UDC 53.081

Ref. No.: ISO/R 31/Part XI - 1961 (E)

ISO

INTERNATIONAL ORGANIZATION FOR STANDARDIZATION

ISO RECOMMENDATION
R 31
PART XI

MATHEMATICAL SIGNS AND SYMBOLS FOR USE IN PHYSICAL SCIENCES AND TECHNOLOGY

1st EDITION

February 1961

Printed in Denmark

Also issued in French and Russian. Copies to be obtained through the national standards organizations.

BRIEF HISTORY

The ISO Recommendation R 31, Part XI, *Mathematical Signs and Symbols for Use in the Physical Sciences and Technology,* was drawn up by Technical Committee ISO/TC 12, *Quantities, Units, Symbols, Conversion Factors and Conversion Tables,* the Secretariat of which is held by the Danish Standards Association, Dansk Standardiseringsråd (DS).

The ISO/TC 12 Secretariat drew up two drafts in succession, the second of which was studied by the Technical Committee during its fourth meeting, held in Copenhagen in November 1957. The Secretariat then prepared a third draft proposal, which was submitted by correspondence to the Members of the Technical Committee and was approved as a Draft ISO Recommendation.

It should be noted that the following international organizations have had these draft proposals sent to them and have taken part in the discussions at the meetings of Technical Committee ISO/TC 12:

International Commission on Illumination
International Committee on Weights and Measures
International Electrotechnical Commission
International Union of Pure and Applied Chemistry
International Union of Pure and Applied Physics
and its Sub-Committee SUN
Organisation Internationale de Métrologie Légale.

On 29 May 1959, the Draft ISO Recommendation (No. 287) was distributed to all the ISO Member Bodies and was approved by the following Member Bodies:

Australia	Greece	Poland
Austria	Hungary	Portugal
Belgium	India	Romania
Burma	Ireland	Spain
Canada	Japan	Sweden
Chile	Netherlands	Switzerland
Denmark	New Zealand	U.S.A.
France	Norway	U.S.S.R.
Germany	Pakistan	

Three Member Bodies opposed the approval of the Draft:
Czechoslovakia, Italy, United Kingdom.

The Draft ISO Recommendation was then submitted by correspondence to the ISO Council, which decided, in February 1961, to accept it as an ISO RECOMMENDATION.

Mathematical Signs and Symbols for Use in the Physical Sciences and Technology

Introduction

This document, containing a selection of *mathematical signs and symbols* recommended for use in the physical sciences and technology, is part of a more comprehensive publication dealing with quantities and units in various fields of science and technology.

This publication is issued as ISO Recommendation R 31/Part I, Part II, etc.

Where several signs or symbols with the same meaning are given, they are on an equal footing.

No recommendation is made or implied about the fount of upright type in which certain mathematical symbols are printed. In this document the same fount is used as in the colomns "Meaning" and "Remarks".

Some principles for printing symbols and numbers are given in ISO Recommendation R 31/Part XII *)

1. Arithmetic, algebra and analysis of real scalar quantities

Item No.	Sign or Symbol	Meaning	Remarks
1.1	$=$	equal to	
1.2	$\neq$	not equal to	
1.3	$\equiv$	identically equal to	It is recommended not to use this sign with the meaning "defined as equal to".
1.4	$\mathrel{\widehat{=}}$	corresponds to	Examples: On the basis of the equation $E = mc^2$, 1 g $\mathrel{\widehat{=}}$ 9×10^{20} erg When 1 cm on a map corresponds to a length of 10 km one may write 1 cm $\mathrel{\widehat{=}}$ 10 km
1.5	$\approx$	approximately equal to	
1.6	$\rightarrow$	approaches	
1.7	$\simeq$	asymptotically equal to	$\sim$ is also used
1.8	$\sim$	proportional to	$\propto$ is also used
1.9	∞	infinity	
1.10	$<$	smaller than	
1.11	$>$	larger than	
1.12	$\leqq$ $\leqslant$ $\leq$	smaller than or equal to	
1.13	$\geqq$ $\geqslant$ $\geq$	larger than or equal to	
1.14	$\ll$	much smaller than	
1.15	$\gg$	much larger than	
1.16	$+$	plus	

*) Draft ISO Recommendation No. 345

Item No.	Sign or Symbol	Meaning	Remarks
1.17	$-$	minus	See also ISO/R 31/Part XII*)
1.18	$\cdot \quad \times \quad .$	multiplied by	
1.19	$\frac{a}{b} \quad a/b$	a divided by b	$a:b$ is also used
1.20	$\lvert a \rvert$	magnitude of a	
1.21	a^n	a raised to the power n	
1.22	$a^{1/2} \quad a^{\frac{1}{2}} \quad \sqrt{a} \quad \sqrt{a}$	square root of a	When the sign $\sqrt{}$ or $\sqrt[n]{}$ is to operate on a composite expression, brackets should be used.
1.23	$a^{1/n} \quad a^{\frac{1}{n}} \quad \sqrt[n]{a} \quad \sqrt[n]{a}$	n'th root of a	
1.24	$\bar{a} \quad \langle a \rangle$	mean value of a	The meaning of "mean value" is not completely defined until the averaging procedure is specified.
1.25	$p!$	factorial p, $1 \times 2 \times 3 \times \ldots \times p$	p is a positive integer
1.26	$\binom{n}{p}$	binomial coefficient, $\frac{n\,(n-1)\ldots(n-p+1)}{1 \times 2 \times \ldots \times p}$	
1.27	Σ	sum	
1.28	$\prod$	product	
1.29	$f(x) \quad \mathrm{f}(x)$	function f (or f) of the variable x	
1.30	$[f(x)]_a^b \quad f(x)\Big\rvert_a^b$	$f(b) - f(a)$	
1.31	$\lim_{x \to a} f(x) \quad \lim_{x \to a} f(x)$	the limit to which $f(x)$ tends as x approaches a	
1.32	Δx	delta x = finite increment of x	
1.33	δx	delta x = variation of x	
1.34	$\frac{df}{dx} \quad \mathrm{d}f/\mathrm{d}x \quad f'(x)$	differential coefficient of $f(x)$ with respect to x	Differentiation of a quantity with respect to time is also indicated by placing a dot above the symbol for the quantity: $\mathrm{d}s/\mathrm{d}t = \dot{s}$
1.35	$\frac{d^n f}{dx^n} \quad f^{(n)}(x)$	differential coefficient of order n of $f(x)$	
1.36	$\frac{\partial f(x, y, \ldots)}{\partial x} \quad \left(\frac{\partial f}{\partial x}\right)_{y, \ldots}$	partial differential coefficient of $f(x, y, \ldots)$ with respect to x, when $y, \ldots$ are held constant	$f_x(x, y, \ldots)$ and $f'_x(x, y, \ldots)$ are also used
1.37	$\mathrm{d}f$	the total differential of f	Example: $\mathrm{d}f(x,y) = \left(\frac{\partial f}{\partial x}\right)_y \mathrm{d}x + \left(\frac{\partial f}{\partial y}\right)_x \mathrm{d}y$
1.38	$\int f(x)\,\mathrm{d}x$	indefinite integral of $f(x)$ with respect to x	
1.39	$\int_a^b f(x)\,\mathrm{d}x \quad \int_a^b f(x)\,\mathrm{d}x$	definite integral of $f(x)$ from $x = a$ to $x = b$	$\int_a^b f(x)\,\mathrm{d}x = \left(\int f(x)\mathrm{d}x\right)_{x=b} - \left(\int f(x)\mathrm{d}x\right)_{x=a}$ For integration over a closed curve, $\oint$ is used
1.40	e	base of natural logarithms	e is also used
1.41	$\mathrm{e}^x \quad \exp x$	e raised to the power x	
1.42	$\log_a x$	logarithm to the base a of x	
1.43	$\ln x \quad \log_e x$	natural logarithm (Napierian logarithm) of x	
1.44	$\lg x \quad \log x \quad \log_{10} x$	common (Briggsian) logarithm of x	
1.45	$\mathrm{lb}\ x \quad \log_2 x$	binary logarithm of x	

*) Some Principles for Printing Symbols and Numbers (Draft ISO Recommendation No. 345)

Item No.	Sign or Symbol	Meaning	Remarks
1.46	sin x	sine of x	
1.47	cos x	cosine of x	
1.48	tan x, tg x	tangent of x	
1.49	cot x, ctg x	cotangent of x, $\frac{1}{\tan x}$	cotg x is also used
1.50	sec x	secant of x, $\frac{1}{\cos x}$	
1.51	cosec x	cosecant of x, $\frac{1}{\sin x}$	The functions 1.46 to 1.51 are called circular functions. See also remark which follows 1.57
1.52	arcsin x	arc sine of x	Sometimes $\sin^{-1} x$ is used
1.53	arccos x	arc cosine of x	Sometimes $\cos^{-1} x$ is used
1.54	arctan x, arctg x	arc tangent of x	Sometimes $\tan^{-1} x$ is used
1.55	arccot x, arcctg x	arc cotangent of x	Sometimes arccotg x or $\cot^{-1} x$ is used
1.56	arcsec x	arc secant of x	Sometimes $\sec^{-1} x$ is used
1.57	arccosec x	arc cosecant of x	Sometimes $\mathrm{cosec}^{-1} x$ is used The notation $\sin^{-1} x$ etc. is derived from the fact that these functions are the inverse of the circular functions. Nevertheless, generally $\sin^n x = (\sin x)^n$, etc.
1.58	sinh x	hyperbolic sine of x	Sometimes sh x is used
1.59	cosh x	hyperbolic cosine of x	Sometimes ch x is used
1.60	tanh x	hyperbolic tangent of x	Sometimes th x is used
1.61	coth x	hyperbolic cotangent of x	
1.62	sech x	hyperbolic secant of x	
1.63	cosech x	hyperbolic cosecant of x	The functions 1.58 to 1.63 are called hyperbolic functions. See also remark which follows 1.69
1.64	arsinh x	inverse hyperbolic sine of x	Sometimes $\sinh^{-1} x$ or arg sh x is used
1.65	arcosh x	inverse hyperbolic cosine of x	Sometimes $\cosh^{-1} x$ or arg ch x is used
1.66	artanh x	inverse hyperbolic tangent of x	Sometimes $\tanh^{-1} x$ or arg th x is used
1.67	arcoth x	inverse hyperbolic cotangent of x	Sometimes $\coth^{-1} x$ or arg coth x is used
1.68	arsech x	inverse hyperbolic secant of x	Sometimes $\mathrm{sech}^{-1} x$ is used
1.69	arcosech x	inverse hyperbolic cosecant of x	Sometimes $\mathrm{cosech}^{-1} x$ is used The notation $\sinh^{-1} x$ etc. is derived from the fact that these functions are the inverse of the hyperbolic functions. Nevertheless, generally $\sinh^n x = (\sinh x)^n$, etc.

2. Arithmetic, algebra and analysis of complex scalar quantities

(in addition to the signs and symbols listed in 1.)

Item No.	Sign or Symbol	Meaning	Remarks
2.1	i, j	imaginary unity, $i^2 = -1$	In electrical technology j is generally used
2.2	Re z	real part of z	$z = \mathrm{Re}\ z + \mathrm{i}\ \mathrm{Im}\ z$
2.3	Im z	imaginary part of z	
2.4	$\lvert z \rvert$	modulus of z	$z = \lvert z \rvert\, e^{\mathrm{i}\arg z}$
2.5	arg z	argument of z	arc z is also used $\arg \mathrm{i} = \pi/2$
2.6	z^*	conjugate of z, complex conjugate of z	$zz^* = \lvert z \rvert^2$ Sometimes $\bar{z}$ is used instead of z^*; but see item 1.24

3. Matrices

Item No.	Sign or Symbol	Meaning	Remarks
3.1	$\tilde{A}$	transpose of matrix A	$\tilde{A}$ is obtained from A by interchanging rows with columns
3.2	A^*	complex conjugate matrix of matrix A	Each element of A^* is the complex conjugate of the corresponding element of A
3.3	$A^\dagger$	Hermitian conjugate matrix of matrix A	$A^\dagger = \tilde{A}^*$

4. Algebra and analysis of vectors and other tensors

Item No.	Sign or Symbol	Meaning	Remarks
4.1	$\boldsymbol{A}$, $\boldsymbol{a}$	vector	$\vec{A}$ and $\mathfrak{A}$ are also used
4.2	$\lvert \boldsymbol{A} \rvert$, A	magnitude of vector	Also called absolute value
4.3	$\boldsymbol{A} \cdot \boldsymbol{B}$	scalar product	
4.4	$\boldsymbol{A} \times \boldsymbol{B}$, $\boldsymbol{A} \wedge \boldsymbol{B}$	vector product	
4.5	∇	differential vector operator	This sign is called nabla; $\frac{\partial}{\partial \boldsymbol{r}}$ is also used
4.6	$\nabla\varphi$, grad φ	gradient of φ	**grad** φ is also used
4.7	$\nabla \cdot \boldsymbol{A}$, div $\boldsymbol{A}$	divergence of $\boldsymbol{A}$	
4.8	$\nabla \times \boldsymbol{A}$, $\nabla \wedge \boldsymbol{A}$, curl $\boldsymbol{A}$, rot $\boldsymbol{A}$	curl of $\boldsymbol{A}$	**curl** $\boldsymbol{A}$ and **rot** $\boldsymbol{A}$ are also used
4.9	$\nabla^2\varphi$, $\Delta\varphi$	Laplacian of φ	
4.10	When second order tensors are represented by single letters, sans serif type may be reserved for that purpose: e. g. A, B		
4.11	Tensors of any order may be represented by letter symbols having indices, e. g. g_{ik}, g_i^k, g^{ik}		