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Geometrical product specifications (GPS) — Filtration —

Part 22:
Linear profile filters: Spline filters

Spécification géométrique des produits (GPS) — Filtrage —

Partie 22: Filtres de profil linéaires: Filtres splines

Reference number
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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: Foreword - Supplementary information

The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This first edition of ISO 16610-22 cancels and replaces ISO/TS 16610-22:2006, which has been technically revised.

ISO 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

— *Part 1: Overview and basic concepts*

— *Part 20: Linear profile filters: Basic concepts*

— *Part 21: Linear profile filters: Gaussian filters*

— *Part 22: Linear profile filters: Spline filters*

— *Part 28: Profile filters: End effects*

— *Part 29: Linear profile filters: Spline wavelets*

— *Part 30: Robust profile filters: Basic concepts*

— *Part 31: Robust profile filters: Gaussian regression filters*

— *Part 32: Robust profile filters: Spline filters*

— *Part 40: Morphological profile filters: Basic concepts*

— *Part 41: Morphological profile filters: Disk and horizontal line-segment filters*

— *Part 49: Morphological profile filters: Scale space techniques*

— *Part 60: Linear areal filters: Basic concepts*

— *Part 61: Linear areal filters: Gaussian filters*

— *Part 71: Robust areal filters: Gaussian regression filters*

— *Part 85: Morphological areal filters: Segmentation*

The following parts are planned:

— *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*

— *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*

— *Part 45: Morphological profile filters: Segmentation*

— *Part 62: Linear areal filters: Spline filters*

— *Part 69: Linear areal filters: Spline wavelets*

— *Part 70: Robust areal filters: Basic concepts*

— *Part 72: Robust areal filters: Spline filters*

— *Part 80: Morphological areal filters: Basic concepts*

— *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*

— *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO 16610 is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO/TR 14638). It influences the feature characteristics and measurement chain links 3 and 5 in the GPS matrix structure.

The ISO/GPS Masterplan given in ISO/TR 14638 gives an overview of the ISO/GPS system of which this part of ISO 16610 is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this part of ISO 16610 and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this part of ISO 16610, unless otherwise indicated.

For more detailed information about the relation of this part of ISO 16610 to the GPS matrix model, see Annex F.

This part of ISO 16610 develops the terminology and concepts of spline filters.

The spline filter has an advantage over a conventional phase correct filter in that for open profiles, the ends of the measured profile are still usable.

Geometrical product specifications (GPS) — Filtration —

Part 22:
Linear profile filters: Spline filters

1 Scope

This part of ISO 16610 specifies spline filters for the filtration of profiles. It specifies in particular how to separate the long- and short-wave component of a profile.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1:2015, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic terminology*

ISO 16610-20:2015, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO/IEC Guide 99:2007, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC Guide 99:2007 (VIM), ISO 16610-1, and ISO 16610-20 and the following apply.

3.1
spline
linear combination of piecewise polynomials, with a smooth fit between the pieces

Note 1 to entry: The degree of the spline is equal to the degree of the polynomial of the highest degree used, e.g. a cubic spline is made of cubic polynomials.

3.2
cardinal spline
basic function of the space of splines (3.1) with infinite support

3.3
natural spline
spline (3.1) which is a straight line beyond the end points

3.4
spline profile filter
linear profile filter based on splines (3.1)

Note 1 to entry: For the purposes of this part of ISO 16610, the result of the low-pass filtering is a spline.

4 Spline profile filters

4.1 General

For a spline filter to conform with this part of ISO 16610, it shall satisfy the filter equations given in 4.3.2, for open profiles, and 4.3.3, for closed profiles.

NOTE A comparison with Gaussian filters is given in Annex B, and illustrative examples are given in Annex C. A concept diagram for the concepts of spline filters is given in Annex D, and the relationship to the filtration matrix model is given in Annex E.

4.2 Weighting function

The weighting function of a spline profile filter cannot be given by a simple closed formula. Filter equations are therefore used instead of weighting functions to describe the spline profile filters. However, where necessary, a numerical calculation of the weighting function for a spline profile filter is always possible. If the sampling interval Δx is small enough, and the spline profile filter is based on cardinal cubic splines, the weighting function $s(x)$, with a default value of $\beta = 0$, can be approximated by the continuous function

$$s(x) = \frac{\pi}{\lambda_C} \sin\left(\sqrt{2}\frac{\pi}{\lambda_C}|x| + \frac{\pi}{4}\right) \exp\left(-\sqrt{2}\frac{\pi}{\lambda_C}|x|\right) \quad (1)$$

NOTE This is the weighting function of the ideal spline profile filter, based on cardinal cubic splines. A graph of this function is shown in Figure 1.

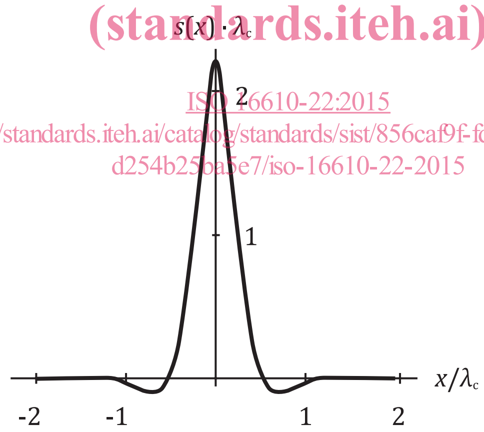


Figure 1 — Weighting function of the ideal spline profile filter, based on cardinal cubic splines

4.3 Filter equations

4.3.1 General

The following filter equations for spline profile filters are based on cardinal cubic splines (for more details, see Reference [4]).

4.3.2 Filter equation for the non-periodic spline profile filter

Non-periodic spline profile filters should be used in cases of filtering open profiles. The filter equation for the non-periodic spline profile filter is given by

$$\left[\boldsymbol{1}+\beta\alpha^{2}\boldsymbol{P}+(1-\beta)\alpha^{4}\boldsymbol{Q}\right]w=z \tag{2}$$

where $\boldsymbol{1}$ is the identity matrix and

$$\boldsymbol{P}=\begin{pmatrix} 1 & -1 & & & & & \\ -1 & 2 & -1 & & & & \\ & -1 & 2 & -1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & -1 & 2 & -1 & & \\ & & & -1 & 2 & -1 & \\ & & & & -1 & 1 & \end{pmatrix} \quad \boldsymbol{Q}=\begin{pmatrix} 1 & -2 & 1 & & & & \\ -2 & 5 & -4 & 1 & & & \\ 1 & -4 & 6 & -4 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & 1 & -4 & 6 & -4 & 1 \\ & & & 1 & -4 & 5 & -2 \\ & & & & 1 & -2 & 1 \end{pmatrix} \tag{3}$$

with n rows, n columns, and the parameters

$$\alpha=\frac{1}{2\sin\frac{\pi\Delta x}{\lambda_{\mathrm{C}}}} \text{ and } 0\le\beta\le 1 \tag{4}$$

where

- n is the number of extracted data values of the profile;
- z is the vector of dimension n of the profile values before filtering;
- w is the vector of dimension n of the profile values in the filtered profile;
- λ_{c} is the cut-off wavelength of the profile filter;
- Δx is the sampling interval.

NOTE 1 The vector w gives the profile values of the long-wave component. The short-wave component r can be obtained by the difference vector $r = z - w$, i.e. by subtracting the long wave-component values obtained by the filtering process from the extracted data values of the profile.

NOTE 2 β is called the tension parameter and controls how tightly the spline curve fits through the data points (see Reference [6]).

NOTE 3 Algorithms to solve matrix equations can be found in Reference [13].

4.3.3 Filter equation for the periodic spline profile filter

Periodic spline profile filters should be used in cases of filtering closed profiles. The filter equation for the periodic spline profile filter is given by

$$\left[\mathbf{1}+\beta\alpha^{2}\tilde{\mathbf{P}}+(1-\beta)\alpha^{4}\tilde{\mathbf{Q}}\right]\tilde{w}=\tilde{z} \tag{5}$$

where $\mathbf{1}$ is the identity matrix and

$$\tilde{\mathbf{P}}=\begin{pmatrix} 2 & -1 & & & & & -1 \\ -1 & 2 & -1 & & & & \\ & -1 & 2 & -1 & & & \\ & \ddots & \ddots & \ddots & \ddots & & \\ & & -1 & 2 & -1 & & \\ & & & -1 & 2 & -1 & \\ -1 & & & & -1 & 2 & \end{pmatrix} \quad \tilde{\mathbf{Q}}=\begin{pmatrix} 6 & -4 & 1 & & & 1 & -4 \\ -4 & 6 & -4 & 1 & & & 1 \\ 1 & -4 & 6 & -4 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & 1 & -4 & 6 & -4 & 1 \\ 1 & & & 1 & -4 & 6 & -4 \\ -4 & 1 & & & 1 & -4 & 6 \end{pmatrix} \tag{6}$$

with n rows, n columns, and the parameters

$$\alpha=\frac{1}{2\sin\frac{\pi\Delta x}{\lambda_{\mathrm{C}}}} \text{ and } 0\le\beta\le 1 \tag{7}$$

where

n is the number of extracted data values of the profile;

$\tilde{z}$ is the vector of dimension n of the profile values before filtering;

$\tilde{w}$ is the vector of dimension n of the profile values in the filtered profile;

λ_{c} is the cut-off wavelength of the profile filter;

Δx is the sampling interval.

NOTE 1 The vector $\tilde{w}$ gives the profile values of the long-wave component. The short-wave component $\tilde{r}$ can be obtained by the difference vector $\tilde{r}=\tilde{z}-\tilde{w}$, i.e. by subtracting the long-wave component values obtained by the filtering process from the extracted data values of the profile.

NOTE 2 β is called the tension parameter and controls how tightly the spline curve fits through the data points (see Reference [6]).

NOTE 3 Algorithms to solve matrix equations can be found in Reference [13].

4.4 Transmission characteristics

4.4.1 General

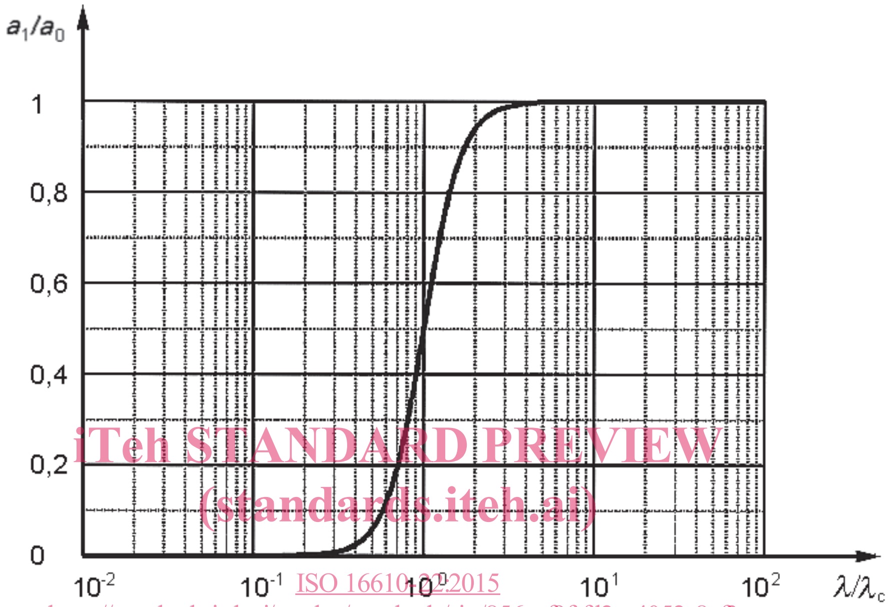
The following transmission characteristics for spline profile filters are based on cardinal cubic splines. For more details, see Reference [3]. For the influence of the sampling interval, see Annex A.

4.4.2 Transmission characteristic of the long-wave profile component

The filter characteristic (see Figure 2) is determined from the filter equations of the spline profile filter by means of the Fourier transformation. The filter characteristic for the long-wave component is approximated (for very small Δx) by the following formula:

$$\frac{a_1}{a_0} = \left[1 + \beta \alpha^2 \sin^2 \frac{\pi \Delta x}{\lambda} + 16(1-\beta)\alpha^4 \sin^4 \frac{\pi \Delta x}{\lambda} \right]^{-1} \tag{8}$$

where



a_0 is the amplitude of the sinusoidal profile before filtering;

a_1 is the amplitude of the sinusoidal profile in the long wave component;

λ is the wavelength of the sine profile ($\lambda \geq 2\Delta x$);

Δx is the sampling interval.

Figure 2 — Transmission characteristic for the long-wave profile component $(\beta = 0; \Delta x = \lambda_c / 200)$

4.4.3 Transmission characteristic of the short-wave profile component

The transmission characteristic of the short-wave profile component is complementary to the transmission characteristic of the long-wave profile component. The short-wave profile component is the difference between the surface profile and the long-wave profile component. The filter characteristic for the short-wave profile component (see Figure 3) has the following formula:

$$\frac{a_2}{a_0} = 1 - \frac{a_1}{a_0} = \left[4\beta\alpha^2 \sin^2 \frac{\pi\Delta x}{\lambda} + 16(1-\beta)\alpha^4 \sin^4 \frac{\pi\Delta x}{\lambda} \right] \left[1 + \beta\alpha^2 \sin^2 \frac{\pi\Delta x}{\lambda} + 16(1-\beta)\alpha^4 \sin^4 \frac{\pi\Delta x}{\lambda} \right]^{-1} \tag{9}$$

where

a_2 is the amplitude of the sinusoidal profile short-wave component.

NOTE The sum of the transmission characteristics of the short-wave profile component and the long-wave profile component equals one.

