Designation: D4015 – 07

# Standard Test Methods for Modulus and Damping of Soils by Resonant-Column Method[1]

This standard is issued under the fixed designation D4015; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

## 1. Scope*

1.1 These test methods cover the determination of shear modulus, shear damping, rod modulus (commonly referred to as Young's modulus), and rod damping for solid cylindrical specimens of soil in the undisturbed and remolded conditions by vibration using the resonant column. The vibration of the specimen may be superposed on a controlled ambient state of stress in the specimen. The vibration apparatus and specimen may be enclosed in a triaxial chamber and subjected to an all-around pressure and axial load. In addition, the specimen may be subjected to other controlled conditions (for example, pore-water pressure, degree of saturation, temperature). These test methods of modulus and damping determination are considered nondestructive when the strain amplitudes of vibration are less than $10^{-4}$ rad ($10^{-4}$ in./in.), and many measurements may be made on the same specimen and with various states of ambient stress.

1.2 These test methods cover only the determination of the modulus and damping, the necessary vibration, and specimen preparation procedures related to the vibration, etc., and do not cover the application, measurement, or control of the ambient stress. The latter procedures may be covered by, but are not limited to, Test Methods D2166 or D2850.

1.3 All recorded and calculated values shall conform to the guide for significant digits and rounding established in Practice D6026.

1.3.1 The procedures used to specify how data are collected/recorded and calculated in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that should generally be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analysis methods for engineering design.

1.3.2 Measurements made to more significant digits or better sensitivity than specified in this standard shall not be regarded a nonconformance with this standard.

1.4 The values stated in SI units are to be regarded as the standard. Reporting test results in units other than SI shall be regarded as conformance with these test methods.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

## 2. Referenced Documents

2.1 *ASTM Standards:*[2]

D653 Terminology Relating to Soil, Rock, and Contained Fluids
D2166 Test Method for Unconfined Compressive Strength of Cohesive Soil
D2216 Test Methods for Laboratory Determination of Water (Moisture) Content of Soil and Rock by Mass
D2850 Test Method for Unconsolidated-Undrained Triaxial Compression Test on Cohesive Soils
D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction
D4767 Test Method for Consolidated Undrained Triaxial Compression Test for Cohesive Soils
D6026 Practice for Using Significant Digits in Geotechnical Data

## 3. Terminology

3.1 *Definitions*—For definitions of other terms used in this Test Method, see Terminology D653.

3.2 *Definitions of Terms Specific to This Standard:*

[1] These test methods are under the jurisdiction of ASTM Committee D18 on Soil and Rock and are the direct responsibility of Subcommittee D18.09 on Cyclic and Dynamic Properties of Soils.

Current edition approved Sept. 1, 2007. Published October 2007. Originally approved in 1981. Last previous edition approved in 2000 as D4015 – 92 (2000). DOI: 10.1520/D4015-07.

[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

*A Summary of Changes section appears at the end of this standard

3.2.1 *ambient stress*—stresses applied to the specimen, during the test, that do not result from the vibration strains. These test methods do not cover the application and measurement of ambient stresses; however, the ambient stress at the time of measurement of the system resonant frequency and system damping shall be measured and recorded in accordance with the Report section of these test methods.

3.2.2 *apparatus model and constants*—the rigidity and mass distribution of the resonant column shall be as required in the following section in order for the resonant-column system to be accurately represented by the model shown in Fig. 1. The apparatus constants are the mass of the passive-end platen, $M_P$, including the mass of all attachments rigidly connected to it; the rotational inertia of the passive-end platen, $J_P$, including the rotational inertia of all attachments rigidly connected to it; similar mass, $M_A$, and rotational inertia, $J_A$, for the active-end platen and all attachments rigidly connected to it, such as portions of the vibration excitation device; the spring and damping constants for both longitudinal and torsional springs and dashpots ($K_{SL}$, $K_{ST}$, $ADC_L$, $ADC_T$); the apparatus resonant frequencies for longitudinal vibration, $f_{oL}$, and torsional vibration, $f_{oT}$; the force/current constant, *FCF*, relating applied vibratory force to the current applied to the longitudinal excitation device; the torque/current constant, *TCF*, relating applied vibratory torque to the current applied to the torsional excitation device; and the motion transducer calibration factors ($LCF_A$, $RCF_A$, $LCF_P$, $RCF_P$) relating the transducer outputs to active- and passive-end longitudinal and rotational motion.

3.2.3 *moduli and damping capacities*—Young's modulus (herein called rod modulus), *E*, is determined from longitudinal vibration, and the shear modulus, *G*, is determined from torsional vibration. The rod and shear moduli shall be defined as the elastic moduli of a uniform, linearly viscoelastic (Voigt model) specimen of the same mass density and dimensions as the soil specimen necessary to produce a resonant column having the measured system resonant frequency and response due to a given vibratory force or torque input. The stress-strain relation for a steady-state vibration in the resonant column is a hysteresis loop. These moduli will correspond to the slope of a line through the end points of the hysteresis loop. The section on calculations provides for computation of rod and shear moduli from the measured system longitudinal and torsional resonant frequencies. The energy dissipated by the system is a measure of the damping of the soil. Damping will be described by the rod damping ratio, $D_L$, and the shear damping ratio, $D_T$, which are analogous to the critical viscous damping ratio, $c/c_r$, for a single-degree-of-freedom system. The damping ratios shall be defined by:

$$D_L = 0.5(\eta\omega/E) \tag{1}$$

where:

$\eta$ = viscous coefficient for rod motion, N·s/m$^2$,
$\omega$ = circular resonant frequency, rad/s, and
$E$ = rod modulus, Pa.

and by:

$$D_T = 0.5(\mu\omega/G) \tag{2}$$

where:

$\mu$ = viscous coefficient for torsional motion, N-s/m$^2$, and
$G$ = shear modulus, Pa.

3.2.3.1 Values of damping determined in this way will correspond to the area of the stress-strain hysteresis loop divided by 4π times the elastic strain energy stored in the specimen at maximum strain. Methods for determining damping ratio are prescribed later. In viscoelastic theory, it is common to use complex moduli to express both modulus and damping. The complex rod modulus is given by:

$$E^* = E(1+2iD_L) \tag{3}$$

and the complex shear modulus is given by:

$$G^* = G(1+2iD_T) \tag{4}$$

where $i=\sqrt{-1}$.

3.2.4 *resonant-column system*—a system consisting of a cylindrical specimen or column of soil that has platens attached to each end as shown in Fig. 1. A sinusoidal vibration excitation device is attached to the active-end platen. The other end is the passive-end platen. It may be rigidly fixed (the criterion for establishing fixity is given later) or its mass and rotational inertia must be known. The vibration excitation device may incorporate springs and dashpots connected to the active-end platen, where the spring constants and viscous damping coefficients are known. Vibration excitation may be longitudinal or torsional. A given apparatus may have the capability of applying one or the other, or both. The mass and rotational inertia of the active-end platen and portions of the vibration excitation device moving with it must be known. Transducers are used to measure the vibration amplitudes for each type of motion at the active end and also at the passive end if it is not rigidly fixed. The frequency of excitation will be adjusted to produce resonance of the system, composed of the specimen and its attached platens and vibration excitation device.

3.2.5 *specimen strain*—for longitudinal motion, the strain, ε, is the average axial strain in the entire specimen. For torsional motion, the strain, γ, is the average shear strain in the specimen. In the case of torsion, shear strain in each cross

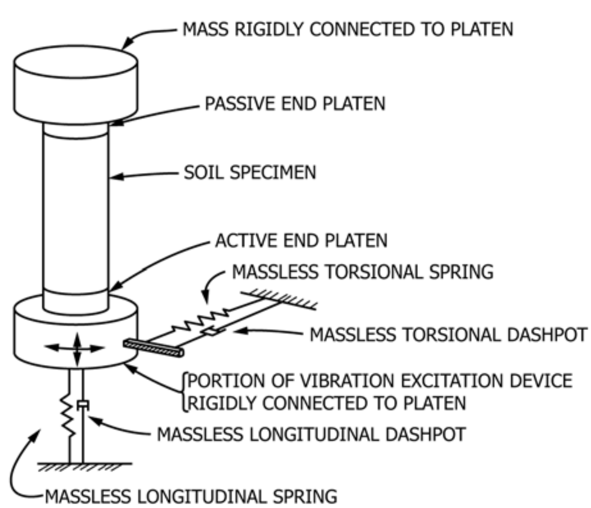


**FIG. 1 Resonant-column Schematic**

section varies from zero along the axis of rotation to a maximum at the perimeter of the specimen, and the average shear strain for each cross section occurs at a radius equal to 80 percent the radius of the specimen. Methods for calculating specimen strain are given later in the calculations section.

3.2.6 *system resonant frequency*—the definition of system resonance depends on both apparatus and specimen characteristics. For the case where the passive-end platen is fixed, motion at the active end is used to establish resonance, which is defined as the lowest frequency for which the sinusoidal excitation force (or moment) is in phase with the velocity of the active-end platen. For the case where the passive-end platen mass (or passive end platen rotational inertia) is greater than 100 times the corresponding value of the specimen and is not rigidly fixed, resonance is the lowest frequency for which the sinusoidal excitation force (or moment) is 180° out of phase with the velocity of the active-end platen. Otherwise, motion at the passive end is used to establish resonance, which is the second lowest frequency for which the sinusoidal excitation force (or moment) is in phase with the velocity of the passive-end platen. (The lowest frequency for this condition is not used because it does not produce significant strains in the specimen.) In general, the system resonant frequency for torsional excitation will be different from the system resonant frequency for longitudinal excitation.

## 4. Summary of Test Method

4.1 In the resonant column test, a cylindrical soil specimen is subjected to an imposed ambient stress condition. Once equilibrium at the imposed stress condition is achieved, torsional or longitudinal, or both, sinusoidal vibrations are applied to the soil specimen and the specimen motions (strains) resulting from the imposed vibrations are measured. The frequency of excitation is varied until resonance is achieved as described in Section 3.2.6. Given the geometry, mass and system parameters, the shear and longitudinal moduli and material (hysteretic) damping may be determined at a measured strain value. The amplitude of vibration is typically varied to measure the variation of moduli and damping as a function of strain. Since the test is usually conducted at strain levels between 0.00001 and 0.5% strain leaving the specimen relatively intact, the test is often conducted at several different sets of ambient stress conditions to measure the variation of moduli and damping with ambient stress.

NOTE 1—The quality of the results produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

## 5. Significance and Use

5.1 The modulus and damping of a given soil, as measured by the resonant-column technique herein described, depend upon the strain amplitude of vibration, the ambient state of effective stress, and the void ratio of the soil, temperature, time, etc. Since the application and control of the ambient stresses and the void ratio are not prescribed in these methods, the applicability of the results to field conditions will depend on the degree to which the application and control of the ambient stresses and the void ratio, as well as other parameters such as soil structure, duplicate field conditions. The techniques used to simulate field conditions depend on many factors and it is up to the engineer to decide on which techniques apply to a given situation and soil type.

## 6. Apparatus

6.1 *General*—The complete test apparatus includes the platens for holding the specimen in the pressure cell, the vibration excitation device, transducers for measuring the response, the control and readout instrumentation, and auxiliary equipment for specimen preparation.

6.2 *Specimen Platens*—Both the active-end and passive-end platens shall be constructed of noncorrosive material having a modulus at least ten times the modulus of the material to be tested. Each platen shall have a circular cross section and a plane surface of contact with the specimen, except that the plane surface of contact may be roughened to provide for more efficient coupling with the ends of the specimen. The diameter of platens shall be equal to or greater than the diameter of the specimen. The construction of the platens shall be such that their stiffness is at least ten times the stiffness of the specimen. The active-end platen may have a portion of the excitation device, transducers, springs, and dashpots connected to it. The transducers and moving portions of the excitation device must be connected to the platen in such a fashion that they are to be considered part of the platen and have the same motion as the platen for the full range of frequencies to be encountered when testing soils. The theoretical model used for the resonant-column system represents the active-end platen, with all attachments, as a rigid mass that is attached to the specimen; this mass may also have massless springs and dashpots attached to it as shown in Fig. 1. If springs are used, the excitation device and active-end platen (without the specimen in place) form a two degree-of-freedom system (one-degree-of-freedom system for devices designed for only longitudinal or only torsional motion) having undamped natural frequencies for longitudinal motion, $f_{oL}$, and torsional motion, $f_{o\,T}$. The device shall be constructed such that these modes of vibration are uncoupled. The passive-end platen may have a mass and transducers rigidly attached to it or it may be rigidly fixed. The passive-end platen may be assumed to be rigidly fixed when the inertia of it and the mass(es) attached to it and the stiffness of the support of the mass(es) provide a dimensionless frequency factor within 1 % of the dimensionless frequency factor for the passive-end inertia ratio equal to infinity. (Use Fig. 2 and the calculations section to get the dimensionless frequency factor.)

6.3 *Vibration Excitation Device*—This shall be an electromagnetic device capable of applying a sinusoidal longitudinal vibration or torsional vibration or both to the active-end platen to which it is rigidly coupled. The frequency of excitation shall be adjustable and controlled to within 0.5 %. The excitation device shall have a means of measuring the current applied to the drive coils that has at least a 5 % accuracy. The voltage

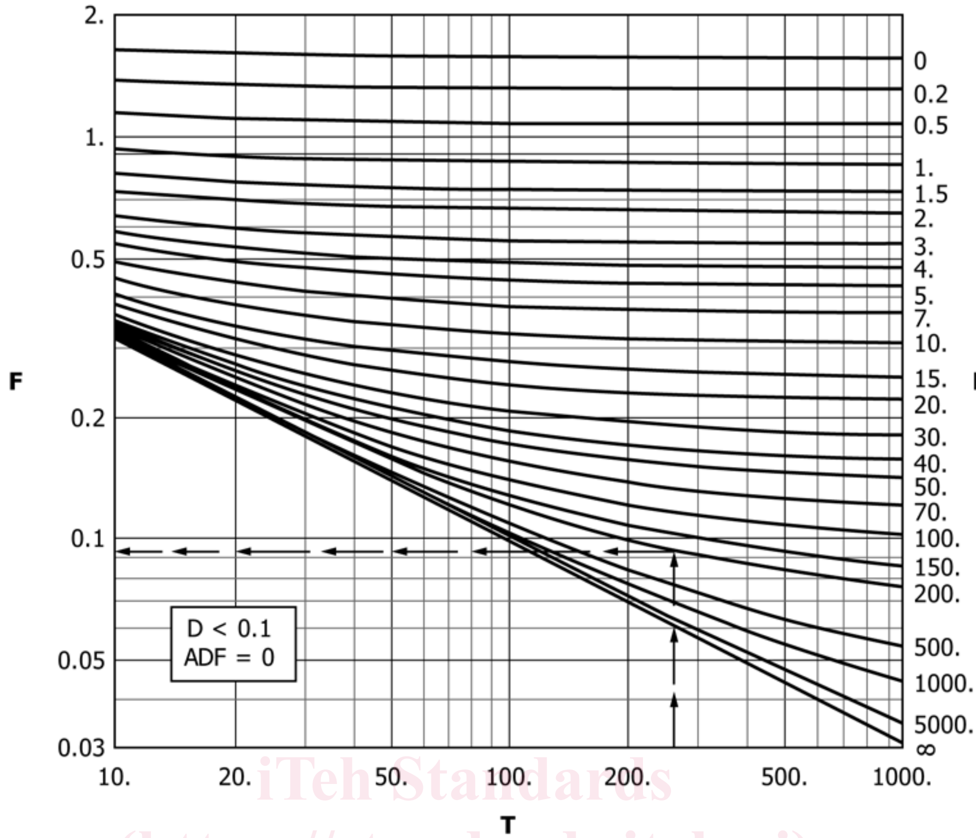


FIG. 2 (a) Dimensionless Frequency Factors[3]



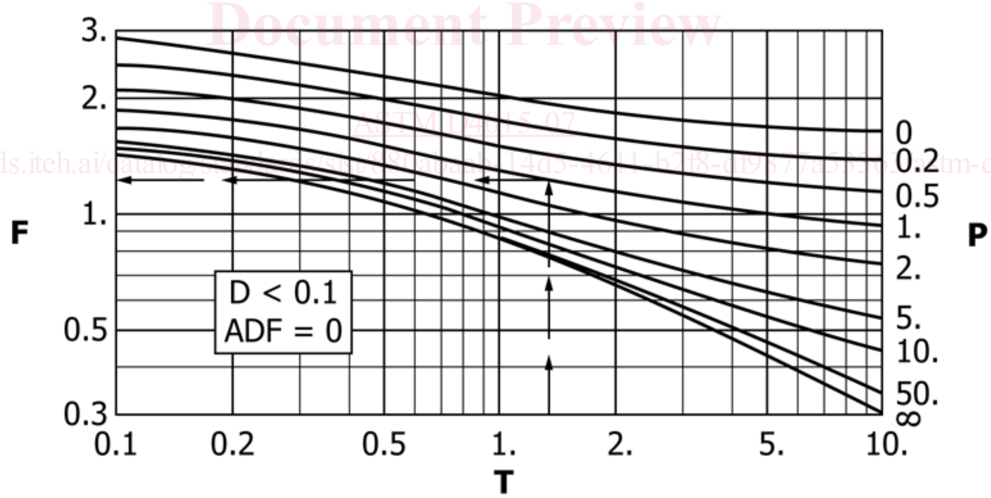


FIG. 2 (b) Dimensionless Frequency Factors[3] *(continued)*

drop across a fixed, temperature-and-frequency-stable power resistor in series with the drive coils may be used for this purpose. The force/current and torque/current factors for the vibration excitation devices must be linear within 5 % for the entire range of operating frequencies anticipated when testing soils.

6.4 *Sine Wave Generator*—The sine wave generator is an electric instrument capable of producing a sinusoidal current with a means of adjusting the frequency over the entire range of operating frequencies anticipated. This instrument shall provide sufficient power to produce the required vibration amplitude, or its output may be electronically amplified to provide sufficient power. The total distortion of the signal applied to the excitation device shall be less than 3 %.

6.5 *Vibration-Measuring Devices and Readout Instruments*—The vibration-measuring devices shall be acceleration, velocity, or displacement transducers that can be attached to and become a part of the active- and passive-end platens. On each platen, one transducer shall be mounted to produce a calibrated electrical output that is proportional to the longitudinal acceleration, velocity, or displacement of that platen (not required for torsion-only apparatus). The other transducer(s) shall be mounted to produce a calibrated electrical output that is proportional to the rotational acceleration, velocity, or displacement (not required for longitudinal-only apparatus). The readout instrument and transducers shall have a sensitivity such that a displacement of $2.5 \times 10^{-6}$ m ($10^{-4}$ in.) and a rotation of $10^{-5}$ rad can be measured with 10 % accuracy for the entire range of frequency anticipated. It is also necessary to have an *x-y*-time oscilloscope available for observing signal waveforms and for establishing the system resonant frequency. This oscilloscope must have at least one amplifier (vertical or horizontal) with sufficient gain to observe the motion transducer output over the entire range of output voltages and frequencies anticipated. For measurement of damping by the free-vibration method, and for calibration of the apparatus damping, the readout instrument shall be capable of recording the decay of free vibration. Either a strip-chart recorder with appropriate response time and chart speed or an oscilloscope and camera or a digital oscilloscope may be used for this purpose.

6.6 *Support for Vibration Excitation Device*—For the special case where the passive end of the specimen is rigidly fixed and the vibration excitation device and active-end platen are placed on top of the specimen, it may be necessary to support all or a portion of the weight of the platen and excitation device to prevent excessive axial stress or compressive failure of the specimen. This support may be provided by a spring, counterbalance weights, or pneumatic cylinder as long as the supporting system does not prevent axial movement of the active-end platen and as long as it does not alter the vibration characteristics of the excitation device.

6.7 *Temporary Platen Support Device*—The temporary support may be any clamping device that can be used to support one or both end platens during attachment of vibration excitation device to prevent specimen disturbance during apparatus assembly. This device is to be removed prior to the application of vibration.

6.8 *Specimen Dimension-Measuring Devices*—Dimension-measuring devices are needed to measure portions of the apparatus during calibration and specimen diameter and length. All dimensions measurements should be accurate to 0.1 %. Any suitable device may be used to make these measurements except that the device(s) used to measure the length and diameter of the specimen must not deform or otherwise affect the specimen. Specially designed perimeter tapes that measure circumference but read out in diameter are preferred for measuring specimen diameters.

6.9 *Balances*—Devices for determining the mass of the soil specimens as well as portions of the device during calibration. All measurements of mass should be accurate to 0.1 %.

6.10 *Specimen Preparation and Triaxial Equipment*—These methods cover specimen preparation and procedures related to the vibration of the specimen and do not cover the application and control of ambient stresses. Any or all of the apparatus described in Test Methods D2166, D2850, or D4767 may be used for specimen preparation and application of ambient stresses. Additional apparatus may be used for these purposes as required.

6.11 *Miscellaneous Apparatus*—The miscellaneous apparatus consists of specimen trimming and carving tools, a membrane expander, remolding apparatus, moisture content cans, and data sheets as required.

## 7. Test Specimen

7.1 *General*—These methods cover only the special specimen preparation procedures related to the vibration and resonant-column technique. Since the resonant-column test may be conducted in conjunction with controlled ambient stresses, the provisions for preparation of specimens in Test Methods D2166, D2850, or D4767 may be applicable or may be used as a guide in connection with other methods of application and control of ambient stresses.

7.2 *Specimen Size*—Specimens shall be of uniform circular cross section with ends perpendicular to the axis of the specimen. Specimens shall have a minimum diameter of 33 mm (1.3 in.). The largest particle contained within the test specimen shall be smaller than one tenth of the specimen diameter except that, for specimens having a diameter of 70 mm (2.8 in.) or larger, the largest particle size shall be smaller than one sixth of the specimen diameter. If, after completion of a test, it is found that larger particles than permitted are present, indicate this information in the report of test data under "Remarks." The length-to-diameter ratio shall be not less than 2 nor more than 7 except that, when an ambient axial stress greater than the ambient lateral stress is applied to the specimen, the ratio of length to diameter shall be between 2 and 3. Measure the length at the third points along the perimeter and average the values. Measure two diameters at each of three elevations and average the values. Determine the mass of the test specimen. For determination of moisture content (Test Method D2216), secure a representative specimen of the cuttings from intact specimens, or of the extra soil for remolded specimens, placing the specimen immediately in a covered container.



7.3 *End Coupling for Torsion*—For torsional motion, complete coupling of the ends of the specimen to the specimen cap and base must be assured. Complete coupling for torsion may be assumed if the mobilized coefficient of friction between the end platens and the specimen is less than 0.2 for all shear strain amplitudes. The coefficient of fraction is approximately given by:

$$\text{Coefficient of friction} = \gamma G / \sigma'_a \tag{5}$$

where:

$\gamma$ = shear strain amplitude (see calculations section),
$G$ = shear modulus (see calculations section), and
$\sigma'_a$ = effective axial stress.

7.3.1 When this criterion is not met, other provisions such as the use of adhesives must be made in order to assure complete coupling. In such cases, the effectiveness of the coupling provisions shall be evaluated by testing two specimens of the same material but of different length. The lengths of these specimens shall differ by at least a factor of 1.5. The provisions for end coupling may be considered satisfactory if the values of the shear modulus for these two specimens of different length do not differ by more than 10 %.

## 8. Calibration

8.1 *Motion Transducers*—Motion transducers shall be calibrated with each other and with an independent method to ensure calibration accuracy within 5 %. Linear motion transducers whose axes are located fixed distances from the axis of rotation may be used to measure rotational motion if the cross-axis sensitivities of the transducers are less than 5 %. For this case the distance between the axis of rotation and the transducer axes shall be known to within 5 %. The calibration factors for longitudinal motion shall be expressed in terms of peak-meters/peak-volt. The calibration factors for rotational motion shall be expressed in terms of peak-radians/peak-volt. This means that for velocity and acceleration transducers the vibration frequency shall be included as a term in the calibration factor. For velocity transducers, the displacement calibration factors are given by:

$$LCF \text{ or } RCF = \text{velocity calibration factor}/(2\pi f) \quad (6)$$

where $f$ = frequency, Hz.
For acceleration transducers, the displacement calibration factors are given by:

$$LCF \text{ or } RCF = \text{acceleration calibration factor}/(2\pi f)^2 \quad (7)$$

8.1.1 Thus, for velocity and acceleration transducers, the displacement calibration factors will not be constants but will vary with measured frequency, $f$. Calibration factors for longitudinal motion are given by the symbol $LCF$ with a subscript $A$ or $P$ denoting whether the transducer is located on the active-end platen or passive-end platen. Likewise, the calibration factors for rotational motion will be given by the symbol $RCF$ and will have subscripts $A$ or $P$ depending on their location.

8.2 *Passive-End Platen Mass and Rotational Inertia*—The mass and rotational inertia of the passive-end platen shall be determined with all transducers and other rigid attachments securely in place. The mass, $M_P$, is determined by use of a balance. The rotational inertia of the concentric solid cylindrical components of the passive-end platen and its attachments is given by:

$$(J_P)_1 = \frac{1}{8}\sum_{i=1}^{n} M_i d_i^{\,2} \quad (8)$$

where:

$M_i$ = mass of $i$th solid cylindrical component,
$d_i$ = diameter of $i$th solid cylindrical component, and
$n$ = number of solid cylindrical components.

Transducers and other masses attached to this platen can be accounted for by:

$$(J_P)_2 = \sum_{i=1}^{n} M_i r_i^{\,2} \quad (9)$$

where:

$M_i$ = mass of ith component,
$r_i$ = distance from the platen axis to center of mass for ith component, and
$n$ = number of components attached to passive-end platen and not covered in determination of $(J_P)_1$.

The total rotational inertia for the passive end is given by:

$$J_P = (J_P)_1 + (J_P)_2 \quad (10)$$

8.3 *Active-End Platen Mass and Rotational Inertia*—The mass, $M_A$, and rotational inertia, $J_A$, of the active-end platen shall be determined with all transducers and rigid attachments, including attached portions of the vibration excitation device, securely in place. The equations just given may be used to obtain the mass and rotational inertia. For rotational inertia, if all components do not have simple geometry, an alternative procedure that involves a metal calibration rod of known torsional stiffness may be used. One end of the rod shall be rigidly fixed and the other end shall be rigidly fastened to the active-end platen. Since it may be very difficult to fasten the calibration rod to the platen without adding rotational inertia, it is recommended that the calibration rod be permanently fastened by welding, etc., to an auxiliary platen. If the auxiliary platen is not identical to the one to be used in testing, the difference between its rotational inertia and that of the platen for soil testing must be taken into account by use of aforementioned equations. (For example, suppose that the value of the active-end rotational inertia with the calibration rod was $J1$ and the rotational inertia of the calibration rod platen was $J2$. If the rotational inertia of the platen for testing soil is $J\,3$, then the value of $J_A$ would be given by $J_A = J1 - J2 + J3$.) The torsional stiffness of the calibration rod should be chosen such that the system resonant frequency with the calibration rod in place is near the middle of the range of system resonant frequencies anticipated for soil testing. Several calibration rods may be necessary to account for different specimen sizes. With the calibration rod in place, determine the low-amplitude system resonant frequency for torsional vibration, $(f_{rod})_T$. The rotational inertia of the active end platen system is calculated from:

$$J_A = \frac{(K_{rod})_T}{(2\pi)^2\left[(f_{rod})_T^{\,2} - f_{oT}^{\,2}\right]} \quad (11)$$

where:

$(K_{rod})_T$ = torsional stiffness of calibration rod, $(I_pG)/L$,
$I_p$ = polar moment of inertia of calibration rod, $(\pi d^4)/32$,
$d$ = calibration rod diameter,
$G$ = shear modulus for calibration rod material, and
$f_{oT}$ = apparatus torsional resonant frequency as described in the following subsection.

8.3.1 The foregoing equations assume that the rotational inertia of the calibration rods is much less than the corresponding values for the active-end platen system. A second alternative procedure is to couple the metal calibration rod to the platens in place of the specimen and then use the procedures of the Calculations section to backfigure the active end inertias from the known moduli of the rod.

8.4 *Apparatus Resonant Frequencies, Spring Constants, and Damping Constants*—Apparatus resonant frequencies and spring constants are defined only for those apparatus that have springs attached to the active-end platen system. To determine the resonant frequencies, set up the apparatus complete with active-end platen and O-rings but no specimen. Vibrate at low amplitude and adjust the frequency of vibration until the input force is in phase with the velocity ofthe active-end platen system. For longitudinal vibration, this apparatus resonant frequency is $f_{oL}$ and for torsional vibration it is $f_{oT}$. The longitudinal and torsional apparatus spring constants ( $K_{SL}$, $K_{ST}$) may be calculated from:

$$K_{SL} = (2\pi f_{oL})^2 M_A \quad (12)$$

$$K_{ST} = (2\pi f_{oT})^2 J_A$$

where $M_A$ and $J_A$ are defined in the previous subsection.

8.4.1 To measure the damping constants for the apparatus, attach the same masses as used for the determination of apparatus resonant frequencies. For apparatus without springs attached to the active-end platen, insert the calibration rod described in the previous subsection. With the apparatus vibrating at the resonant frequency, cut off the power to the excitation device and record the decay curve for the vibration of the apparatus. From the decay curve, compute the logarithmic decrement, δ, as follows:

$$\delta = (1/n)\ln(A_1/A_{n+1}) \quad (13)$$

where:

$A_1$ = amplitude of vibration for first cycle after power is cut off, and
$A_{n+1}$ = amplitude for (n + 1)th cycle.

The apparatus damping coefficient, $ADC_L$, from longitudinal vibration shall be given by:

$$ADC_{oL} = 2 f_{oL} M_A \delta_L \quad (14)$$

where:

$f_{oL}$ = longitudinal motion resonant frequency measured during apparatus damping determination,
$M_A$ = active-end platen mass from previous subsection, and
$\delta_L$ = logarithmic decrement for longitudinal motion.

For torsional motion, the apparatus damping coefficient, $ADC_{oT}$is given by:

$$ADC_{oT} = 2 f_{oT} J_A \delta_T \quad (15)$$

where:

$f_{oT}$ = torsional motion resonant frequency measured during apparatus damping determination,
$J_A$ = active-end rotational inertia from previous subsection, and
$\delta_T$ = logarithmic decrement for torsional motion.

8.5 *Force/Current and Torque/Current*—For apparatus without springs attached to the active-end platen, insert the calibration rod as described earlier. Determine the resonant frequency of this single-degree-of-freedom system consisting of the active-end platen and apparatus spring (or calibration rod) by use of the same procedure as described later in the procedures section. Then set the frequency to 0.707 times the resonant frequency and apply sufficient current to the vibration excitation device so that the vibration transducer output to the readout device has a signal of at least ten times the signal due to ambient vibrations and electrical noise when no power is applied to the excitation device. Read and record the output of both the vibration transducer and the current measuring instrument. Next, set the frequency to 1.414 times the system resonant frequency and obtain the vibration transducer and current instrument readings in a similar fashion to those at 0.707 times the resonant frequency. Calculate $C_1$ and $C_2$ from:

$$C_1 = 0.5(VTCF)(TO1)/CR1 \quad (16)$$

$$C_2 = (VTCF)(TO2)/CR2$$

where:

*VTCF* = active-end vibration transducer displacement calibration factor (LCF or RCF) (Note 2) depending on whether vibration is longitudinal or torsional,
*TO*1 = active-end transducer output at 0.707 times resonant frequency,
*CR*1 = current instrument reading at 0.707 times resonant frequency (Note 3),
*TO*2 = active-end transducer output at 1.414 times resonant frequency, and
*CR*2 = current instrument reading at 1.414 times resonant frequency (Note 3).

NOTE 2—*LCF* and *RCF* will be functions of frequency for velocity and acceleration measuring transducers (see 8.1).

NOTE 3—If a current-measuring instrument is used, the units will be amperes. Alternatively, voltage drop across a fixed resistance may also be measured and the units will then be volts.



$C_1$ and $C_2$ should agree within 10 %. By use of $C_1$ and $C_2$ from longitudinal vibration, the force/current calibration factor, *FCF*, is obtained from:

$$FCF = 0.5(C_1 + C_2)K \quad (17)$$



where $K$ = apparatus spring constant (or for apparatus without springs, the calibrating rod spring constant) for longitudinal motion.

By use of $C_1$ and $C_2$ from torsional vibration, the torque/current calibration factor, *TCF*, is obtained from:

$$TCF = 0.5(C_1 + C_2)K \quad (18)$$

where $K$ = apparatus spring constant (or for apparatus without springs, the calibrating rod spring constant) for torsional motion.

## 9. Procedure

9.1 *Test Setup*—The exact procedure to be followed during test setup will depend on the apparatus and electronic equipment used and on methods used for application, measurement, and control of the ambient stresses. However, the specimen shall be placed in the apparatus by procedures that will minimize the disturbance of the specimen. Particular care must be exercised when attaching the end platens to the specimen and when attaching the vibration excitation device to the platens. A temporary support as discussed earlier may be needed. For cases where ambient isotropic stresses are to be applied to a membrane-enclosed specimen, liquid- or air-confining media may be used for dry or partially saturated specimens. For tests where complete saturation is important, a liquid-confining medium should be used. Where the vibration excitation device is located within the pressure chamber, an air-liquid interface is acceptable as long as the liquid covers the entire membrane that encloses the specimen.

9.2 *Electronic Equipment*—Connect the vibration excitation device to the sine wave generator (with amplifier, if required). The power supplied to the vibration excitation device should be very low in order not to exceed the amplitude of vibration prescribed later. Connect the vibration transducers to the readout instruments for the type of motion (longitudinal or torsional) to be applied. Adjust the readout instruments according to the instruction manuals for these instruments.

9.3 *Measurement of Resonant Frequency*—The procedure for measuring system resonant frequency is the same for both longitudinal and torsional vibration except that the longitudinal motion transducer is used for longitudinal motion and the rotational motion transducer is used for torsional motion. If the passive end is fixed or if $P > 100$ (see the calculations section for definition of $P$), motion of the active-end platen is used to establish resonance. Otherwise, motion of the passive-end platen is used. With the power as low as practical, increase the frequency of excitation from a very low value (for example, 10 Hz) until the system resonant frequency is obtained. The phase relationship describing resonance can be established by observing the Lissajous figure formed on an *x-y* oscilloscope with the voltage proportional to the driving current applied to the horizontal amplifier and the output from the transducer applied to the vertical amplifier. If a velocity transducer is used for vibration measurement, the system resonant frequency occurs when the figure formed is a straight, sloping line. If a displacement or acceleration transducer is used, the frequency should be adjusted to produce an ellipse with axes vertical and horizontal. (Refer to Definitions section to establish which resonant frequency should be recorded.) It is recommended that the frequency be measured with a digital electronic frequency meter and be recorded to at least three significant figures. The system resonant frequency for longitudinal motion shall be designated $f_L$ and that for torsional motion shall be designated $f_T$.

9.4 *Measurement of Strain Amplitude*— The strain amplitude measurements shall be made only at the system resonant frequencies. Thus, for a given current applied to the excitation device, the vibration motion transducer outputs recorded at the system resonant frequency give sufficient information to calculate strain amplitude. To increase or decrease strain amplitude, the current to the vibration excitation device must be increased or decreased. After making a change in current applied to the vibration excitation device, the procedure of the previous subsection must be followed to establish the corresponding system resonant frequency before the transducer outputs can be used to establish the new strain amplitude value.

9.5 *Measurement of System Damping*—Associated with each strain amplitude and system resonant frequency is a value of damping. Two methods are available for measuring system damping: the steady-state vibration method and the amplitude decay method. Theoretically, both methods should give identical results. In practice, results of each method are usually close to each other. The steady-state method is easier and quicker. It is generally always used and the amplitude decay method is used for occasional spot-checking. The procedures for both methods are independent of whether longitudinal or torsional motion is under consideration. For the steady-state method, the active-end or the passive-end vibration transducer output (depending on which end is used to establish resonance) and the current applied to the vibration excitation device must be measured at each resonant frequency. The calculations are outlined in the following section. For the free-vibration method, with the system vibrating at the system resonant frequency, cut off the power to the vibration excitation device and record the output of the transducer used in establishing resonance as a function of time. This gives the decay curve for free vibration. The calculations for damping are also outlined in the following section.

## 10. Calculation

10.1 *General*—Calculations require the apparatus calibration factors and the physical dimensions and mass of the specimen. In addition, for each ambient stress condition, one data set is required for each vibration strain amplitude. A data set consists of the type of vibration (longitudinal or torsional), duration of vibration (this time can be used to calculate the number of vibration cycles), system resonant frequency, active- or passive-end transducer outputs (depending on which end is used to establish resonance), the reading associated with the current applied to the vibration excitation device, and the free-vibration amplitude decay curve (if the amplitude decay method of measuring damping is also going to be used). The calculations outlined in this section may all be made by computer. A program for making some of the calculations is provided in Appendix X1. Other programs may be used to make a portion or all of the calculations as long as they provide identical results.



10.2 *Soil Mass Density*—The soil mass density, ρ, is given by:

$$\rho = \frac{M}{V} \quad (19)$$

where:

$M$ = total mass of specimen, and
$V$ = volume of specimen.

10.3 *Specimen Rotational Inertia*—The specimen rotational inertia about the axis of rotation is given by:

$$J = \frac{Md^2}{8} \quad (20)$$

where $d$ = diameter of specimen.

10.4 *Active-End Inertia Factors:*

10.4.1 The active-end inertia factor for longitudinal motion, $T_L$, is calculated from:

$$T_L = \frac{M_A}{M}\left[1 - \left(f_{oL}/f_L\right)^2\right] \quad (21)$$

where:

$M_A$ = mass of active-end platen system as calculated earlier,

$f_{oL}$ = apparatus resonant frequency for longitudinal motion (for apparatus without springs attached to the active end platen, this term is zero), and
$f_L$ = system resonant frequency for longitudinal motion.

10.4.2 The active-end inertia factor for torsional motion, $T_T$, is given by:

$$T_T = \frac{J_A}{J}\left[1 - (f_{oT}/f_T)^2\right] \tag{22}$$

where:
$J_A$ = rotational inertia of active-end platen system as calculated earlier,
$J$ = specimen rotational inertia as calculated earlier,
$f_{oT}$ = apparatus resonant frequency for torsional motion (for apparatus without springs attached to the active-end platen, this factor is zero), and
$f_T$ = system resonant frequency for torsional motion.

10.5 *Passive-End Inertia Ratios:*

10.5.1 For longitudinal motion, the passive-end inertia ratio, $P_L$, is given by:

$$P_L = \frac{M_P}{M} \tag{23}$$

where $M_P$ = mass of passive-end platen system as described earlier.

10.5.2 For torsional motion, the passive-end inertia ratio, $P_T$, is given by:

$$P_T = J_p/J \tag{24}$$

where $J_P$ = rotational inertia of passive-end platen system as calculated earlier.

10.5.3 For the special case where the passive end of the specimen is rigidly fixed, $P_L$ and $P_T$ are equal to infinity.

10.6 *Apparatus Damping Factors:*

10.6.1 For longitudinal motion, the apparatus damping factor, $ADF_L$, is calculated from

$$ADF_L = ADC_{oL}/[2\pi f_L M] \tag{25}$$

where $ADC_{oL}$ = apparatus damping coefficient for longitudinal motion as described earlier.

10.6.2 For rotational motion, the apparatus damping factor, $ADF_{oT}$, is calculated from:

$$ADF_T = ADC_{0T}/[2\pi f_T J] \tag{26}$$

where $ADC_{oT}$ = apparatus damping coefficient for torsional motion as described earlier.

10.7 *Dimensionless Frequency Factor*—The dimensionless frequency factor, $F$, is used in calculating modulus. It is a function of system factors $T$, $P$, and $ADF$ and of specimen damping ratio, $D$. Values of $F$ are provided by the computer program in Appendix X1, which is written in FORTRAN IV. For cases where $ADF$ is zero and specimen damping ratio is less than 10 % values of $F$ can be obtained from Fig. 2. Fig. 2*b* is similar to Fig. 2*a* except that the range of $T$ is different. This figure is independent of which end of the specimen is used to determine resonance.

10.8 *Magnification Factors:*

10.8.1 These factors are used in calculating damping. For longitudinal motion, the magnification factor is calculated from:

$$MMF_L = \left[\frac{(LCF)(LTO)}{(FCF)(CR_L)}\right] M(2\pi f_L)^2 \tag{27}$$

where:
$LCF$ = longitudinal motion transducer displacement calibration factor for transducer used in establishing resonance (Note 4),[3]
$LTO$ = longitudinal motion transducer output of transducer used in establishing resonance,
$FCF$ = force/current factor given earlier, and
$CR_L$ = current reading to longitudinal excitation system (Note 5).

NOTE 4—$LCF$ and $RCF$ will be functions of frequency for velocity- and acceleration-measuring transducers (see 8.1).

NOTE 5—If a current-measuring instrument is used, the units will be amperes. Alternatively, this may be a voltage measurement across a fixed resistance and in units of volts. In the latter case, the same fixed resistance use in calibration (see 8.5) must be used for these measurements.

10.8.2 For torsional motion, the magnification factor is calculated from:

$$MMF_T = \left[\frac{(RCF)(RTO)}{(TCF)(CR_T)}\right] J(2\pi f_T)^2 \tag{28}$$

where:
$RCF$ = rotational transducer displacement calibration factor for transducer used in establishing resonance (Note 4)
$RTO$ = rotational transducer output for transducer used in establishing resonance,
$TCF$ = torque/current factor given earlier, and
$CR_T$ = current reading to torsional excitation system (Note 5)

10.9 *Moduli:*

10.9.1 The rod modulus is calculated from:

$$E = \rho(2\pi L)^2(f_L/F_L)^2 \tag{29}$$

where:
$\rho$ = specimen mass density given earlier,
$f_L$ = system resonant frequency for longitudinal motion given earlier,
$F_L$ = dimensionless frequency factor given earlier, and
$L$ = specimen length.

10.9.2 The shear modulus is calculated from:

$$G = \rho(2\pi L)^2(f_T/F_T)^2 \tag{30}$$

where:
$f_T$ = system resonant frequency for torsional motion given earlier, and
$F_T$ = dimensionless frequency factor given earlier.

10.10 *Strain Amplitude:*

10.10.1 The average rod strain amplitude, ε, for longitudinal vibration shall be calculated from:

[3] Enlarged and detailed copies of Figs. 2*a*, 2*b*, 3*a*, 3*b*, 4*a*, and 4 *b* may be obtained at a nominal charge from ASTM Headquarters, 1916 Race St., Philadelphia, Pa. 19103. Request Adjunct No. 12-440150-00.

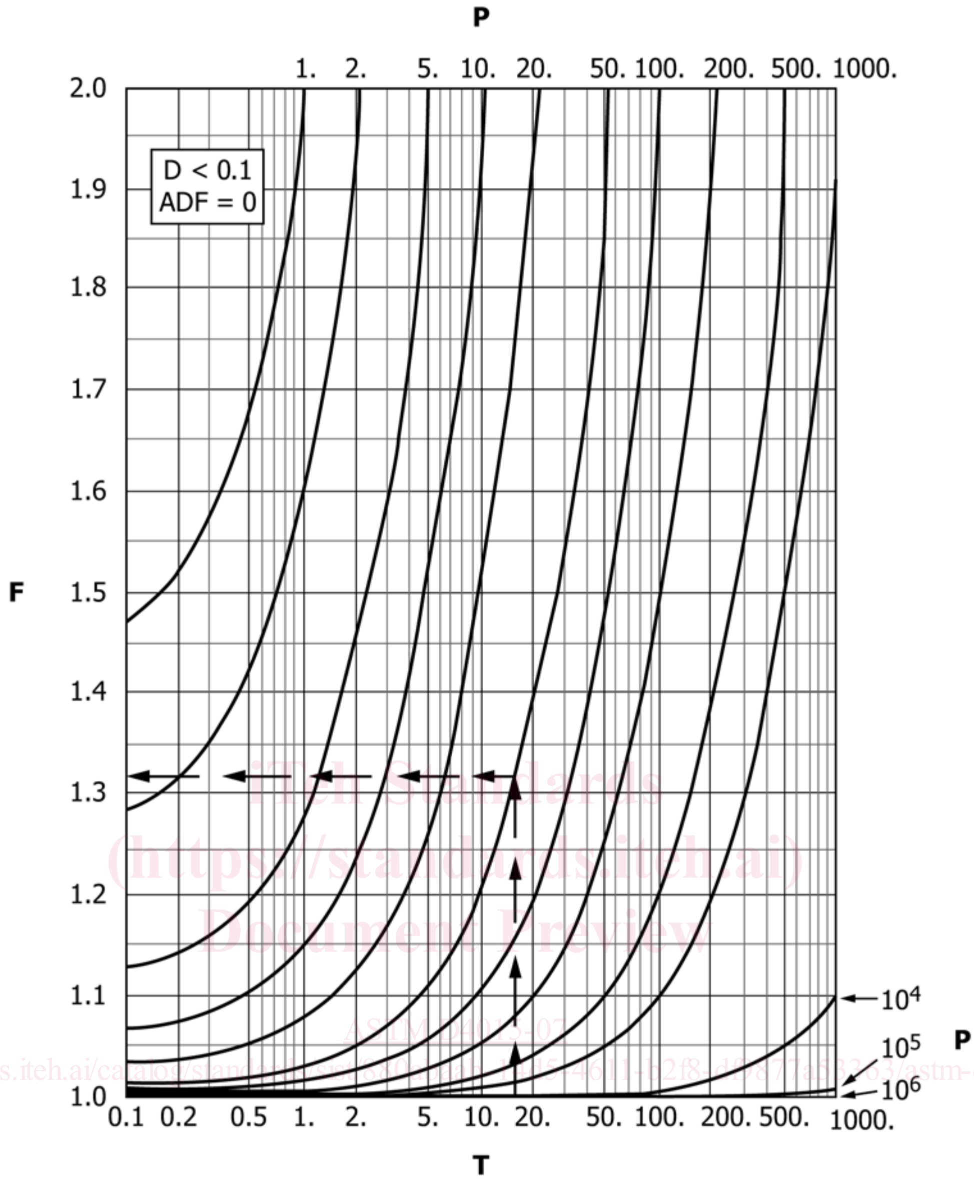


FIG. 3 (a) Strain Factors for Resonance Determined from Motion at Active End[3]