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Standard Practice for Process and Measurement Capability Indices[1]

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1. Scope

1.1 This practice provides guidance for the use of capability indices for evaluating process capability and performance. Process capability indices compare the variability of a process quality measure against product specifications or tolerances and assume the process is in a state of statistical control. Process performance indices are useful in situations when the process is not in a state of statistical control.

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E 456 Terminology Relating to Quality and Statistics

E 2586 Practice for Calculating and Using Basic Statistics

2.2 *ISO Standard:*

ISO 3534-2 Statistics—Vocabulary and Symbols-Statistical Quality Control[3]

2.3 *Other Document:*

MNL 7 Manual on Presentation of Data and Control Chart Analysis[4]

3. Terminology

3.1 *Definitions*—Unless otherwise noted, all statistical terms are defined in Terminology E 456.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *average standard deviation,* $\bar{s}$, *n*—arithmetic average of sample standard deviations.

3.2.2 *long term standard deviation,* σ_{LT}, *n*—sample standard deviation of all individual (observed) values taken over a long period of time.

3.2.2.1 *Discussion*—A long period of time may be defined as shifts, weeks, or months, etc.

3.2.3 *lower process capability index,* C_{pkl}, *n*—index describing process capability in relation to the lower specification limit.

3.2.4 *lower process performance index,* P_{pkl}, *n*—index describing process performance in relation to the lower specification limit.

3.2.5 *minimum process capability index,* C_{pk}, *n*—smaller of the upper process capability index and the lower process capability index.

3.2.6 *minimum process performance index,* P_{pk}, *n*—smaller of the upper process performance index and the lower process performance index.

3.2.7 *process capability, PC*, *n*—statistical estimate of the outcome of a characteristic from a process that has been demonstrated to be in a state of statistical control.

3.2.8 *process capability index,* C_p, *n*—an index describing process capability in relation to specified tolerance.

3.2.9 *process performance, PP*, *n*—statistical measure of the outcome of a characteristic from a process that may not have been demonstrated to be in a state of statistical control.

3.2.10 *process performance index,* P_p, *n*—index describing process performance in relation to specified tolerance.

3.2.11 *range, R*, *n*—maximum value minus the minimum value in a sample. **E 2586**

3.2.12 *short term standard deviation,* σ_{ST}, *n*—the inherent variation present when a process is operating in a state of statistical control, expressed in terms of standard deviation.

3.2.12.1 *Discussion*—This may also be stated as the inherent process variation.

3.2.13 *special cause*, *n*—source of intermittent variation in a process. **ISO 3534-2**

3.2.13.1 *Discussion*—Sometimes "special cause" is taken to be synonymous with "assignable cause." However a distinction should be recognized. A special cause is assignable only when it is specifically identified. Also a common cause may be assignable.

3.2.13.2 *Discussion*—A special cause arises because of specific circumstances which are not always present. As such, in a process subject to special causes, the magnitude of the variation from time to time is unpredictable.

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[1] This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.30 on Statistical Quality Control.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, http://www.ansi.org.

[4] Available from ASTM Headquarters, 100 Barr Harbor Drive, W. Conshohocken, PA 19428.

3.2.14 *stable process*, *n*—process in a state of statistical control; process condition when all special causes of variation have been removed. **ISO 3534-2**

3.2.14.1 *Discussion*—Observed variation can then be attributed to random (common) causes. Such a process will generally behave as though the results are simple random samples from the same population.

3.2.14.2 *Discussion*—This state does not imply that the random variation is large or small, within or outside of specification, but rather that the variation is predictable using statistical techniques.

3.2.14.3 *Discussion*—The process capability of a stable process is usually improved by fundamental changes that reduce or remove some of the random causes present and/or adjusting the mean towards the preferred value.

3.2.14.4 *Discussion*—Continual adjustment of a stable process will increase variation.

3.2.15 *upper process capability index,* C_{pku}, *n*—index describing process capability in relation to the upper specification limit.

3.2.16 *upper process performance index* (P_{pku}), *n*—index describing process performance in relation to the upper specification limit.

4. Significance and Use

4.1 *Process Capability*—Process capability can be defined as the natural or inherent behavior of a stable process that is in a state of statistical control (**1**).[5] A "state of statistical control" is achieved when the process exhibits no detectable patterns or trends, such that the variation seen in the data is believed to be random and inherent to the process. Process capability is linked to the use of control charts and the state of statistical control. A process must be studied to evaluate its state of control before evaluating process capability.

4.2 *Process Control*—There are many ways to implement control charts, but the most popular choice is to achieve a state of statistical control for the process under study. Special causes are identified by a set of rules based on probability theory. The process is investigated whenever the chart signals the occurrence of special causes. Taking appropriate actions to eliminate identified special causes and preventing their reappearance will ultimately obtain a state of statistical control. In this state, a minimum level of variation may be reached, which is referred to as common cause or inherent variation. For the purpose of this standard, this variation is a measure of the uniformity of process output, typically a product characteristic.

4.3 *Process Capability Indices*—The behavior of a process (as related to inherent variability) in the state of statistical control is used to describe its capability. To compare a process with customer requirements (or specifications), it is common practice to think of capability in terms of the proportion of the process output that is within product specifications or tolerances. The metric of this proportion is the percentage of the process spread used up by the specification. This comparison becomes the essence of all process capability measures. The manner in which these measures are calculated defines the different types of capability indices and their use. Two process capability indices are defined in 5.2 and 5.3. In practice, these indices are used to drive process improvement through continuous improvement efforts. These indices may be used to identify the need for management actions required to reduce common cause variation, compare products from different sources, and to compare processes.

4.4 *Process Performance Indices*—When a process is not in a state of statistical control, the process is subject to special cause variation, which can manifest itself in various ways on the process variability. Special causes can give rise to changes in the short-term variability of the process or can cause long-term shifts or drifts of the process mean. Special causes can also create transient shifts or spikes in the process mean. Even in such cases, there may be a need to assess the long-term variability of the process against customer specifications using process performance indices, which are defined in 6.2 and 6.3. These indices are similar to those for capability indices and differ only in the estimate of variability used in the calculation. This estimated variability includes additional components of variation due to special causes. Since process performance indices have additional components of variation, process performance usually has a wider spread than the process capability spread. These measures are useful in determining the role of measurement and sampling variability when compared to product uniformity.

5. Process Capability Analysis

5.1 It is common practice to define process behavior in terms of its variability. Process capability, PC, is calculated as:

$$PC = 6\sigma_{ST} \quad (1)$$

where σ_{ST} is the inherent variability of a controlled process (**2,7**). Since control charts can be used to achieve and verify control for many different types of processes, the assumption of a normal distribution is not necessary to affect control, but complete control is required to establish the capability of a process (**2**). Thus, what is required is a process in control with respect to its measures of location and spread. Once this is achieved, the inherent variability of the process can be estimated from the control charts. The estimate obtained is an estimate of variability over a short time interval (minutes, hours, or a few batches). From control charts, σ_{ST} may be estimated from the short-term variation within subgroups depending on the type of control chart deployed, for example, average-range ($\bar{X} - R$) or individual-moving range ($\bar{X} - MR$). The estimate is:

$$\hat{\sigma}_{ST} = \frac{\bar{R}}{d_2} \text{ or } \frac{\overline{MR}}{d_2} \quad (2)$$

where, $\bar{R}$ is the average range, $\overline{MR}$ is the average moving range, d_2 is a factor dependent on the subgroup size, *n*, of the control chart, (see ASTM MNL 7, Part 3). If an average-standard deviation ($\bar{X} - s$) chart is used, the estimate becomes:

$$\hat{\sigma}_{ST} = \frac{\bar{s}}{c_4} \quad (3)$$

[5] The boldface numbers in parentheses refer to the list of references at the end of this standard.

where $\bar{s}$ is the average standard deviation, and c_4 is a factor dependent on the subgroup size, *n*, of the control chart, (see ASTM MNL 7, Part 3).

5.1.1 Therefore, PC is estimated by:

$$6\,\hat{\sigma}_{ST} = \frac{6\bar{R}}{d_2} \text{ or } \frac{6\bar{s}}{c_4} \tag{4}$$

5.2 *Process Capability Index, C_P:*

5.2.1 The process capability index relates the process capability to the customer's specification tolerance. The process capability index, C_p, is:

$$C_p = \frac{\text{Specification Tolerance}}{\text{Process Capability}} = \frac{USL - LSL}{6\sigma_{ST}} \tag{5}$$

where USL = upper specification limit and LSL = lower specification limit. For a process that is centered with an underlying normal distribution, Fig. 1, Fig. 2, and Fig. 3 denotes three cases where PC, the process capability, is wider than (Fig. 1), equal to (Fig. 2), and narrower than (Fig. 3) the specification tolerance.

5.2.2 Since the tail area of the distribution beyond specification limits measures the proportion of product defectives, a larger value of C_p is better. The relationship between C_p and the percent defective product produced by a centered process (with a normal distribution) is:

C_p	Percent Defective	Parts per Million	C_p	Percent Defective	Parts per Million
0.6	7.19	71900	1.1	0.0967	967
0.7	3.57	35700	1.2	0.0320	318
0.8	1.64	16400	1.3	0.0096	96
0.9	0.69	6900	1.33	0.00636	64
1.0	0.27	2700	1.67	0.00006	0.57

5.2.3 From these examples, one can see that any process with a C_p< 1 is not as capable of meeting customer requirements (as indicated by % defectives) as a process with values of $C_p \geq$ 1. Values of C_p progressively greater than 1 indicate more capable processes. The current focus of modern quality is on process improvement with a goal of increasing product uniformity about a target. The implementation of this focus is to create processes with C_p> 1. Some industries consider C_p= 1.33 (an $8\sigma_{ST}$ specification tolerance) a minimum with a C_p= 1.66 preferred (**3**). Improvement of C_p should depend on a company's quality focus, marketing plan, and their competitor's achievements, etc.

5.3 *Process Capability Indices Adjusted For Process Shift, C_{pk}:*

5.3.1 The above examples depict process capability for a process centered within its specification tolerance. Process centering is not a requirement since process capability is independent of any specifications that may be applied to it. The amount of shift present in a process depends on how far the process average is from the center of the specification spread. In the last part of the above example (C_p > 1), suppose that the process is actually centered above the USL. The C_p has a value >1, but clearly this process is not producing as much conforming product as it would have if it were centered on target.

5.3.2 For those cases where the process is not centered, deliberately run off-center for economic reasons, or only a single specification limit is involved, C_p is not the appropriate process capability index. For these situations, the C_{pk} index is used. C_{pk} is a process capability index that considers the process average against a single or double-sided specification limit. It measures whether the process is capable of meeting the customer's requirements by considering:

5.3.2.1 The specification limit(s),

5.3.2.2 The current process average, and

5.3.2.3 The current $\hat{\sigma}_{ST}$

5.3.3 Under the assumption of normality,[6] C_{pk} is calculated as:

$$C_{pk} = \min[C_{pku}, C_{pkl}] \tag{6}$$

and is estimated by:

$$\hat{C}_{pk} = \min[\hat{C}_{pku}, \hat{C}_{pkl}] \tag{7}$$

where the estimated upper process capability index is defined as:

$$\hat{C}_{pku} = \frac{USL - \bar{X}}{3\,\hat{\sigma}_{ST}} \tag{8}$$

and the estimated lower process capability index is defined as:

$$\hat{C}_{pkl} = \frac{\bar{X} - LSL}{3\,\hat{\sigma}_{ST}} \tag{9}$$

5.3.4 These one-sided process capability indices (C_{pku} and C_{pkl}) are useful in their own right with regard to single-sided

[6] Testing for the normality of a set of data may range from simply plotting the data on a normal probability plot (2) to more formal tests, e.g., Anderson-Darling test (which can be found in many statistical software programs, for example, Minitab).

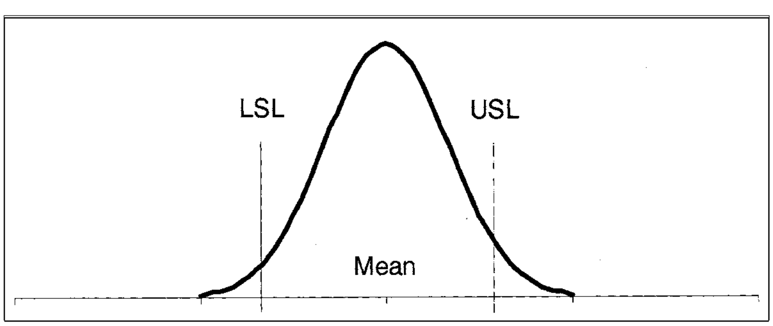

FIG. 1 Process Capability Wider Than Specifications, C_p< 1

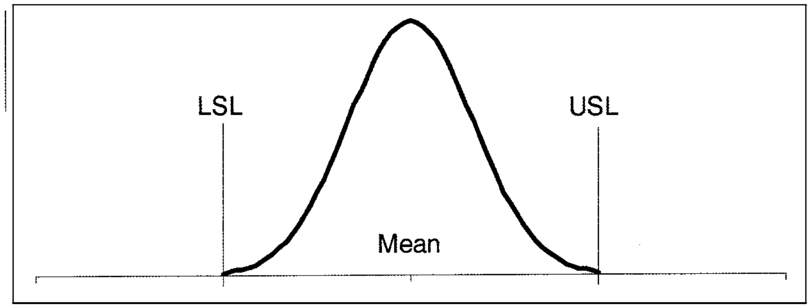

FIG. 2 Process Capability Equal to Specification Tolerance, $C_p = 1$

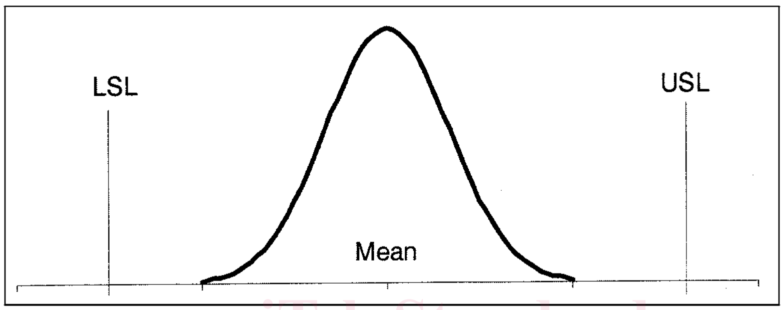

FIG. 3 Process Capability Narrower Than Specifications, $C_p > 1$

specification limits. Examples of this type of use would apply to impurities, by-products, bursting strength of bottles, etc. Once again, the meaning of C_{pk} is best viewed pictorially in Fig. 4.

5.3.5 The relationship between C_p and C_{pk} can be summarized (**2**) as:

5.3.5.1 C_{pk} can be equal to but never larger than C_p,

5.3.5.2 C_p and C_{pk} are equal only when the process is centered on target,

5.3.5.3 If C_p is larger than C_{pk}, then the process is not centered on target,

5.3.5.4 If both C_p and C_{pk} are >1, the process is capable and performing within the specifications,

5.3.5.5 If both C_p and C_{pk} are <1, the process is not capable and not performing within the specifications, and

5.3.5.6 If C_p is >1 and C_{pk} is <1, the process is capable, but not centered and not performing within the specifications.

5.4 *Caveats on the Practical Use of Process Capability Indices*:

5.4.1 One must keep the theoretical aspects and assumptions underlying the use of process capability indices in mind when calculating and interpreting the corresponding values of these indices. To review:

5.4.1.1 For interpretability, C_{pk} requires a Gaussian (normal or bell-shaped) distribution or one that can be transformed to a normal. Definition of C_{pk} requires a normal distribution with a spread of three standard deviations on either side of the mean (**2**,**4**).

5.4.1.2 The process must be in a state of statistical control (stable over time with constant short-term variability).

5.4.1.3 Large sample sizes (preferably >200 or a minimum of 100) are required to estimate C_{pk} with a high level of confidence (at least 95%).

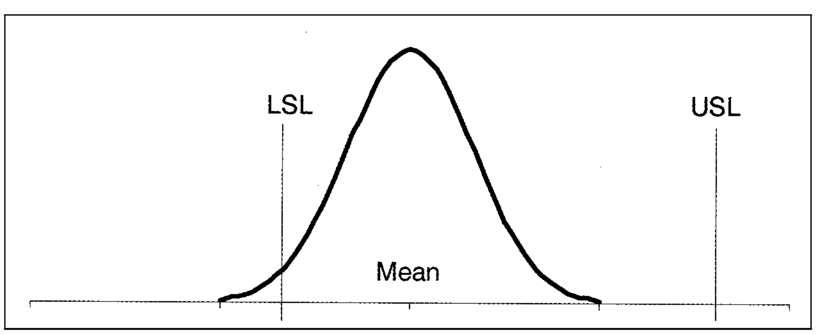

FIG. 4 Noncentered Process, $C_p > 1$ and $C_{pk} < 1$