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## Worm gears — Worm profiles and gear mesh geometry

*Engrenages à vis cylindriques — Géométrie des profils de vis et  
des engrenements*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: [Foreword - Supplementary information](#)

The committee responsible for this document is ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This second edition cancels and replaces the first edition (ISO/TR 10828:1997), which has been technically revised.

This edition includes the formulation for the geometrical dimensions of the worm and worm wheel, and that for the determination of gear mesh geometry (path of contact, zone and lines of contact) with the details to determine the non-dimensional parameters used to apply load capacity calculations (radius of curvature, sliding velocities).

## Introduction

Thread forms of the worms of worm gear pairs are commonly related to the following machining processes:

- the type of machining process (turning, milling, grinding);
- the shapes of edges or surfaces of the cutting tools used;
- the tool position relative to an axial plane of the worm;
- where relevant, the diameters of disc type tools (grinding wheel diameter).

This Technical Report introduces all the aspects concerning the gear mesh geometry to define conjugate worm wheel, path of contact, lines of contact and other associated geometrical characteristics.

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# Worm gears — Worm profiles and gear mesh geometry

## 1 Scope

In this Technical Report, thread profiles of the five most common types of worms at the date of publication are described and formulae of their axial profiles are given.

The five worm types covered in this technical report are designated by the letters A, C, I, K and N.

The formulae to calculate the path of contact, the conjugate profile of the worm wheel, the lines of contact, the radius of curvature and the velocities at points of contact are provided. At the end the application of those formulae to calculate parameters used in load capacity calculations are provided.

## 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1122-2, *Vocabulary — Worm gears*

ISO 701, *International gear notations — Symbols for geometrical data*

ISO/TR 14521, *Worm gears — Load capacity of worm gears*

## 3 Symbols and abbreviated terms

For the purposes of this document, Tables 1 to 3 give the symbols the indices and the description.

**Table 1 — Symbols for worm gears from Clause 4 of this document**

Symbols	Description	Units	Figures	Formula number
$A$	distance from the worm axis to virtual point of the cutter (see ref.[1])	mm	Fig. A.4	
$a$	centre distance	mm	Fig. 3	41/42
$a_0$	refers to the worm/tool centre distance (length of the common perpendicular to the worm/tool axes)	mm	Fig. 18	54
$a_1$ to $a_4$	coefficient for A, I and N profile			
$b_1$	facewidth of worm	mm		24
$b_{2H}$	effective wheel facewidth	mm	Fig. 4	39
$b_{2R}$	wheel rim width	mm	Fig. 4	
$c_1, c_2$	tip clearance	mm		46/47

Symbols	Description	Units	Figures	Formula number
$d_{a1}$	worm tip diameter	mm		14
$d_{a2}$	worm wheel throat diameter	mm		35
$d_{b1}$	base diameter of involute helicoid (for I profile)	mm		22
$d_{e2}$	worm wheel outside diameter	mm		36
$d_{f1}$	worm root diameter	mm		15
$d_{f2}$	worm wheel root diameter	mm		34
$d_{m1}$	worm reference diameter	mm	Fig. 1/3	10
$d_{m2}$	worm wheel reference diameter	mm	Fig 2/3	25
$d_{w1}$	worm pitch diameter	mm		43
$d_{w2}$	worm wheel pitch diameter	mm	Fig 5	44
$e_{mx1}$	worm reference tooth space width in axial section	mm	Fig. 1	17
$e_{n1}$	worm normal tooth space width in normal section	mm		19
$e_{m2}$	worm wheel tooth space width in mid-plane section	mm		28
$h_1$	worm tooth depth	mm		11
$h_2$	worm wheel tooth depth	mm		32
$h_{am1}$	worm tooth reference addendum in axial section	mm	Fig. 3	12
$h_{am2}$	worm wheel tooth reference addendum in mid-plane section	mm	Fig. 3	30
$h_{am1}^*$	worm tooth reference addendum coefficient in axial section	-		
$h_{am2}^*$	worm wheel tooth reference addendum coefficient in mid-plane section	-		
$h_{e2}$	worm wheel tooth external addendum	mm		33
$h_{fm1}$	worm tooth reference dedendum in axial section	mm		13
$h_{fm2}$	worm wheel tooth reference dedendum in mid-plane section	mm		31
$h_{fm1}^*$	worm tooth reference dedendum coefficient in axial section	-		
$h_{fm2}^*$	worm wheel tooth reference dedendum coefficient in mid-plane section	-		
$j_x$	axial backlash	mm		
$m_n$	normal module	mm		9
$m_{x1}$	axial module	mm		2/G.1
$p_{bn1}$	normal pitch on base cylinder	mm		23
$p_{n1}$	normal pitch	mm		8
$p_{t2}$	transverse pitch	mm		26
$p_{x1}$	axial pitch	mm	Fig. 1	1
$p_{z1}$	lead (of worm)	mm		3
$p_{zu1}$	unit lead (lead of worm per radian)	mm/rad		4
$q_1$	diameter quotient	mm		5
$r_{g2}$	worm wheel throat form radius	mm		40



Symbols	Description	Units	Figures	Formula number
$r_{b1}$	base radius for involute profile	mm	Fig. A.4 and A.5	
$r'_{b1}$	base radius of a notional base circle	mm	Fig. A.4 and A.5	
$r_t$	radius at cusp	mm	Fig. 29	
$s_{m2}$	tooth thickness at the reference diameter of the worm wheel	mm	Fig. 2	27
$s_K$	rim thickness	mm	Fig. 12	
$s_{mx1}$	worm thread thickness in axial section	mm	Fig. 1	16
$s_{mx1}^*$	worm thread thickness in axial section coefficient	-		
$s_{n1}$	normal worm thread thickness in normal section	mm		18
$u$	gear ratio			45
$x_2$	worm wheel profile shift coefficient	-		29
$z_1$	number of threads in worm	-		
$z_2$	number of teeth in worm wheel	-		
$\alpha_{0n}$	tool normal pressure angle	°		
$\alpha_{0t}$	tool transverse pressure angle for A and I profiles	°	Fig. 7	
$\alpha_n$	normal pressure angle	°		20
$\beta_{m1}$	reference helix angle of worm	°		7
$\gamma_{m1}$	reference lead angle of worm	°		6
$\gamma_{b1}$	base lead angle of worm thread (for I profile)	°	Fig. A.1	21
$\gamma'_{b1}$	base lead angle of the notional base helix	°	Fig. A.4 and A.5	
$\rho_{Gm}$	Radius of curvature of grinding wheel (C profile)			

In calculation, when a radius is derived, the symbol  $d$  for diameter shall be replaced by  $r$  for radius.

**Table 2 — Subscripts for worm gears**

Symbols	Description
0	cutting tool
1	worm
2	Wheel
G	grinding wheel

**Table 3 — Coordinate of remarkable points**

Symbols	Description
$x_G(y_G), y_G, \alpha_G(y_G)$	Coordinates of a point on the tool flank when the origin is at the point of intersection of the tool axis and the tool median plane, with the x-axis as the tool spindle axis and the

	abscissa on the trace of the median plane;
$x_x(y_r), y_x(y_r), \alpha_x(y_r)$	Coordinates of axial profile and axial pressure angle for A, I, N worm profiles
$x_x(y_G), y_x(y_G), \alpha_x(y_G)$	Coordinates of axial profile and axial pressure angle for K and C worm profiles
$x_D(y_p, D), y_D(y_p, D), \alpha_D(y_p, D)$	Coordinates of worm profile and pressure angle of worm profile in an offset plane
$x'_D(y_p, D), y_D(y_p, D), \alpha_D(y_p, D)$	Coordinates of worm profile and pressure angle of worm profile in an offset plane with origin on pitch axis
$x_{ID}(y_p, D), y_{ID}(y_p, D)$	Coordinates of path of contact in an offset plane with origin on pitch axis
$xR_D(y_p, D), yR_D(y_p, D)$	Coordinates of conjugate worm wheel profile of the worm in an offset plane with origin on worm wheel axis
$xT_D(r_{t2D}, D), yT_D(r_{t2D}, D)$	Coordinates of trochoid profile of the worm wheel profile in an offset plane with origin on worm wheel axis
$x_D(ycusp, D), y_D(ycusp, D)$	Coordinates of cusp point in an offset plane with origin on pitch axis
$C_{eq1D}(y_p, D)$	Curvature for the worm at a point in an offset plane
$C_{eq2D}(y_p, D)$	Curvature for the worm wheel at a point in an offset plane
$R_{eqD}(y_p, D)$	Equivalent radius of curvature in an offset plane
$r_{e2D}(D)$	outside radius of the worm wheel in the offset plane D
$r_{t2D}$	root radius of the worm wheel in the offset plane D
$\overline{M}_1(y_p, D)$	Coordinate of a point of contact for the worm (Eq 118)
$\overline{M}_2(y_p, D)$	Coordinate of a point of contact for the worm wheel (Eq 119)
$\overline{TN1}_{cont}(y_p, D)$	Tangent unit vector to a line of contact (Eq 128)
$\overline{NormalNxy}(y_p, D)$	Normal unit vector to the lines of contact (Eq 114)
$\overline{NORMAL}(y_p, D)$	Normal unit vector to the lines of contact (Eq 116)
$\overline{Re}q(y_p, D)$	Radius of curvature along the line the contact (Eq 135)
$\overline{V}_1(y_p, D)$	Velocity of a point of the thread of the worm (Eq 138)
$\overline{V}_2(y_p, D)$	Velocity of a point of the tooth flank of the worm (Eq 140)
$V_{cDn}(y_p, D)$	Velocity at the contact point along the path of contact (Eq 150)
$V_{SUMn}(y_p, D)$	Sum of velocities at the point of contact (Eq 153) for method B in ISO/TR 14521

## 4 Formulae for calculation of dimensions

### 4.1 Parameters for a cylindrical worm

#### 4.1.1 Axial pitch

$$p_{x1} = \pi \cdot m_{x1} \quad (1)$$

#### 4.1.2 Axial module

$$m_{x1} = \frac{p_{x1}}{\pi} \quad (2)$$

#### 4.1.3 Lead

$$p_{z1} = z_1 \cdot p_{x1} \quad (3)$$

#### 4.1.4 Unit lead

$$p_{zu1} = \frac{p_{z1}}{2 \cdot \pi} \quad (4)$$

#### 4.1.5 Diameter quotient

$$q_1 = \frac{d_{m1}}{m_{x1}} \quad (5)$$

#### 4.1.6 Reference lead angle

$$\tan \gamma_{m1} = \frac{m_{x1} \cdot z_1}{d_{m1}} = \frac{z_1}{q_1} \quad (6)$$

#### 4.1.7 Reference helix angle

$$\beta_{m1} = 90^\circ - \gamma_{m1} \quad (7)$$

#### 4.1.8 Normal pitch on reference cylinder

$$p_{n1} = p_{x1} \cdot \cos \gamma_{m1} \quad (8)$$

#### 4.1.9 Normal module

$$m_n = m_{x1} \cdot \cos \gamma_{m1} \quad (9)$$

#### 4.1.10 Reference diameter

$$d_{m1} = q_1 \cdot m_{x1} \quad (10)$$

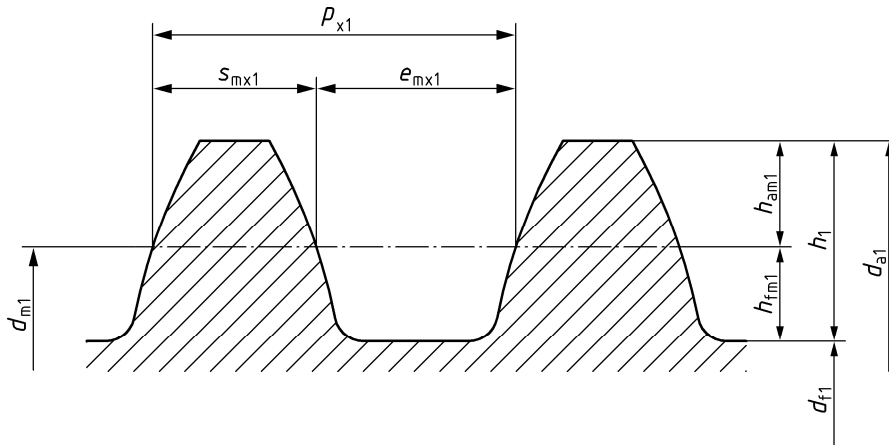


Figure 1 — Axial parameters for worm

#### 4.1.11 Reference tooth depth

$$h_1 = h_{am1} + h_{fm1} = \frac{1}{2} \cdot (d_{a1} - d_{f1}) \quad (11)$$

#### 4.1.12 Reference addendum

$$h_{am1} = h_{am1}^* \cdot m_{x1} = \frac{1}{2} \cdot (d_{a1} - d_{m1}) \quad (12)$$

where  $h_{am1}^*$  is the addendum coefficient; normally  $h_{am1}^* = 1$

#### 4.1.13 Reference dedendum

$$h_{fm1} = h_{fm1}^* \cdot m_{x1} = \frac{1}{2} \cdot (d_{m1} - d_{f1}) \quad (13)$$

where  $h_{fm1}^*$  = dedendum coefficient; generally  $1,1 < h_{fm1}^* < 1,3$ , the recommended value is 1,2

#### 4.1.14 Tip diameter

$$d_{a1} = d_{m1} + 2 \cdot h_{am1} \quad (14)$$

#### 4.1.15 Root diameter

$$d_{f1} = d_{m1} - 2 \cdot h_{fm1} \quad (15)$$

#### 4.1.16 Thread thickness coefficient $s_{mx1}^*$

A recommended value is  $s_{mx1}^* = 0,5$

In general practice, this coefficient is very often less than 0,5 when there is a wish to increase the worm wheel thread thickness to extend durability against wear of worm wheel.

See Figure 1.

**4.1.17 Reference thread thickness in the axial section**

$$s_{mx1} = s_{mx1}^* \cdot p_{x1} \quad (16)$$

**4.1.18 Reference space width in the axial section**

$$e_{mx1} = p_{x1} - s_{mx1} \quad (17)$$

**4.1.19 Normal thread thickness**

$$s_{n1} = s_{mx1} \cdot \cos \gamma_{m1} \quad (18)$$

**4.1.20 Normal space width**

$$e_{n1} = e_{mx1} \cdot \cos \gamma_{m1} \quad (19)$$

**4.1.21 Profile flank form**

It is specified by a letter:

- A is the envelope of straight line in the axial section;
- N is the envelope of straight line in the normal section of the space width;
- I is the involute helicoid (the envelope of straight line in a plane tangent to the base cylinder);
- K is a milled helicoid by double cone form;
- C is a milled helicoid by circular convex form.

**4.1.22 Normal pressure angle**

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For type-A

$$\tan \alpha_n = \tan \alpha_{0t} \cdot \cos \gamma_{m1} \quad (20)$$

For other type

$$\alpha_n = \alpha_{0n}$$

where  $\alpha_{0n}$  is defined in 6.3, 6.4 and 6.5 for I and N and in 6.6 and 6.7 for K and C.

**4.1.23 Base lead angle for I profile**

$$\cos \gamma_{b1} = \cos \gamma_{m1} \cdot \cos \alpha_{0n} \quad (21)$$

**4.1.24 Base diameter for I profile**

$$d_{b1} = d_{m1} \cdot \frac{\tan \gamma_{m1}}{\tan \gamma_{b1}} = \frac{m_{x1} \cdot z_1}{\tan \gamma_{b1}} \quad (22)$$

NOTE For I profile if the root diameter is less than the base diameter attention should be taken in order that the diameter of start of active profile (SAP) is greater than  $d_{b1}$ .

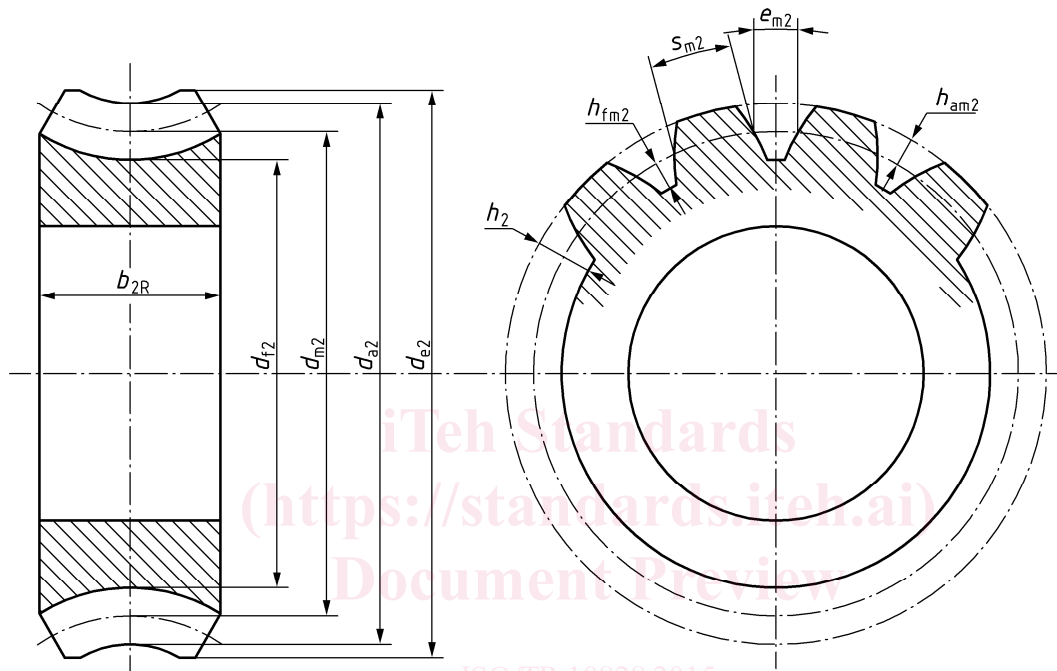
#### 4.1.25 Normal pitch on base cylinder

$$p_{bn1} = p_{x1} \cdot \cos \gamma_{b1} \quad (23)$$

#### 4.1.26 Worm face width

$$b_1 \geq \sqrt{(d_{e2})^2 - (2 \cdot a - d_{a1})^2} \quad (24)$$

### 4.2 Parameters for a worm wheel



NOTE On Figure 2 the profile shift coefficient  $x_2$  is negative.

Figure 2 — Parameters for worm wheel

#### 4.2.1 Reference diameter

$$d_{m2} = d_{w2} + 2 \cdot x_2 \cdot m_{x1} \quad \text{or} \quad d_{m2} = 2 \cdot a - d_{m1} \quad (25)$$

#### 4.2.2 Transverse pitch

$$p_{t2} = p_{x1} \quad (26)$$

#### 4.2.3 Transverse tooth thickness at reference diameter

This value can be calculated only for a worm wheel without profile shift as follows:

$$s_{m2} = e_{mx1} - j_x \quad (27)$$

where  $j_x$  = axial backlash.

See Figure 2.