

SLOVENSKI STANDARD SIST-TP CEN/TR 16988:2016

01-november-2016

Ocena negotovosti s preskusom enega samega gorečega predmeta

Estimation of uncertainty in the single burning item test

Messunsicherheit - Thermische Beanspruchung durch einen einzelnen brennenden Gegenstand (SBI)

iTeh STANDARD PREVIEW (standards.iteh.ai)

Ta slovenski standard je istoveten z: CEN/TR 16988:2016

https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4ee7a144b6fbad/sist-tp-cen-tr-16988-2016

ICS:

13.220.40 Sposobnost vžiga in obnašanje materialov in proizvodov pri gorenju Ignitability and burning behaviour of materials and products

SIST-TP CEN/TR 16988:2016

en,fr,de

iTeh STANDARD PREVIEW (standards.iteh.ai)

SIST-TP CEN/TR 16988:2016 https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4ee7a144b6fbad/sist-tp-cen-tr-16988-2016

TECHNICAL REPORT RAPPORT TECHNIQUE TECHNISCHER BERICHT

CEN/TR 16988

July 2016

ICS 17.200.01

English Version

Estimation of uncertainty in the single burning item test

Messunsicherheit - Thermische Beanspruchung durch einen einzelnen brennenden Gegenstand (SBI)

This Technical Report was approved by CEN on 4 July 2016. It has been drawn up by the Technical Committee CEN/TC 127.

CEN members are the national standards bodies of Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, Former Yugoslav Republic of Macedonia, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey and United Kingdom.

iTeh STANDARD PREVIEW (standards.iteh.ai)

<u>SIST-TP CEN/TR 16988:2016</u> https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4ee7a144b6fbad/sist-tp-cen-tr-16988-2016



EUROPEAN COMMITTEE FOR STANDARDIZATION COMITÉ EUROPÉEN DE NORMALISATION EUROPÄISCHES KOMITEE FÜR NORMUNG

CEN-CENELEC Management Centre: Avenue Marnix 17, B-1000 Brussels

© 2016 CEN All rights of exploitation in any form and by any means reserved worldwide for CEN national Members.

Ref. No. CEN/TR 16988:2016 E

Contents

European foreword4			
1	Scope	5	
1.1	General	5	
1.2	Calculation procedure		
1.2.1	Introduction		
1.2.2	Synchronization of data	-	
1.2.3	Heat output		
	1.		
2	Uncertainty		
2.1	Introduction		
2.2	Elaboration of terms and concepts		
2.2.1	Mean and variance		
2.2.2	Estimation of the confidence interval for the population mean		
2.2.3	Sources of uncertainty		
2.2.4	Standard uncertainties for different distributions	12	
2.2.5	Combined uncertainty	15	
2.2.6	Expanded uncertainty	16	
2.2.7	Uncorrected bias	16	
2.3	Combined standard uncertainties	17	
2.3.1	Combined standard uncertainty on sumarcls.itch.ai)	17	
2.3.2	Combined standard uncertainty on averages	18	
2.3.3	Combined standard uncertainty of a product and a division		
2.3.4	Combined standard uncertainty on the heat release rate (Q)		
2.3.5	Combined standard uncertainty on the depletion factor (ϕ)		
2.3.6	Combined standard uncertainty on the initial O ₂ -concentration (X ^{D°} ₀₂)		
2.3.7	Combined standard uncertainty on the volume flow rate (V_{D298})		
2.3.8	Combined standard uncertainty on the air density (ρ_{air})		
2.3.9	Combined standard uncertainty on specimen heat release rate (Q _{specimen})		
	Combined standard uncertainty on the average heat release rate (Q_{av})		
	Combined standard uncertainty on FIGRA		
	Combined standard uncertainty on THR600s		
2.3.13	Combined standard uncertainty on the volume flow (V(t))	25	
	Combined standard uncertainty on the smoke production rate (SPR)		
	Combined standard uncertainty on specimen smoke production rate (SPR)		
	Combined standard uncertainty on the average smoke production rate (SPR _{av})		
	Combined standard uncertainty on SMOGRA		
	Combined standard uncertainty on TSP600s		
2.4	Confidence interval classification parameters		
2.5	Standard uncertainty on the different components		
2.5.1	Uncertainty on the data acquisition (DAQ)		
2.5.2	Transient error		
2.5.3	Aliasing error		
2.5.4	Uncertainty on data synchronicity		
2.5.5	Uncertainty on the component E and E'		
2.5.6	Uncertainty on the component ϕ		
2.5.7	Uncertainty on the component p _{atm}		
2.5.8	Uncertainty on the component T _{room}		
2.5.9	Uncertainty on the component α	38	

2.5.10	Uncertainty on the component c	38
	Uncertainty on the component A and L	
2.5.12	Uncertainty on the component q _{gas}	40
	Uncertainty on the component kt	
2.5.14	Uncertainty on the component k _p	43
2.5.15	Uncertainty on the component Δp	44
2.5.16	Uncertainty on the component T _{ms}	44
2.5.17	Uncertainty on the component I	46
Annex	A (informative) List of symbols and abbreviations	48

iTeh STANDARD PREVIEW (standards.iteh.ai)

SIST-TP CEN/TR 16988:2016 https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4ee7a144b6fbad/sist-tp-cen-tr-16988-2016

European foreword

This document (CEN/TR 16988:2016) has been prepared by Technical Committee CEN/TC 127 "Fire Safety in Buildings", the secretariat of which is held by BSI.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. CEN shall not be held responsible for identifying any or all such patent rights.

This document has been prepared under a mandate given to CEN by the European Commission and the European Free Trade Association.

iTeh STANDARD PREVIEW (standards.iteh.ai)

<u>SIST-TP CEN/TR 16988:2016</u> https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4ee7a144b6fbad/sist-tp-cen-tr-16988-2016

1 Scope

1.1 General

The measuring technique of the SBI (single burning item) test instrument is based on the observation that, in general, the heats of combustion per unit mass of oxygen consumed are approximately the same for most fuels commonly encountered in fires (Huggett [12]). The mass flow, together with the oxygen concentration in the extraction system, suffices to continuously calculate the amount of heat released. Some corrections can be introduced if CO_2 , CO and/or H_2O are additionally measured.

1.2 Calculation procedure

1.2.1 Introduction

The main calculation procedures for obtaining the HRR and its derived parameters are summarized here for convenience. The formulas will be used in the following clauses and especially in the clause on uncertainty.

The calculations and procedures can be found in full detail in the SBI standard [1].

1.2.2 Synchronization of data

The measured data are synchronized making use of the dips and peaks that occur in the data due to the switch from 'primary' to 'main' burner around t = 300 s, i.e. at the start of the thermal attack to the test specimen. Synchronization is necessary due to the delayed response of the oxygen and carbon dioxide analysers. The filters, long transport lines, the cooler, etc. in between the gas sample probe and the analyser unit, cause this shift in time.

After synchronization, all data are shifted so that the 'main' burner ignites – by definition – at time t = 300 s. <u>SIST-TP CEN/TR 16988:2016</u>

1.2.3 Heat output https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4e-

e7a144b6fbad/sist-tp-cen-tr-16988-2016

1.2.3.1 Average heat release rate of the specimen (HRR_{30s})

A first step in the calculation of the HRR contribution of the specimen is the calculation of the global HRR. The global HRR is constituted of the HRR contribution of both the specimen and the burner and is defined as

HRR_{total}(t) = E'V_{D298}(t)x_{a_02}
$$\left(\frac{\phi(t)}{1+0,105\phi(t)}\right)$$
 (1)

where

 $\text{HRR}_{\text{total}}(t)$ is the total heat release rate of the specimen and burner (kW);

E' is the heat release per unit volume of oxygen consumed at 298 K, = 17 200 (kJ/m3);

 $\dot{V}_{D298}(t)$ is the volume flow rate of the exhaust system, normalized at 298 K (m3/s);

 $x_{a O2}$ is the mole fraction of oxygen in the ambient air including water vapour;

 $\varphi(t)$ is the oxygen depletion factor.

The last two terms x_{a_02} and $\left(\frac{\phi(t)}{1+0,105\phi(t)}\right)$ express the amount of moles of oxygen, per unit volume,

that have chemically reacted into some combustion gases. Multiplication with the volume flow gives the

amount of moles of oxygen that have reacted away. Finally this value is multiplied with the 'Huggett' factor. Huggett stated that regardless of the fuel burnt roughly a same amount of heat is released.

The volume flow of the exhaust system, normalized at 298 K, $V_{D298}(t)$ is given by

$$V_{D298}(t) = cA \frac{k_t}{k_{\rho}} \sqrt{\frac{\Delta p(t)}{T_{\rm ms}(t)}}$$
⁽²⁾

where

c
$$\sqrt{(2T_0 / \rho_0)} = 22,4 [K^{0.5} \cdot m^{1.5} \cdot kg^{-0.5}]$$

A is the area of the exhaust duct at the general measurement section (m2);

 k_t is the flow profile correction factor; converts the velocity at the height of the bi-directional probe in the axis of the duct to the mean velocity over the cross section of the duct;

 k_{o} is the Reynolds number correction for the bidirectional probe, taken as 1,08;

 $\Delta p(t)$ is the pressure difference over the bi-directional probe (Pa);

 $T_{\rm ms}(t)$ is the temperature in the measurement section (K).

The oxygen depletion factor $\varphi(t)$ is defined as

$$\phi(t) = \frac{\bar{x}O_2(30s...90s)\{1 - \bar{x}O_2(t)\} - \bar{x}O_2(t)\{1 - \bar{x}O_2(30s..90s)\}}{\bar{x}O_2(30s...90s)\{1 - \bar{x}O_2(t) - \bar{x}O_2(t)\} + \bar{x}O_2(t)\}}$$
(3)

where

SIST-TP CEN/TR 16988:2016

 $xO_2(t)$ is the oxygen concentration in mole fraction, rds/sist/1 feff798-3550-4d84-be4e-

e7a144b6fbad/sist-tp-cen-tr-16988-2016

xCO₂(t) is the carbon dioxide concentration in mole fraction;

Ys...Zs mean taken over interval Y s to Z s.

The mole fraction of oxygen in ambient air, taking into account the moisture content, is given by

$$x_{a_{02}} = \bar{x}O_{2}(30 \,\mathrm{s...90\,s}) \left[1 - \frac{H}{100p} \exp\left\{ 23, 2 - \frac{3816}{\bar{T}_{\mathrm{ms}}(30 \,\mathrm{s...90\,s}) - 46} \right\} \right]$$
(4)

where

 $xO_2(t)$ is the oxygen concentration in mole fraction;

- H is the relative humidity (%);
- p is the ambient pressure (Pa);

Tms(t) is the temperature in the general measurement section (K).

Since we are interested in the HRR contribution of the specimen only, the HRR contribution of the burner should be subtracted. An estimate of the burner contribution $\text{HRR}_{\text{burner}}(t)$ is taken as the $\text{HRR}_{\text{total}}(t)$ during the base line period preceding the thermal attack to the specimen. A mass flow controller ensures an identical HRR through the burners before and after switching from primary to the main burner. The average HRR of the burner is calculated as the average HRR_{total}(t) during the base line period with the primary burner on (210 s $\leq t \leq$ 270 s):

$$HRR_{av burner} = \overline{HRR}_{total}(210 \, \text{s}...270 \, \text{s}) \tag{5}$$

where

HRRav_burner is the average heat release rate of the burner (kW);

HRRtotal(t) is the total heat release rate of specimen and burner (kW).

HRR of the specimen

In general, the HRR of the specimen is taken as the global HRR, $HRR_{total}(t)$, minus the average HRR of the burner, HRR_{av_burner} :

For *t* > 312 s:

$$HRR(t) = HRR_{total}(t) - HRR_{av \ burner}$$
(6)

where:

HRR(t) is the heat release rate of the specimen (kW);

HRRtotal(t) is the global heat release rate of specimen and burner (kW);

HRRav_burner is the average heat release rate of the burner (kW).

During the switch from the primary to the main burner at the start of the exposure period, the total heat output of the two burners is less than HRR_{av_burner} (it takes some time for the gas to be directed from one burner to the other). Formula (24) gives negative values for HRR(t) for at most 12 s (burner switch response time). Such negative values and the value for t = 300 s are set to zero, as follows:

HRR(300) = 0 kW SIST-TP CEN/TR 16988:2016 https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4e-For 300 s < t ≤ 312 s: e7a144b6fbad/sist-tp-cen-tr-16988-2016 (7)

$$HRR(t) = \max \{0 \text{ kW}, HRR_{total}(t) - HRR_{av \text{ burner}}\}$$
(8)

where

max.[a, b] is the maximum of two values a and b.

Calculation of HRR_{30s}

In view of the calculation of the FIGRA index, the HRR data are smoothened with a 'flat' 30 s running average filter using 11 consecutive measurements:

$$HRR_{30s}(t) = \frac{0.5HRR(t-15) + HRR(t-12) + ... + HRR(t+12) + 0.5HRR(t+15)}{10}$$
(9)

where

 $HRR_{30s}(t)$ is the average of HRR(t) over 30 s (kW);

HRR(t) is the heat release rate at time t (kW).

1.2.3.2 Calculation of THR(t) and THR_{600s}

The total heat release of the specimen THR(t) and the total heat release of the specimen in the first 600 s of the exposure period (300 s $\leq t \leq$ 900 s), THR_{600s}, are calculated as follows:

THR
$$(t_a) = \frac{1}{1000} \sum_{300s}^{t_a} HRR(t) \times 3$$
 (10)

THR_{600s} =
$$\frac{1}{1000} \sum_{300s}^{900s} \text{HRR}(t) \times 3$$
 (11)

whereby the factor 1 000 is introduced to convert the result from kJ into MJ and the factor 3 stands for the time interval in-between 2 consecutive measurements,

and where

- THR(t_a) is the total heat release of the specimen during the period 300 s \leq $t \leq$ t_a (MJ);
- HRR(*t*) is the heat release rate of the specimen (kW);
- THR_{600s} is the total heat release of the specimen during the period $300 \text{ s} \le t \le 900 \text{ s}$ (MJ); (equal to THR(900)).

1.2.3.3 Calculation of FIGRA_{0.2MJ} and FIGRA_{0.4MJ} (Fire growth rate indices)

The FIGRA is defined as the maximum of the ratio $HRR_{av}(t)/(t - 300)$, multiplied by 1 000. The ratio is calculated only for that part of the exposure period in which the threshold levels for HRR_{av} and THR have been exceeded. If one or both threshold values are not exceeded during the exposure period, FIGRA is equal to zero. Two combinations of threshold values are used, resulting in FIGRA_{0,2MJ} and FIGRA_{0,4MJ}.

a) The average of HRR, HRR_{av}, used to calculate the FIGRA is equal to HRR_{30s}, with the exception of the first 12 s of the exposure period. For data points in the first 12 s, the average is taken only over the widest possible symmetrical range of data points within the exposure period:

https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4e-

For
$$t = 300$$
 s: HRR_{av} (300 s) = 0 e^{7a144b6fbad/sist-tp-cen-tr-16988-2016} (12)

For
$$t = 303 \text{ s}$$
: HRR_{av}(303 s) = HRR(300 s...306 s) (13)

For
$$t = 306 \text{ s: } \text{HRR}_{av}(306 \text{ s}) = \text{HRR}(300 \text{ s}...312 \text{ s})$$
 (14)

For
$$t = 309 \text{ s}$$
: HRR_{av}(309 s) = HRR(300 s...318 s) (15)

For
$$t = 312$$
 s: HRR_{av} (312 s) = HRR(300 s...324 s) (16)

For
$$t \ge 315$$
 s: HRR_{av} $(t) = HRR_{30s}(t)$ (17)

b) Calculate FIGRA_{0,2MJ} for all *t* where:

 $(HRR_{av}(t) > 3 \text{ kW})$ and (THR(t) > 0,2 MJ) and $(300 \text{ s} < t \le 1 500 \text{ s})$;

and calculate FIGRA_{0,4MJ} for all t where:

$$(HRR_{av}(t) > 3 \text{ kW})$$
 and $(THR(t) > 0.4 \text{ MJ})$ and $(300 \text{ s} < t \le 1500 \text{ s})$;

both using:

FIGRA = 1000 × max
$$\left(\frac{\text{HRR}_{av}(t)}{t - 300}\right)$$

where:

FIGRA is the fire growth rate index

HRR_{av}(*t*) is the average of HRR(*t*) as specified in a) (kW);

As a consequence, specimens with a HRR_{av} not exceeding 3 kW during the total test have FIGRA values FIGRA_{0,2MJ} and FIGRA_{0,4MJ} equal to zero. Specimens with a THR not exceeding 0,2 MJ over the total test period have a FIGRA_{0,2MJ} equal to zero and specimen with a THR not exceeding 0,4 MJ over the total test period have a FIGRA_{0,4MJ} equal to zero.

2 Uncertainty

2.1 Introduction

According to EN ISO/IEC 17025 [3], which sets out the general requirements for the competence of testing and calibration laboratories, and EN ISO 10012 [7], which sets out the requirements for assuring the quality of measuring equipment, uncertainties shall be reported in both testing and calibration reports.

The general principles for evaluating and reporting uncertainties are given in the ISO Guide to the Expression of Uncertainty in Measurement (GUM) [6], but need to be applied to the specific case of fire testing. Due to the harmonization of fire testing in the European Community (EUROCLASSES; EN 13501-1 [21]) and the pressure on testing laboratories to operate under accreditation, this is becoming even more important.

It is of common knowledge that measurement results are never perfectly accurate. In practice the sources of systematic and random errors which can affect the results of measurement are numerous, even for the most careful operators. To describe this lack of perfection, the term 'uncertainty' is used. Although the concept of uncertainty may be related to a 'doubt', in the real sense the knowledge of uncertainty implies increased confidence in the validity of results.

The qualitative concept of accuracy is quantified by the uncertainty which varies inversely 'proportioned' to it. Accuracy consists of both trueness and precision as shown in Figure 1. A numerical measure for precision is the standard deviation, while trueness is expressed numerically by the systematic error or the bias.

It is considered good practice to eliminate any systematic errors. However, if the value of a systematic error is unknown it may be regarded as a random error. Random errors result in a spread of the values and can usually be reduced by increasing the number of observations. Its expectation or expected value is zero.

(18)



Figure 1 — Concepts of accuracy (uncertainty), precision (standard deviation) and trueness (bias)

iTeh STANDARD PREVIEW

In general, the result of a measurement is only an approximation or estimate of the value of the specific quantity subject to measurement, that is, the measurement and so the result is complete only when accompanied by a quantitative statement of its uncertainty.

Without knowledge of the accuracy (trueness and precision) of measurement methods and/or the uncertainty of measurement results, it can appear very easy to make decisions. But, in practice, these decisions might be incorrect and sometimes lead to serious consequences, if the measurement uncertainty is not taken into account.

For example, in fire testing, when rejecting instead of accepting a good product during a certification process or, conversely, when accepting a bad product by error. So, it is vital to quantify the reliability of the measurement results to greatly reduce any disputes and adverse consequences of legal proceedings. This is of particular importance if the growing number of cases of litigation in Europe and the liability problems of manufacturers in case of accidents are considered.

The difference between error and uncertainty should always be borne in mind. For example, the result of a measurement after correction can unknowably be very close to the unknown value of the measurand, and thus have negligible error, even though it might have a large uncertainty.



2.2 Elaboration of terms and concepts

2.2.1 Mean and variance

A population with a 'normal' probability density function is characterized by its mean value μ and its variance σ^2 : N(μ , σ^2). When both μ and σ^2 are unknown, they can be estimated by taking a number n of samples and by calculating the estimated mean \overline{x} , the estimated variance s^2 and the estimated standard deviation s.

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{19}$$

$$s^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - \bar{x})^{2}$$
(20)

If a covariance exists between two variables x and y, it is given by

$$s_{ij}^{2} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{i} - x)(y_{i} - y)$$
(21)

2.2.2 Estimation of the confidence interval for the population mean

Often the standard deviation σ is unknown. To evaluate the confidence interval, some estimate of σ shall be made. The most obvious candidate is the sample standard deviation s. But the use of s introduces an additional source of unreliability, especially if the sample is small. To retain the confidence interval, the interval shall therefore be widened. This is done by using the t distribution instead of the standard normal distribution. For a sample size larger than 100, the t-distribution approaches the normal distribution. For a 95 % (two tails of 2,5 %) confidence interval – which we strive for – the uncertainty is estimated by

$$t_{0.025} \frac{S}{\sqrt{n}} \tag{22}$$

The value $t_{0.025}$ depends on the amount of information used in calculating s², i.e. on the degrees of freedom. For large sample sizes, $t_{0.025}$ approaches 1,96 which is the value for a normal distribution. For a normal distribution, a coverage factor 2 (1,96) corresponds to a 95 % confidence interval (see 2.2.6).

2.2.3 Sources of uncertainty

According to GUM [6] any detailed report of the uncertainty should consist of a complete list of the components, specifying for each the method used to obtain its numerical value. The components may be grouped into two categories based on their method of evaluation:

Туре А	The components in category A are characterized by the estimated variances s _i ² or by the estimated standard deviation s derived from data by statistical methods. Where appropriate the covariance s _{ij} ² should be given. For such a component, the standard uncertainty is ui = si.
Туре В	The standard uncertainty of a Type B evaluation is approximated based on specifications, calibrations, handbooks, experience, judgements etc. and is represented by a quantity uj. It is obtained from an assumed probability distribution based on all the available information.
	Where appropriate the covariance should be given and should be treated in a similar way.

The 'type' classification does not indicate any difference in the nature of the components resulting from the two types of evaluation. Both are based on probability distributions, and the uncertainty components resulting from either type are quantified by standard deviations. It should be recognized that a Type B evaluation of standard uncertainty can be as reliable as a Type A evaluation.

The standard deviation of a Type B evaluation is based on the shape of the distribution. Distributions used in this dcoument are the rectangular, the triangular, the trapezoidal and the normal distribution. For the rectangular and triangular also asymmetric distributions are discussed.

2.2.4 Standard uncertainties for different distributions

Normal distribution

Often calibration certificates, handbooks, manufacturer's specifications, etc. state a particular multiple of a standard deviation. In this case, a normal distribution is assumed to obtain the standard uncertainty.

Rectangular distribution

In other cases the probability that the value of X_i lies within the interval a- to a+ for all practical purposes is equal to one and the probability that X_i lies outside this interval is essentially zero. If there is no specific knowledge about the possible values of X_i within the interval, a uniform or rectangular

distribution of values is assumed. The associated standard deviation is function of the width of the distribution as:

$$u_{rect} = \frac{a}{\sqrt{3}}$$
(23)

Indeed, for a rectangular distribution, the variance is obtained as in 24. Given the probability function of the rectangular distribution



This can be written in terms of the Heaviside step function H(x) as

$$P(x) = \frac{H(x-a) - H(x-b)}{b-a} (standards.iteh.ai)$$
(25)

This makes that the variance σ^2 with population mean β for an asymmetric distribution becomes https://standards.iteh.ai/catalog/standards/sist/1feff798-3550-4d84-be4e-

$$\mu = \int_{-\infty}^{\infty} P(x)xdx = \int_{a}^{b} \frac{x}{(b-a)} dx = \frac{e^{7a1} \frac{b}{46a} \frac{b}{a} \frac{dx}{2}}{2}$$
(26)

$$\sigma^2 = \int_{-\infty}^{\infty} P(x)(x-\mu)^2 dx$$
(27)

$$\sigma^{2} = \int_{-\infty}^{\infty} \frac{H(x-a) - H(x-b)}{b-a} (x - \frac{a+b}{2})^{2} dx$$
(28)

$$\sigma^{2} = \int_{a}^{b} \frac{\left(x - \frac{a+b}{2}\right)^{2}}{b-a} dx = \frac{\left(a+b\right)^{2}}{12}$$
(29)

So for a symmetric rectangular interval a- to a+, the variance reduces to

$$\sigma^2 = \frac{a^2}{3}.$$
(30)

The sample estimate of the standard deviation thus is:

$$u_{rect} = \frac{a}{\sqrt{3}}.$$
(31)