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Geometrical product specifications (GPS) — Filtration —

Part 20:
Linear profile filters: Basic concepts

Spécification géométrique des produits (GPS) — Filtrage —

Partie 20: Filtres de profil linéaires: Concepts de base

Reference number
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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary Information

The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This first edition cancels and replaces ISO/TS 16610-20:2006 which has been technically revised.

ISO 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

- *Part 1: Overview and basic concepts*
- *Part 20: Linear profile filters: Basic concepts*
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- *Part 41: Morphological profile filters: Disk and horizontal line-segment filters*
- *Part 49: Morphological profile filters: Scale space techniques*
- *Part 60: Linear areal filters: Basic concepts*
- *Part 61: Linear areal filters: Gaussian filters*

— *Part 71: Robust areal filters: Gaussian regression filters*

— *Part 85: Morphological areal filters: Segmentation*

The following parts are planned:

— *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*

— *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*

— *Part 45: Morphological profile filters: Segmentation*

— *Part 62: Linear areal filters: Spline filters*

— *Part 69: Linear areal filters: Spline wavelets*

— *Part 70: Robust areal filters: Basic concepts*

— *Part 72: Robust areal filters: Spline filters*

— *Part 80: Morphological areal filters: Basic concepts*

— *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*

— *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO 16610 is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO/TR 14638). It influences chain links 3 and 5 in the GPS matrix structure.

The ISO/GPS Masterplan given in ISO 14638 gives an overview of the ISO/GPS system of which this part of ISO 16610 is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this part of ISO 16610 and the default decision rules given in ISO 14253-1 apply to the specifications made in accordance with this part of ISO 16610, unless otherwise indicated.

For more detailed information about the relation of this part of ISO 16610 to the GPS matrix model, see Annex C.

This part of ISO 16610 develops the basic concepts of linear filters, which include spline filters and spline wavelets, and the Gaussian filters.

Geometrical product specifications (GPS) — Filtration —

Part 20:
Linear profile filters: Basic concepts

1 Scope

This part of ISO 16610 describes the basic concepts of linear profile filters.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC Guide 99, ISO 16610-1, and the following apply.

3.1
linear profile filter
profile filter which separates profiles into long wave and short wave components and is also a linear function

Note 1 to entry: If F is a function and X and Y are profiles, then F being a linear function implies F(aX + bY) = aF(X) + bF(Y).

3.2
phase correct profile filter
phase correct linear profile filter
linear profile filter (3.1) which does not cause phase shifts leading to asymmetrical profile distortions

Note 1 to entry: Phase correct filters are a particular kind of the so called linear phase filters because any linear phase filter can be transformed (simply by shifting its weighting function) to a zero phase filter which is a phase correct filter.

3.3
weighting function
function for calculating the mean line which indicates, for each point, the weight attached by the profile in the vicinity of that point

Note 1 to entry: The transmission characteristic of the mean line is the Fourier transformation of the weighting function.

3.4
transmission characteristic of a filter
characteristic that indicates the amount by which the amplitude of a sinusoidal profile is attenuated as a function of its wavelength

Note 1 to entry: The transmission characteristic is the Fourier transformation of the weighting function.

3.5
cut-off wavelength
wavelength of a sinusoidal profile of which 50 % of the amplitude is transmitted by the profile filter

Note 1 to entry: Linear profile filters are identified by the filter type and the cut-off wavelength value.

Note 2 to entry: The cut-off wavelength is the recommended nesting index for linear profile filters.

3.6
filter bank
set of high-pass and low-pass filters arranged in a specified structure

Note 1 to entry: See 5.4 for further details.

3.7
multiresolution analysis
decomposition of a profile by a *filter bank* (3.6) into portions of different scales

Note 1 to entry: The portions at different scales are also referred to as resolutions.

4 Basic concepts

4.1 General

For a filter to conform with this part of ISO 16610, it shall exhibit the characteristics described in 5.1, 5.2, 5.3, and 5.4.

NOTE A concept diagram for linear profile filters is given in Annex A. The relationship to the filtration matrix model is given in Annex B.

The most general linear profile filter is defined by

$$y(x)=\int K(x,\xi)z(\xi)\mathrm{d}\xi \tag{1}$$

where

$z(\xi)$ is the unfiltered profile;

$y(x)$ is the filtered profile;

$K(x,\xi)$ is a real symmetric and spatial invariant kernel.

If $K(x,\xi)=K(x-\xi)$, the filtering is a convolution,

$$y(x)=\int K(x-\xi)z(\xi)\mathrm{d}\xi \tag{2}$$

and the kernel is called the weighting function of the filter.

However, extracted data are always discrete. Consequently, the filters described here are also discrete. If the weighting function is not discrete (see 4.4, Example 2), it shall be converted into a discrete representation.

4.2 Discrete representation of data

An extracted profile can be represented by a vector. The length *n* of this vector is equal to the number of data points. The sampling is assumed to be uniform, i.e. the sampling interval is constant. The *i*th data point of the profile is therefore the *i*th component of the vector.

$$z = (a_1 a_2 ... a_i ... a_{n-1} a_n) \tag{3}$$

4.3 Discrete representation of the linear profile filter

A linear profile filter is represented by a square matrix. The dimension of this matrix is equal to the number of data points to be filtered. If the filter is non-periodic, the matrix is a constant diagonal (Toeplitz) matrix:

$$S = \begin{pmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & & & & \\ & a & b & c & b & c & & & \\ & & c' & b' & a & b & c & & \\ & & & c' & b' & c' & b' & a & \\ & & & & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \tag{4}$$

Otherwise, if the filter is periodic, the matrix is a circulant matrix:

$$S = \begin{pmatrix} a & b & c & \dots & \dots & c' & b' \\ b' & a & b & c & \dots & \dots & c' \\ c' & b' & a & b & c & \dots & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & \dots & c' & b' & a & b & c \\ c & \dots & \dots & c' & b' & a & b \\ b & c & \dots & \dots & c' & b' & a \end{pmatrix} \tag{5}$$

If the filter is phase correct, the matrix representing the filter is symmetrical, i.e. $b = b', c = c', \dots$ (generally $a_{ij} = a_{ji}$) is valid. The sum of the matrix elements a_{ij} of each row i is constant and for low-pass filters equals one, i.e.

$$\sum_j a_{ij} = 1 \tag{6}$$

NOTE 1 In the case of a symmetrical matrix, the sum of the matrix elements a_{ij} of each column j is also constant and equals one, i.e. $\sum_i a_{ij} = 1$ is also valid.

NOTE 2 The relationship between the matrix *S*, the input data by the vector z, and the output data by the vector w is given in Formula (13).

4.4 Discrete representation of the weighting function

Given that each row of the matrix representation of the filter is identical after being shifted accordingly, the matrix elements may be represented by one single row, i.e.

$$a_{ij} = s_k \text{ with } k = i - j \tag{7}$$

The values s_k form a vector s of a dimension equal to the length of the input or output data vector respectively. This vector is the discrete representation of the weighting function of the filter.

NOTE 1 The length of the weighting function is usually much smaller than the length of the data set. In this case, *s* contains zeros at each end.

EXAMPLE 1 The moving average filter is frequently used for easy smoothing of a data set which is not necessarily an optimal method. In the following example of a filter with a discrete weighting function, where a length of 3 has been taken, the weighting function is given by

$$\left(...0,0,\frac{1}{3},\frac{1}{3},\frac{1}{3},0,0...\right) \tag{8}$$

NOTE 2 The weighting function is often also called the impulse response function because it is the output data set of the filter if the input data set is only a single unity impulse $(...0,0,0,1,0,0,0...)$.

If the weighting function is given as a continuous function, it shall be sampled in order to obtain a discrete data set. The sampling interval used shall be equal to the sampling interval of the extracted data. It is mandatory to renormalize the sampled data of the weighting function subsequently in order to fulfil the condition that they shall sum to unity, thus, avoiding bias effects (for details concerning bias effects, see Reference [3]).

EXAMPLE 2 The Gaussian filter, in accordance with ISO 16610-21, is an example of a continuous weighting function $s(x)$ defined by Formula (9):

$$s(x)=\frac{1}{\alpha\lambda_c}\exp\left[-\pi\left(\frac{x}{\alpha\lambda_c}\right)^2\right] \tag{9}$$

where

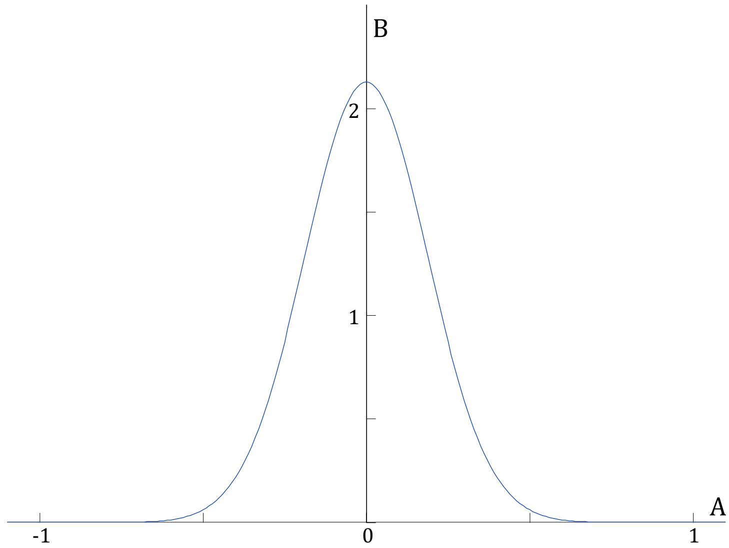
x is the distance from the centre (maximum) of the weighting function;

λ_c is the cut-off wavelength;

α is a constant given by the following equation:

$$\alpha=\sqrt{\frac{\ln 2}{\pi}}=0,4697... \tag{10}$$

The graph of this weighting function is shown in Figure 1.



Key

A = $\frac{X}{\lambda_c}$

B = $\lambda_c Z$

Figure 1 — Example of a continuous weighting function (Gaussian filter)

The sample data s_k of the weighting function after a renormalization are given by

$$s_k = \frac{1}{C} \exp\left[-\pi \left(\frac{\Delta x}{\alpha \lambda_c} \right)^2 k^2 \right] \tag{11}$$

with the sampling interval Δx, and the normalization constant

$$C = \sum_k \exp\left[-\pi \left(\frac{\Delta x}{\alpha \lambda_c} \right)^2 k^2 \right] \tag{12}$$