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МЕЖДУНАРОДНАЯ ОРГАНИЗАЦИЯ ПО СТАНДАРТИЗАЦИИ

Flight dynamics — Concepts, quantities and symbols —

Part 1: Aircraft motion relative to the air

Mécanique du vol — Concepts, grandeurs et symboles —

Partie 1: Mouvement de l'avion par rapport à l'air

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work.

Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council. They are approved in accordance with ISO procedures requiring at least 75 % approval by the member bodies voting.

International Standard ISO 1151-1 was prepared by Technical Committee ISO/TC 20, *Aircraft and space vehicles.*

This fourth edition cancels and replaces the third edition (ISO 1151-1 : 1985) of which it constitutes a technical revision; this fourth edition includes Draft Addenda ISO/DIS 1151-1/DAD 1, circulated in 1986 (addition of a symbol to 1.5.4, 1.6.1.3 and 1.6.2.9; new sub-clauses 1.4.10, 1.4.11, 1.5.10 to 1.5.13 and 1.10) and ISO/DIS 1151-1/DAD 2, circulated in 1986 (addition of annex B).

Users should note that all International Standards undergo revision from time to time and that any reference made herein to any other International Standard implies its latest edition, unless otherwise stated.

ISO 1151, *Flight dynamics — Concepts, quantities and symbols*, comprises, at present, seven parts:

Part 1 : Aircraft motion relative to the air.

Part 2 : Motions of the aircraft and the atmosphere relative to the Earth.

Part 3 : Derivatives of forces, moments and their coefficients.

Part 4 : Parameters used in the study of aircraft stability and control.

Part 5 : Quantities used in measurements.

Part 6 : Aircraft geometry.

Part 7 : Flight points and flight envelopes.

ISO 1151 is intended to introduce the main concepts, to include the more important terms used in theoretical and experimental studies and, as far as possible, to give corresponding symbols.

In all the parts comprising ISO 1151, the term "aircraft" denotes a vehicle intended for atmosphere or space flight. Usually, it has an essentially port and starboard symmetry with respect to a plane. That plane is determined by the geometric characteristics of the aircraft. In that plane, two orthogonal directions are defined: fore-and-aft and dorsal-ventral. The transverse direction, on the perpendicular to that plane, follows.

When there is a single plane of symmetry, it is the reference plane of the aircraft. When there is more than one plane of symmetry, or when there is none, it is necessary to choose a reference plane. In the former case, the reference plane is one of the planes of symmetry. In the latter case, the reference plane is arbitrary. In all cases, it is necessary to specify the choice made.

Angles of rotation, angular velocities and moments about any axis are positive clockwise when viewed in the positive direction of that axis.

All the axis systems used are three-dimensional, orthogonal and right-handed, which implies that a positive rotation through $\pi/2$ around the x-axis brings the y-axis into the position previously occupied by the z-axis.

The centre of gravity coincides with the centre of mass if the field of gravity is homogeneous. If this is not the case, the centre of gravity can be replaced by the centre of mass in the definitions of ISO 1151; in which case, this should be indicated.

Numbering of sections and clauses

With the aim of easing the indication of references from a section or a clause, a decimal numbering system has been adopted such that the first figure is the number of the part of ISO 1151 considered.

Contents

Flight dynamics — Concepts, quantities and symbols —

Part 1: Aircraft motion relative to the air

1.0 Introduction

This part of ISO 1151 gives basic definitions and deals with aircraft motion relative to the atmosphere, assumed to be at rest or in translational motion at constant velocity relative to the Earth. [1)]

The aircraft is assumed to be rigid. However, most of the definitions can be applied to the case of a flexible aircraft.

When account is taken of the variations at the Earth's surface in the direction of the vertical (local direction of acceleration due to gravity), the term given in the sub-clauses and figures in question is qualified by the term "local".

1.1 Axis systems

No.	Term	Definition	Symbol
1.1.1	Earth-fixed axis system	A system with both the origin and axes fixed relative to the Earth, and chosen as appropriate.	$x_o y_o z_o$
1.1.2	Normal earth-fixed axis system	An earth-fixed axis system (1.1.1) in which the z_o-axis is oriented according to the downward vertical passing through the origin.	$x_o y_o z_o$ NOTE — However, $x_g y_g z_g$ is an acceptable alternative.
1.1.3	Aircraft-carried earth axis system	A system in which each axis has the same direction as the corresponding earth-fixed axis, with the origin fixed in the aircraft, usually the centre of gravity.	$x_o y_o z_o$
1.1.4	Aircraft-carried normal earth axis system	A system in which each axis has the same direction as the corresponding normal earth-fixed axis, with the origin fixed in the aircraft, usually the centre of gravity.	$x_o y_o z_o$ NOTE — However, $x_g y_g z_g$ is an acceptable alternative.
1.1.5	Body axis system[2)]	A system fixed in the aircraft, with the origin, usually the centre of gravity, consisting of the following axes:	xyz
	Longitudinal axis	An axis in the reference plane (see foreword on p. iii) or, if the origin is outside that plane, in the plane through the origin, parallel to the reference plane.	x
	Transverse axis	An axis normal to the reference plane and positive to starboard.	y
	Normal axis	An axis completing the system. NOTE — This axis lies in the reference plane or is parallel to that plane. It is positive in the ventral sense.	z

1) The motions of the atmosphere for which this assumption does not hold true will be examined in another part of ISO 1151.

2) Usually, the origins of the axis systems defined in 1.1.5, 1.1.6 and 1.1.7 coincide. If that is not the case, it is necessary to distinguish the different origins by appropriate suffixes.

No.	Term	Definition	Symbol
1.1.6	Air-path axis system[1]	A system with the origin fixed in the aircraft, usually the centre of gravity, consisting of the following axes:	$x_a y_a z_a$
	x_a-axis; air-path axis	An axis in the direction of the aircraft velocity (1.3.1).	x_a
	y_a-axis; lateral air-path axis; cross-stream axis	An axis normal to the air-path axis and the z_a-axis defined below; it is positive to starboard.	y_a
	z_a-axis; normal air-path axis	An axis — in the reference plane or, if the origin is outside that plane, parallel to the reference plane, and — normal to the air-path axis. The positive direction of this axis is chosen so as to complete the orthogonal, right-handed system $x_a y_a z_a$.	z_a
1.1.7	Intermediate axis system[1]	A system with the origin fixed in the aircraft, usually the centre of gravity, consisting of the following axes:	$x_e y_e z_e$
	x_e-axis	The projection of the air-path axis on the reference plane, or, if the origin is outside that plane, on the plane through the origin, parallel to the reference plane.	x_e
	y_e-axis	An axis normal to the reference plane and positive to starboard. NOTE — This axis coincides with the transverse axis (1.1.5) or is parallel to it.	y_e
	z_e-axis	An axis completing the axis system. NOTE — This axis coincides with the normal air-path axis (1.1.6) or is parallel to it.	z_e

1) Usually, the origins of the axis systems defined in 1.1.5, 1.1.6 and 1.1.7 coincide. If that is not the case, it is necessary to distinguish the different origins by appropriate suffixes.

1.2 Angles

1.2.1 Orientation of the aircraft velocity with respect to the body axis system (see figure 1)

No.	Term	Definition	Symbol
1.2.1.1	Angle of sideslip	The angle which the aircraft velocity (1.3.1) makes with the reference plane of the aircraft. It is positive when the aircraft velocity component along the transverse axis (1.1.5) is positive. According to convention, it has the range: $-\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2}$	β
1.2.1.2	Angle of attack	The angle between the longitudinal axis (1.1.5) and the projection of the aircraft velocity (1.3.1) on the reference plane. It is positive when the aircraft velocity component along the normal axis (1.1.5) is positive. According to convention, it has the range: $-\pi < \alpha \leqslant \pi$	α

1.2.2 Transition from the aircraft-carried normal earth axis system to the body axis system

This transition is achieved by three rotations, defined below, performed in the following order: Ψ, Θ, Φ (see figure 2).

NOTE — Similar angles may be defined with respect to any aircraft-carried earth axis system. The same symbols, Ψ, Θ, Φ, with appropriate suffixes as necessary, may then be used. However, the terms "azimuth angle", "inclination angle" and "back angle" refer only to the specific case where the z_o-axis is vertical.

No.	Term	Definition	Symbol
1.2.2.1	Azimuth angle	The rotation (positive, if clockwise) about the $z_o(z_g)$-axis which brings the $x_o(x_g)$-axis into coincidence with the projection of the longitudinal axis (1.1.5) on the horizontal plane through the origin.	Ψ
1.2.2.2	Inclination angle	The rotation in a vertical plane, following the Ψ rotation (1.2.2.1), which brings the displaced $x_o(x_g)$-axis into coincidence with the longitudinal axis (1.1.5). It is positive when the positive portion of the x-axis lies above the horizontal plane through the origin. According to convention, it has the range: $-\frac{\pi}{2} \leqslant \Theta \leqslant \frac{\pi}{2}$	Θ
1.2.2.3	Bank angle	The rotation (positive, if clockwise) about the longitudinal axis (1.1.5), brings the displaced $y_o(y_g)$-axis into its final position y from the position it reached after the Ψ rotation (1.2.2.1).	Φ

1.2.3 Transition from the aircraft-carried normal earth axis system to the air-path axis system

This transition is achieved by three rotations, defined below, performed in the following order: χ_a, γ_a, μ_a (see figure 3).

No.	Term	Definition	Symbol
1.2.3.1	Air-path azimuth angle; air-path track angle	The rotation (positive, if clockwise) about the $z_o(z_g)$-axis which brings the $x_o(x_g)$-axis into coincidence with the projection of the air-path x_a-axis (1.1.6) on the horizontal plane through the origin.	χ_a
1.2.3.2	Air-path inclination angle; air-path climb angle	The rotation in a vertical plane, following the χ_a rotation (1.2.3.1), which brings the displaced $x_o(x_g)$-axis into coincidence with the air-path x_a-axis (1.1.6). It is positive when the positive portion of the x_a-axis lies above the horizontal plane through the origin. According to convention, it has the range: $-\frac{\pi}{2} \leqslant \gamma_a \leqslant \frac{\pi}{2}$	γ_a
1.2.3.3	Air-path bank angle	The rotation (positive, if clockwise) about the air-path x_a-axis (1.1.6) which brings the displaced $y_o(y_g)$-axis into its final position y_a from the position it reached after the χ_a rotation (1.2.3.1).	μ_a

1.3 Velocities and angular velocities

No.	Term	Definition	Symbol
1.3.1	Aircraft velocity Airspeed	The velocity of the origin of the body axis system (1.1.5) (usually the centre of gravity), relative to the air, unaffected by the aerodynamic field of the aircraft. The magnitude of the aircraft velocity.	$\vec{V}$ V
1.3.2	Speed of sound	The velocity of propagation of a sound wave in the ambient air, unaffected by the aerodynamic field of the aircraft.	a

No.	Term	Definition	Symbol
1.3.3	Mach number	The ratio of the airspeed (1.3.1) to the speed of sound (1.3.2). Equal to V/a.	M is recommended. The symbols Ma and $\mathscr{M}$ may be used if there is likely to be any confusion.
1.3.4	Aircraft velocity components	The components of the aircraft velocity (1.3.1), $\vec{V}$, for any of the axis systems used.	
		In the axis systems 1.1.1 to 1.1.4:	
		component along the x_o-axis	u_o
		component along the y_o-axis	v_o
		component along the z_o-axis	w_o
		In the body axis system (1.1.5):	
		component along the longitudinal axis	u
		component along the transverse axis	v
		component along the normal axis	w
		NOTE — In the air-path axis system (1.1.6), the component along the x_a-axis is $u_a = V$.	The velocity components may be written V_i, where i is a number or letter subscript.
1.3.5	Aircraft angular velocity	The angular velocity of the body axis system (1.1.5) relative to the Earth.	$\vec{\Omega}$
	Aircraft angular speed	The magnitude of aircraft angular velocity.	Ω
1.3.6	Angular velocity components	The components of the aircraft angular velocity (1.3.5), $\vec{\Omega}$, for any of the axis systems used.	
		In the axis systems 1.1.1 to 1.1.4:	
		component along the x_o-axis	p_o
		component along the y_o-axis	q_o
		component along the z_o-axis	r_o
		In the body axis system (1.1.5):	
	Rate of roll	component along the longitudinal axis	p
	Rate of pitch	component along the transverse axis	q
	Rate of yaw	component along the normal axis	r
			The angular velocity components may be written Ω_i, where i is a number or letter subscript.

No.	Term	Definition	Symbol
1.3.7	Normalized angular velocities	The normalized form of the components of the aircraft angular velocity (1.3.5), defined as follows: In the body axis system (1.1.5):	
	Normalized rate of roll	$\frac{pl}{V}$	p^*
	Normalized rate of pitch	$\frac{ql}{V}$	q^*
	Normalized rate of yaw	$\frac{rl}{V}$	r^*
		where l is the reference length (1.4.6); V is the airspeed (1.3.1). Similar normalized quantities can be formed for the other axis systems.	Similar quantities using a constant reference speed in place of V (1.3.1) may also be defined. These require different symbols.

1.4 Aircraft inertia, reference quantities and reduced parameters

No.	Term	Definition	Symbol
1.4.1	Aircraft mass	The current mass of the aircraft.	m
1.4.2	Moments of inertia	The moments of inertia of the aircraft with respect to the body axes xyz (1.1.5).	
		Moment of inertia about the longitudinal axis: $\int(y^2 + z^2)\mathrm{d}m$	I_x
		Moment of inertia about the transverse axis: $\int(z^2 + x^2)\mathrm{d}m$	I_y
		Moment of inertia about the normal axis: $\int(x^2 + y^2)\mathrm{d}m$	I_z
			NOTE — A, B, C are acceptable alternatives.
1.4.3	Products of inertia	The products of inertia of the aircraft with respect to the body axes xyz (1.1.5). These are:	
		$\int yz\,\mathrm{d}m$	I_{yz}
		$\int zx\,\mathrm{d}m$	I_{zx}
		$\int xy\,\mathrm{d}m$	I_{xy}
			NOTE — D, E, F are acceptable alternatives.

No.	Term	Definition	Symbol
1.4.4	Radius of gyration	The square root of the ratio of the moment of inertia to the aircraft mass (1.4.1): for the longitudinal axis (1.1.5): $\sqrt{I_x/m}$ for the transverse axis (1.1.5): $\sqrt{I_y/m}$ for the normal axis (1.1.5): $\sqrt{I_z/m}$	 r_x r_y r_z
1.4.5	Reference area	An area used to define the aerodynamic coefficients and various normalized quantities. In a given document, one single reference area will be used the value of which shall be specified. NOTE — Although hinge moment coefficients are usually defined using specific reference areas, it may be more appropriate, in some cases, to use the single reference area adopted for the aircraft.	S
1.4.6	Reference length	A length used to define the aerodynamic moment coefficients and various normalized quantities. It is recommended, in a given document, that one single reference length be used the value of which shall be specified. NOTES 1 It is, however, acceptable to introduce two different reference lengths as regards the longitudinal motion and the lateral motion. These lengths shall also be specified. 2 Although hinge moment coefficients are usually defined using specific reference lengths, it may be more appropriate, in some cases, to use the single reference length adopted for the aircraft.	l
1.4.7	Normalized mass	A non-dimensional coefficient defined as follows: $\frac{m}{\frac{1}{2}\varrho_e S l}$ where m is the aircraft mass (1.4.1); ϱ_e is a datum (air) density (3.3.2); S is the reference area (1.4.5); l is the reference length (1.4.6).	μ or m^*
1.4.8	Dynamic unit of time	A quantity defined as follows: $\frac{m}{\frac{1}{2}\varrho_e V_e S} = \frac{\mu l}{V_e}$ where m is the aircraft mass (1.4.1); ϱ_e is a datum (air) density (3.3.2); V_e is a datum speed (3.3.1); S is the reference area (1.4.5); l is the reference length (1.4.6); μ is the normalized mass (1.4.7).	τ

No.	Term	Definition	Symbol
1.4.9	Aerodynamic unit of time	A quantity defined as follows: $\frac{l}{V_e}$ where l is the reference length (1.4.6); V_e is a datum speed (3.3.1).	τ_A
1.4.10	Inertia matrix	A symmetrical matrix the structure of which is as follows: $\boldsymbol{I} = \begin{pmatrix} I_x & -I_{xy} & -I_{zx} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{zx} & -I_{yz} & I_z \end{pmatrix}$	$\boldsymbol{I}$
1.4.11	Inverse inertia matrix	The inverse of the inertia matrix (1.4.10). A symmetrical matrix the structure of which is as follows: $\boldsymbol{I}^{-1} = \boldsymbol{J} = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$ where $J_{11} = (I_y I_z - I_{yz}^2)/\Delta$ $J_{22} = (I_z I_x - I_{zx}^2)/\Delta$ $J_{33} = (I_x I_y - I_{xy}^2)/\Delta$ $J_{12} = (I_{xy} I_z + I_{yz} I_{zx})/\Delta$ $J_{23} = (I_{yz} I_x + I_{zx} I_{xy})/\Delta$ $J_{31} = (I_{zx} I_y + I_{xy} I_{yz})/\Delta$ $J_{21} = J_{12}$ $J_{32} = J_{23}$ $J_{13} = J_{31}$ $\Delta = I_x I_y I_z - 2 I_{xy} I_{yz} I_{zx} - I_x I_{yz}^2 - I_y I_{zx}^2 - I_z I_{xy}^2$	$\boldsymbol{J}$

1.5 Forces, moments, coefficients and load factors

No.	Term	Definition	Symbol
1.5.1	Resultant force	The resultant vector of the system of forces acting on aircraft, including the airframe aerodynamic forces and propulsion forces, but excluding the gravitational, inertial and reaction forces due to contact with the Earth's surface.	$\vec{R}$
1.5.2	Components of the resultant force	The components of the resultant force vector, $\vec{R}$. In the body axis system (1.1.5): component along the longitudinal axis component along the transverse axis component along the normal axis In the air-path axis system (1.1.6): component along the x_a-axis component along the y_a-axis component along the z_a-axis	 X Y Z X_a Y_a Z_a

No.	Term	Definition	Symbol
1.5.3	Force coefficients	Non-dimensional coefficients of the components of the resultant force (1.5.2), defined as follows: In the body axis system (1.1.5): *X* force coefficient: $\frac{X}{\frac{1}{2}\varrho V^2 S}$	C_X
		Y force coefficient: $\frac{Y}{\frac{1}{2}\varrho V^2 S}$	C_Y
		Z force coefficient: $\frac{Z}{\frac{1}{2}\varrho V^2 S}$	C_Z
		In the air-path axis system (1.1.6): X_a force coefficient: $\frac{X_a}{\frac{1}{2}\varrho V^2 S}$	C_{Xa}
		Y_a force coefficient: $\frac{Y_a}{\frac{1}{2}\varrho V^2 S}$	C_{Ya}
		Z_a force coefficient: $\frac{Z_a}{\frac{1}{2}\varrho V^2 S}$	C_{Za}
		where ϱ is the density (5.1.3) of the ambient air, unaffected by the aerodynamic field of the the aircraft; V is the airspeed (1.3.1); S is the reference area (1.4.5). NOTE — These definitions are not usually used in helicopter studies.	
1.5.4	Resultant moment	The resultant moment of the system of forces, forming the resultant force (1.5.1), about a reference point, usually the centre of gravity.	$\vec{Q}$