



Designation: E2022-01 Designation: E 2022 – 06^{ε1}

Standard Practice for Calculation of Weighting Factors for Tristimulus Integration¹

This standard is issued under the fixed designation E 2022; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

^{ε1} NOTE—The units statement in subsection 1.3 was added editorially in May 2008.

1. Scope

1.1 This practice describes the method to be used for calculating tables of weighting factors for tristimulus integration using custom spectral power distributions of illuminants or sources, or custom color-matching functions.

1.2 This practice provides methods for calculating tables of values for use with spectral reflectance or transmittance data, which are corrected for the influences of finite bandpass. In addition, this practice provides methods for calculating weighting factors from spectral data which has not been bandpass corrected. In the latter case, a correction for the influence of bandpass on the resulting tristimulus values is built in to the tristimulus integration through the weighting factors.

1.3

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to its use.

2. Referenced Documents

2.1 ASTM Standards:²

E 284 Terminology of Appearance

E 308 Practice for Computing the Colors of Objects by Using the CIE System

2.2 CIE Standard:

CIE Standard S 002 Colorimetric Observers³

3. Terminology

3.1 *Definitions*—Appearance terms in this practice are in accordance with Terminology E 284.

3.2 *Definitions of Terms Specific to This Standard:* [STM E2022-06ε1](#)

3.2.1 *illuminant, n*—real or ideal radiant flux, specified by its spectral distribution over the wavelengths that, in illuminating objects, can affect their perceived colors.

3.2.2 *source, n*—an object that produces light or other radiant flux, or the spectral power distribution of that light.

3.2.2.1 *Discussion*—A source is an emitter of visible radiation. An illuminant is a table of agreed spectral power distribution that may represent a source; thus, Illuminant A is a standard spectral power distribution and Source A is the physical representation of that distribution. Illuminant D65 is a standard illuminant that represents average north sky daylight but has no representative source.

3.2.3 *spectral power distribution, SPD, S(λ), n*—specification of an illuminant by the spectral composition of a radiometric quantity, such as radiance or radiant flux, as a function of wavelength.

4. Summary of Practice

4.1 CIE color-matching functions are standardized at 1-nm wavelength intervals. Tristimulus integration by multiplication of abridged spectral data into sets of weighting factors occurs at larger intervals, typically 10-nm or 20-nm; therefore, intermediate 1-nm interval spectral data are missing, but needed.

¹ This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.04 on Color and Appearance Analysis.

Current edition approved June 10, 2001. Published August 2001.

Current edition approved July 1, 2006. Published July 2006. Originally approved in 1999. Last previous edition approved in 2001 as E 2022 - 01.

² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For Annual Book of ASTM Standards, Vol 06.01, volume information, refer to the standard's Document Summary page on the ASTM website.

³ Available from USNC-CIE Publications Office, TLA Lighting Consultants, 7 Pond Street, Salem, MA 01970.

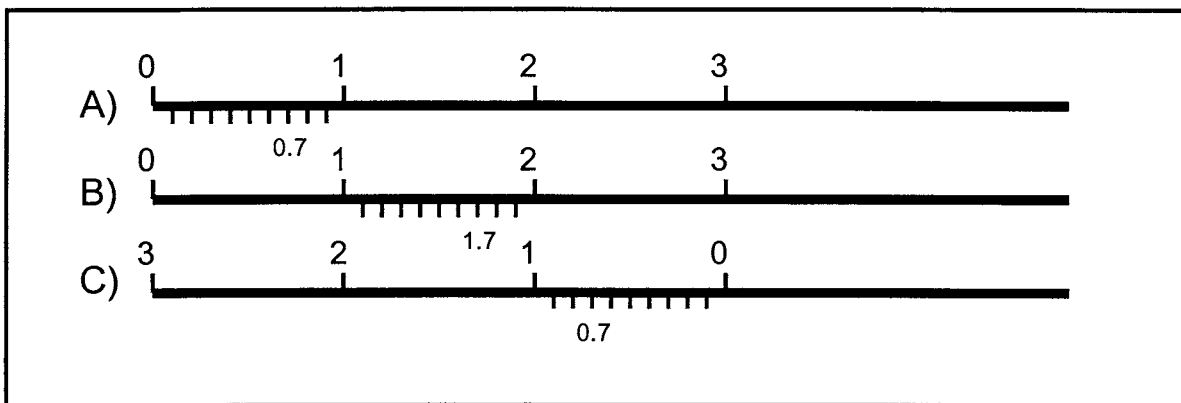


FIG. 1— The Values of i in Eq 1 are p Plotted a Above the a Abscissa and the v Values of r are p Plotted b Below for A) the f First m Measurement i Interval; B) the i Intermediate m Measurement i Intervals; and, C) the i Last m Measurement i Interval b Being i Interpolated.

4.2 Lagrange interpolating coefficients are calculated for the missing wavelengths. The Lagrange coefficients, when multiplied into the appropriate measured spectral data, interpolate the abridged spectrum to 1-nm interval. The 1-nm interval spectrum is then multiplied into the CIE 1-nm color-matching data, and into the source spectral power distribution. Each separate term of this multiplication is collected into a value associated with a measured spectral wavelength, thus forming weighting factors for tristimulus integration.

4.3 A correction may be applied to the resulting table of weighting factors to incorporate a correction for the spectral data's bandpass dependence.

5. Significance and Use

5.1 This practice is intended to provide a method that will yield uniformity of calculations used in making, matching, or controlling colors of objects. This uniformity is accomplished by providing a method for calculation of weighting factors for tristimulus integration consistent with the methods utilized to obtain the weighting factors for common illuminant-observer combinations contained in Practice E 308.

5.2 This practice should be utilized by persons desiring to calculate a set of weighting factors for tristimulus integration who have custom source, or illuminant spectral power distributions, or custom observer response functions.

5.3 This practice assumes that the measurement interval is equal to the spectral bandwidth integral when applying correction for bandwidth.

6. Procedure

6.1 *Calculation of Lagrange Coefficients* —Obtain by calculation, or by table look-up, a set of Lagrange interpolating coefficients for each of the missing wavelengths.⁴

6.1.1 The coefficients should be quadratic (three-point) in the first and last missing interval, and cubic (four-point) in all intervals between the first and the last missing interval.

6.1.2 *Generalized Lagrange Coefficients* —Lagrange coefficients may be calculated for any interval and number of missing wavelengths by Eq 1:

$$L_j(r) = \prod_{i=0, i \neq j}^n \frac{(r - r_i)}{(r_j - r_i)}, \text{ for } j = 0, 1, \dots, n \quad (1)$$

⁴ Hildebrand, F. B., *Introduction to Numerical Analysis*, Second Edition, Dover, New York, 1974, Chapter 3.

where:

- n = degree of coefficients being calculated,⁵
- i and j = indices denoting the location along the abscissa,
- π = repetitive multiplication of the terms in the numerator and the denominator, and
- indices of the interpolant, r = chosen on the same scale as the values i and j .

6.1.2.1 Fig. 1 assist the user in selecting the values of i , j , and r for these calculations.

6.1.2.2 Eq 1 is general and is applicable to any measurement interval or interpolation interval, regular or irregular.

6.1.3 *10 and 20-nm Lagrange Coefficients* —Where the measured spectral data have a regular or constant interval, the equation reduces to the following:

$$L_0 = \frac{(r-1)(r-2)(r-3)}{-6} \tag{2}$$

$$L_1 = \frac{(r)(r-2)(r-3)}{2} \tag{3}$$

$$L_2 = \frac{(r-1)(r)(r-3)}{-2} \tag{4}$$

$$L_3 = \frac{(r-1)(r-2)(r)}{6} \tag{5}$$

for the cubic case, and to

$$L_0 = \frac{(r-1)(r-2)}{2} \tag{6}$$

$$L_1 = \frac{(r)(r-2)}{-1} \tag{7}$$

$$L_2 = \frac{(r-1)(r)}{2} \tag{8}$$

for the quadratic case. In each of the above equations, as many or as few values of r as required are chosen to generate the necessary coefficients.

6.1.3.1 Eq 2-8 are applicable when the spectral data are abridged at 10-nm or 20-nm intervals, and the interpolated interval is regular with respect to the measurement interval, presumably 1-nm.

6.1.4 Tables 4-41-4 provide both quadratic and cubic Lagrange coefficients for 10-nm and 20-nm intervals.

6.2 With the Lagrange coefficients provided, the intermediate missing spectral data may be predicted as follows:

$$P(\lambda) = \sum_{i=0}^n L_i m_i \tag{9}$$

⁵ Fairman, H. S., "The Calculation of Weight Factors for Tristimulus Integration," *Color Research and Application*, Vol 10, 1985, pp. 199–203.

TABLE 1 The Lagrange Quadratic Interpolation Coefficients Applicable to the First and Last Missing Interval for Calculation of 10-nm Weighting Factors for Tristimulus Integration

Index of Missing Wavelength	L_0	L_1	L_2
1	0.855	0.190	-0.045
2	0.720	0.360	-0.080
3	0.595	0.510	-0.105
4	0.480	0.640	-0.120
5	0.375	0.750	-0.125
6	0.280	0.840	-0.120
7	0.195	0.910	-0.105
8	0.120	0.960	-0.080
9	0.055	0.990	-0.045

**TABLE 2 The Lagrange Cubic Interpolation Coefficients
Applicable to the Interior Missing Intervals for Calculation of
10-nm Weighting Factors for Tristimulus Integration**

Index of Missing Wavelength	L_0	L_1	L_2	L_3
1	-0.0285	0.9405	0.1045	-0.0165
2	-0.0480	0.8640	0.2160	-0.0320
3	-0.0595	0.7735	0.3315	-0.0455
4	-0.0640	0.6720	0.4480	-0.0560
5	-0.0625	0.5625	0.5625	-0.0625
6	-0.0560	0.4480	0.6720	-0.0640
7	-0.0455	0.3315	0.7735	-0.0595
8	-0.0320	0.2160	0.8640	-0.0480
9	-0.0165	0.1045	0.9405	-0.0285

**TABLE 3 The Lagrange Quadratic Interpolating Coefficients
Applicable to the First and Last Missing Interval for Calculation
of 20-nm Weighting Factors for Tristimulus Integration.**

Index of Missing Wavelength	L_0	L_1	L_2
1	0.92625	0.0975	-0.02375
2	0.85500	0.1900	-0.04500
3	0.78625	0.2775	-0.06375
4	0.72000	0.3600	-0.08000
5	0.65625	0.4375	-0.09375
6	0.59500	0.5100	-0.10500
7	0.53625	0.5775	-0.11375
8	0.48000	0.6400	-0.12000
9	0.42675	0.6975	-0.12375
10	0.37500	0.7500	-0.12500
11	0.32625	0.7975	-0.12375
12	0.28000	0.8400	-0.12000
13	0.23625	0.8775	-0.11375
14	0.19500	0.9100	-0.10500
15	0.15625	0.9375	-0.09375
16	0.12000	0.9600	-0.08000
17	0.08625	0.9775	-0.06375
18	0.05500	0.9900	-0.04500
19	0.02625	0.9975	-0.02375

ASTM E2022-06e1

<https://standards.iteh.ai/catalog/standards/sist/1ae8effbbb/astm-e2022-06e1>

**TABLE 4 The Lagrange Cubic Interpolating Coefficients
Applicable to the Interior Missing Intervals for Calculation of
20-nm Weighting Factors for Tristimulus Integration**

Index of Missing Wavelength	L_0	L_1	L_2	L_3
1	-0.0154375	0.9725625	0.0511875	-0.0083125
2	-0.028500	0.940500	0.104500	-0.016500
3	-0.0393125	0.9041875	0.1595625	-0.0244375
4	-0.048000	0.864000	0.216000	-0.032000
5	-0.0546875	0.8203125	0.2734375	-0.0390625
6	-0.059500	0.773500	0.331500	-0.045500
7	-0.0625625	0.7239375	0.3898125	-0.0511875
8	-0.064000	0.672000	0.448000	-0.056000
9	-0.0639375	0.6180625	0.5056875	-0.0598125
10	-0.062500	0.562500	0.562500	-0.062500
11	-0.0598125	0.5056875	0.6180625	-0.0639375
12	-0.056000	0.448000	0.672000	-0.064000
13	-0.0511875	0.3898125	0.7239375	-0.0625625
14	-0.045500	0.331500	0.773500	-0.059500
15	-0.0390625	0.2734375	0.8203125	-0.0546875
16	-0.032000	0.216000	0.864000	-0.048000
17	-0.0244375	0.1595625	0.9041875	-0.0393125
18	-0.016500	0.104500	0.940500	-0.028500
19	-0.0083125	0.0511875	0.9725625	-0.0154375

where:

P = the value being interpolated at interval λ ,

L = the Lagrange coefficients, and

m = the measured abridged spectral values.

Because the measured spectral values are as yet unknown, it may be best to consider this equation in its expanded form:

$$P(\lambda) = L_0m_0 + L_1m_1 + L_2m_2 + L_3m_3 \quad (10)$$

6.3 Multiply each $P(\lambda)$ by the 1-nm interval relative spectral power of the source or illuminant being considered.

6.3.1 It may be necessary to interpolate missing values of the source spectral power distribution $S(\lambda)$, if the source has been measured at other than 1-nm intervals.

6.3.2 Doing so results in the following equation:

$$S(\lambda)P(\lambda) = S(\lambda)L_0m_0 + S(\lambda)L_1m_1 + S(\lambda)L_2m_2 + S(\lambda)L_3m_3 \quad (11)$$

6.4 Multiply the weighted power at each 1-nm wavelength by the appropriate custom color-matching function value for that wavelength. Using the CIE color-matching functions as an example, obtain the CIE 1-nm data from CIE Standard S 002, Colorimetric Observers. Doing so results in the following equation:

$$\bar{x}(\lambda)S(\lambda)P(\lambda) = [\bar{x}(\lambda)S(\lambda)L_0]m_0 + [\bar{x}(\lambda)S(\lambda)L_1]m_1 + [\bar{x}(\lambda)S(\lambda)L_2]m_2 + [\bar{x}(\lambda)S(\lambda)L_3]m_3 \quad (12)$$

where:

$\bar{x}(\lambda)$ = the value of the CIE X color-matching function at wavelength λ , and the calculations are carried out for each of the three CIE color-matching functions, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$.

6.5 In the four terms on the right-hand side of this equation, the numerical values of the three factors in the brackets are known and should be multiplied into a single coefficient. The fourth factor, m_i , in each of the four additive terms is associated with a different measured wavelength.

6.6 Add all multiplicative coefficients dependent upon each different measured wavelength into a single coefficient applicable to that wavelength. This results in a single set of weighting factors that then will contain one value for each measured wavelength in each of three color-matching functions. The partial contribution to the tristimulus value at wavelength m_0 is:

$$[(\bar{x}(\lambda_0)S(\lambda_0)L_0) + (\bar{x}(\lambda_1)S(\lambda_1)L_0) + \dots]m_0 = wt_0m_0 \quad (13)$$

6.7 Normalize the weighting factors by calculating the following normalizing coefficient:

$$k = \frac{100}{\sum S(\lambda)y(\lambda)} \quad (14)$$

where:

k = the normalizing coefficient,

$S(\lambda)$ = the power in the 1-nm spectrum, and

$y(\lambda)$ = the CIE Y color-matching function.

6.8 Multiply the weighting factors by k to normalize the set to $Y = 100$ for the perfect reflecting diffuser.

6.9 *Correction for Bandpass Dependence* —If it is desired to correct the resulting weighting factors for the bandpass dependence of the measured spectral data, apply the following correction to the interior passbands.⁶

$$W_c(i) = -0.083 \cdot W_M(i-1) + 1.166 \cdot W_M(i) - 0.083 \cdot W_M(i+1) \quad (15)$$

where:

W = the indexed weight,

c = a corrected weight, and

m = a weight calculated without bandpass correction.

The index i varies from the second measured passband to the next to last measured passband. The following correction applies to the first and last measured passband:

$$W_c(i) = 1.166 \cdot W_M(i) - 0.083 \cdot W_M(i \pm 1) \quad (16)$$

$$W_c(i) = 1.083 \cdot W_M(i) - 0.083 \cdot W_M(i \pm 1) \quad (16)$$

where the symbols are the same as those of Eq 16.15 and the index i and \pm refers to the first and last measured passbands, respectively.

7. Precision

7.1 The precision of the practice is limited only by the precision of the data provided for the source spectral power distribution. The CIE color-matching functions are precise to six digits by definition. The Lagrange coefficients are precise to seven digits.

8. Keywords

8.1 color-matching functions; illuminant; illuminant-observer weights; source; tristimulus weighting factors

⁶ Stearns, E. I. and Stearns, R. E., "Influence of Spectrophotometer Slits on Tristimulus Calculations," *Color Research and Application*, Vol 13, 1988, pp. 257–259.

APPENDIX

(Nonmandatory Information)

X1.EXAMPLE OF THE CALCULATIONS

TABLE X1.1 Spectral Power Distribution of Typical 3-Band Fluorescent Lamp with Correlated Color Temperature of 3000 K (1-nm measurement interval)

λ	SPD	λ	SPD	λ	SPD	λ	SPD	λ	SPD	λ	SPD
360	0.004880	450	0.014870	540	0.162400	630	0.111200	720	0.004410	810	0.000000
361	0.004595	451	0.015040	541	0.277600	631	0.102900	721	0.003505	811	0.000000
362	0.004310	452	0.015210	542	0.392800	632	0.094620	722	0.002600	812	0.000000
363	0.020290	453	0.014980	543	0.353900	633	0.062350	723	0.002470	813	0.000000
364	0.036270	454	0.014750	544	0.315100	634	0.030080	724	0.002340	814	0.000000
365	0.047350	455	0.014370	545	0.429800	635	0.027420	725	0.002375	815	0.000000
366	0.058440	456	0.014000	546	0.544600	636	0.024770	726	0.002410	816	0.000000
367	0.031870	457	0.014060	547	0.383500	637	0.023050	727	0.002450	817	0.000000
368	0.005300	458	0.014110	548	0.222500	638	0.021330	728	0.002490	818	0.000000
369	0.004700	459	0.013930	549	0.182100	639	0.020750	729	0.001795	819	0.000000
370	0.004100	460	0.013760	550	0.141700	640	0.020170	730	0.001100	820	0.000000
371	0.003785	461	0.013470	551	0.113500	641	0.019920	731	0.001120	821	0.000000
372	0.003470	462	0.013180	552	0.085290	642	0.019660	732	0.001140	822	0.000000
373	0.003540	463	0.013470	553	0.070050	643	0.019740	733	0.001750	823	0.000000
374	0.003610	464	0.013750	554	0.054810	644	0.019810	734	0.002360	824	0.000000
375	0.003615	465	0.014000	555	0.046030	645	0.019550	735	0.002190	825	0.000000
376	0.003620	466	0.014250	556	0.037250	646	0.019280	736	0.002020	826	0.000000
377	0.004210	467	0.013810	557	0.034310	647	0.019080	737	0.003930	827	0.000000
378	0.004800	468	0.013370	558	0.031360	648	0.018880	738	0.005840	828	0.000000
379	0.005170	469	0.012870	559	0.030480	649	0.030460	739	0.003355	829	0.000000
380	0.005540	470	0.012370	560	0.029590	650	0.042050	740	0.000870	830	0.000000
381	0.005240	471	0.012640	561	0.029650	651	0.034870	741	0.002235		
382	0.004940	472	0.012900	562	0.029700	652	0.027690	742	0.003600		
383	0.004615	473	0.012640	563	0.029530	653	0.024990	743	0.002500		
384	0.004290	474	0.012380	564	0.029360	654	0.022290	744	0.001400		
385	0.003750	475	0.011680	565	0.029200	655	0.020120	745	0.002155		
386	0.003210	476	0.010970	566	0.029040	656	0.017950	746	0.002910		
387	0.003050	477	0.011050	567	0.029500	657	0.019130	747	0.002970		
388	0.002890	478	0.011130	568	0.029960	658	0.020320	748	0.003030		
389	0.002980	479	0.012680	569	0.029480	659	0.017400	749	0.003615		
390	0.003070	480	0.014240	570	0.029000	660	0.014470	750	0.004200		
391	0.002795	481	0.019080	571	0.029140	661	0.020750	751	0.003470		
392	0.002520	482	0.023910	572	0.029280	662	0.027030	752	0.002740		
393	0.002395	483	0.035600	573	0.029390	663	0.022910	753	0.002225		
394	0.002270	484	0.047290	574	0.029500	664	0.018790	754	0.001710		
395	0.002285	485	0.064030	575	0.040510	665	0.015270	755	0.000855		
396	0.002300	486	0.080770	576	0.051530	666	0.011740	756	0.000000		
397	0.002420	487	0.082540	577	0.060840	667	0.012890	757	0.000310		
398	0.002540	488	0.084310	578	0.070160	668	0.014040	758	0.000620		
399	0.002640	489	0.073870	579	0.079050	669	0.013040	759	0.000310		
400	0.002740	490	0.063440	580	0.087930	670	0.012030	760	0.000000		
401	0.002845	491	0.059500	581	0.090370	671	0.012230	761	0.000000		
402	0.002950	492	0.055560	582	0.092820	672	0.012430	762	0.000000		
403	0.062430	493	0.049350	583	0.098470	673	0.011550	763	0.000000		
404	0.121900	494	0.043140	584	0.104100	674	0.010680	764	0.000000		
405	0.085640	495	0.038320	585	0.102800	675	0.010140	765	0.000000		
406	0.049360	496	0.033490	586	0.101400	676	0.009600	766	0.000000		
407	0.032040	497	0.030100	587	0.113700	677	0.009705	767	0.000000		
408	0.014720	498	0.026710	588	0.126000	678	0.009810	768	0.000000		
409	0.009680	499	0.023390	589	0.097210	679	0.010690	769	0.000000		
410	0.004640	500	0.020080	590	0.068430	680	0.011560	770	0.000000		
411	0.005120	501	0.017300	591	0.085320	681	0.010990	771	0.000000		
412	0.005600	502	0.014520	592	0.102200	682	0.010420	772	0.000000		
413	0.005835	503	0.012700	593	0.103800	683	0.010040	773	0.000000		
414	0.006070	504	0.010870	594	0.105400	684	0.009650	774	0.000000		
415	0.006515	505	0.009670	595	0.083490	685	0.012730	775	0.000000		
416	0.006960	506	0.008470	596	0.061600	686	0.015810	776	0.000000		
417	0.007105	507	0.008350	597	0.064520	687	0.021660	777	0.000000		
418	0.007250	508	0.008230	598	0.067430	688	0.027500	778	0.000000		
419	0.007345	509	0.007905	599	0.077740	689	0.018370	779	0.000000		
420	0.007440	510	0.007580	600	0.088050	690	0.009240	780	0.000000		
421	0.007790	511	0.007370	601	0.068570	691	0.008135	781	0.000000		
422	0.008140	512	0.007160	602	0.049080	692	0.007030	782	0.000000		