60	4.42	15.30	4.51	16.00	4.61	16.70	4.69	17.40	4.78
14.61	4.42	15.31	4.52	16.01	4.61	16.71	4.70	17.41	4.78
14.62	4.42	15.32	4.52	16.02	4.61	16.72	4.70	17.42	4.78
14.63	4.43	15.33	4.52	16.03	4.61	16.73	4.70	17.43	4.79
14.64	4.43	15.34	4.52	16.04	4.61	16.74	4.70	17.44	4.79
14.65	4.43	15.35	4.52	16.05	4.61	16.75	4.70	17.45	4.79
14.66	4.43	15.36	4.52	16.06	4.61	16.76	4.70	17.46	4.79
14.67	4.43	15.37	4.52	16.07	4.61	16.77	4.70	17.47	4.79
14.68	4.43	15.38	4.53	16.08	4.62	16.78	4.70	17.48	4.79
14.69	4.43	15.39	4.53	16.09	4.62	16.79	4.71	17.49	4.79
14.70	4.43	15.40	4.53	16.10	4.62	16.80	4.71	17.50	4.79

**TABLE 1** *Continued*

Y	V	Y	V	Y	V	Y	V	Y	V
17.51	4.79	18.21	4.88	18.91	4.96	19.61	5.04	20.31	5.12
17.52	4.80	18.22	4.88	18.92	4.96	19.62	5.04	20.32	5.12
17.53	4.80	18.23	4.88	18.93	4.96	19.63	5.04	20.33	5.12
17.54	4.80	18.24	4.88	18.94	4.96	19.64	5.04	20.34	5.12
17.55	4.80	18.25	4.88	18.95	4.97	19.65	5.05	20.35	5.12
17.56	4.80	18.26	4.89	18.96	4.97	19.66	5.05	20.36	5.12
17.57	4.80	18.27	4.89	18.97	4.97	19.67	5.05	20.37	5.13
17.58	4.80	18.28	4.89	18.98	4.97	19.68	5.05	20.38	5.13
17.59	4.80	18.29	4.89	18.99	4.97	19.69	5.05	20.39	5.13
17.60	4.81	18.30	4.89	19.00	4.97	19.70	5.05	20.40	5.13
17.61	4.81	18.31	4.89	19.01	4.97	19.71	5.05	20.41	5.13
17.62	4.81	18.32	4.89	19.02	4.97	19.72	5.05	20.42	5.13
17.63	4.81	18.33	4.89	19.03	4.98	19.73	5.05	20.43	5.13
17.64	4.81	18.34	4.89	19.04	4.98	19.74	5.06	20.44	5.13
17.65	4.81	18.35	4.90	19.05	4.98	19.75	5.06	20.45	5.13
17.66	4.81	18.36	4.90	19.06	4.98	19.76	5.06	20.46	5.14
17.67	4.81	18.37	4.90	19.07	4.98	19.77	5.06	20.47	5.14
17.68	4.82	18.38	4.90	19.08	4.98	19.78	5.06	20.48	5.14
17.69	4.82	18.39	4.90	19.09	4.98	19.79	5.06	20.49	5.14
17.70	4.82	18.40	4.90	19.10	4.98	19.80	5.06	20.50	5.14
17.71	4.82	18.41	4.90	19.11	4.98	19.81	5.06	20.51	5.14
17.72	4.82	18.42	4.90	19.12	4.99	19.82	5.07	20.52	5.14
17.73	4.82	18.43	4.91	19.13	4.99	19.83	5.07	20.53	5.14
17.74	4.82	18.44	4.91	19.14	4.99	19.84	5.07	20.54	5.14
17.75	4.82	18.45	4.91	19.15	4.99	19.85	5.07	20.55	5.15
17.76	4.83	18.46	4.91	19.16	4.99	19.86	5.07	20.56	5.15
17.77	4.83	18.47	4.91	19.17	4.99	19.87	5.07	20.57	5.15
17.78	4.83	18.48	4.91	19.18	4.99	19.88	5.07	20.58	5.15
17.79	4.83	18.49	4.91	19.19	4.99	19.89	5.07	20.59	5.15
17.80	4.83	18.50	4.91	19.20	4.99	19.90	5.07	20.60	5.15
17.81	4.83	18.51	4.91	19.21	5.00	19.91	5.08	20.61	5.15
17.82	4.83	18.52	4.92	19.22	5.00	19.92	5.08	20.62	5.15
17.83	4.83	18.53	4.92	19.23	5.00	19.93	5.08	20.63	5.15
17.84	4.83	18.54	4.92	19.24	5.00	19.94	5.08	20.64	5.16
17.85	4.84	18.55	4.92	19.25	5.00	19.95	5.08	20.65	5.16
17.86	4.84	18.56	4.92	19.26	5.00	19.96	5.08	20.66	5.16
17.87	4.84	18.57	4.92	19.27	5.00	19.97	5.08	20.67	5.16
17.88	4.84	18.58	4.92	19.28	5.00	19.98	5.08	20.68	5.16
17.89	4.84	18.59	4.92	19.29	5.01	19.99	5.08	20.69	5.16
17.90	4.84	18.60	4.93	19.30	5.01	20.00	5.09	20.70	5.16
17.91	4.84	18.61	4.93	19.31	5.01	20.01	5.09	20.71	5.16
17.92	4.84	18.62	4.93	19.32	5.01	20.02	5.09	20.72	5.16
17.93	4.85	18.63	4.93	19.33	5.01	20.03	5.09	20.73	5.17
17.94	4.85	18.64	4.93	19.34	5.01	20.04	5.09	20.74	5.17
17.95	4.85	18.65	4.93	19.35	5.01	20.05	5.09	20.75	5.17
17.96	4.85	18.66	4.93	19.36	5.01	20.06	5.09	20.76	5.17
17.97	4.85	18.67	4.93	19.37	5.01	20.07	5.09	20.77	5.17
17.98	4.85	18.68	4.93	19.38	5.02	20.08	5.09	20.78	5.17
17.99	4.85	18.69	4.94	19.39	5.02	20.09	5.10	20.79	5.17
18.00	4.85	18.70	4.94	19.40	5.02	20.10	5.10	20.80	5.17
18.01	4.86	18.71	4.94	19.41	5.02	20.11	5.10	20.81	5.17
18.02	4.86	18.72	4.94	19.42	5.02	20.12	5.10	20.82	5.18
18.03	4.86	18.73	4.94	19.43	5.02	20.13	5.10	20.83	5.18
18.04	4.86	18.74	4.94	19.44	5.02	20.14	5.10	20.84	5.18
18.05	4.86	18.75	4.94	19.45	5.02	20.15	5.10	20.85	5.18
18.06	4.86	18.76	4.94	19.46	5.02	20.16	5.10	20.86	5.18
18.07	4.86	18.77	4.95	19.47	5.03	20.17	5.10	20.87	5.18
18.08	4.86	18.78	4.95	19.48	5.03	20.18	5.11	20.88	5.18
18.09	4.86	18.79	4.95	19.49	5.03	20.19	5.11	20.89	5.18
18.10	4.87	18.80	4.95	19.50	5.03	20.20	5.11	20.90	5.18
18.11	4.87	18.81	4.95	19.51	5.03	20.21	5.11	20.91	5.18
18.12	4.87	18.82	4.95	19.52	5.03	20.22	5.11	20.92	5.19
18.13	4.87	18.83	4.95	19.53	5.03	20.23	5.11	20.93	5.19
18.14	4.87	18.84	4.95	19.54	5.03	20.24	5.11	20.94	5.19
18.15	4.87	18.85	4.95	19.55	5.03	20.25	5.11	20.95	5.19
18.16	4.87	18.86	4.96	19.56	5.04	20.26	5.11	20.96	5.19
18.17	4.87	18.87	4.96	19.57	5.04	20.27	5.11	20.97	5.19
18.18	4.88	18.88	4.96	19.58	5.04	20.28	5.12	20.98	5.19
18.19	4.88	18.89	4.96	19.59	5.04	20.29	5.12	20.99	5.19
18.20	4.88	18.90	4.96	19.60	5.04	20.30	5.12	21.00	5.19