
Bevel and hypoid gear geometry

Géométrie des engrenages coniques et hypoïdes

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

The committee responsible for this document is ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

This second edition ~~cancels and replaces the first edition (ISO 23509:2006)~~, which has been technically revised with the following changes: www.iso.org/iso/23509-2016

- minor corrections of several formulae;
- the figures have been reworked;
- explanations have been added in [4.4](#);
- the structure of [Formula \(129\)](#) has been changed to cover the case $\zeta_m = 0^\circ$;
- a formula for the calculation of c_{be2} has been added as [Formula \(F.160\)](#);
- the values for α_{nC} and α_{nD} in [Formulae \(F.318\)](#) and [\(F.319\)](#) have been extended to three decimal digits to prevent rounding errors.

Introduction

For many decades, information on bevel, and especially hypoid, gear geometry has been developed and published by the gear machine manufacturers. It is clear that the specific formulae for their respective geometries were developed for the mechanical generation methods of their particular machines and tools. In many cases, these formulae could not be used in general for all bevel gear types. This situation changed with the introduction of universal, multi-axis, CNC-machines, which in principle are able to produce nearly all types of gearing. The manufacturers were, therefore, asked to provide CNC programs for the geometries of different bevel gear generation methods on their machines.

This document integrates straight bevel gears and the three major design generation methods for spiral bevel gears into one complete set of formulae. In only a few places do specific formulae for each method have to be applied. The structure of the formulae is such that they can be programmed directly, allowing the user to compare the different designs.

The formulae of the three methods are developed for the general case of hypoid gears and to calculate the specific case of spiral bevel gears by entering zero for the hypoid offset. Additionally, the geometries correspond such that each gear set consists of a generated or non-generated wheel without offset and a pinion which is generated and provided with the total hypoid offset.

An additional objective of this document is that, on the basis of the combined bevel gear geometries, an ISO hypoid gear rating system can be established in the future.

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Bevel and hypoid gear geometry

1 Scope

This document specifies the geometry of bevel gears.

The term bevel gears is used to mean straight, spiral, zerol bevel and hypoid gear designs. If the text pertains to one or more, but not all, of these, the specific forms are identified.

The manufacturing process of forming the desired tooth form is not intended to imply any specific process, but rather to be general in nature and applicable to all methods of manufacture.

The geometry for the calculation of factors used in bevel gear rating, such as ISO 10300 (all parts), is also included.

This document is intended for use by an experienced gear designer capable of selecting reasonable values for the factors based on his/her knowledge and background. It is not intended for use by the engineering public at large.

[Annex A](#) provides a structure for the calculation of the methods provided in this document.

2 Normative references

There are no normative references in this document.

3 Terms, definitions and symbols

For the purposes of this document, the terms and definitions given in ISO 1122-1 and the following apply.

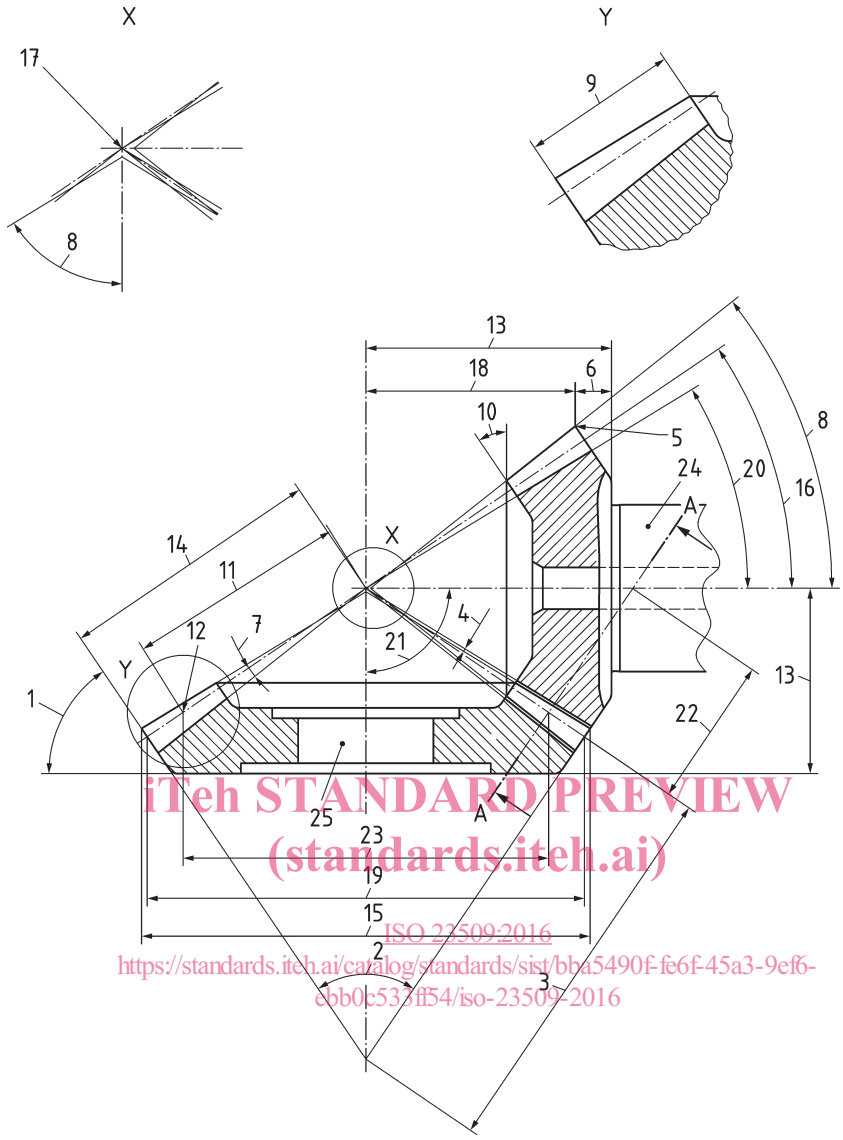
ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <http://www.iso.org/obp>

NOTE 1 The symbols, terms and definitions used in this document are, wherever possible, consistent with other International Standards. It is known, because of certain limitations, that some symbols, their terms and definitions, as used in this document, are different from those used in similar literature pertaining to spur and helical gearing.

NOTE 2 Bevel gear nomenclature used throughout this document is illustrated in [Figure 1](#), the axial section of a bevel gear, and in [Figure 2](#), the mean transverse section. Hypoid nomenclature is illustrated in [Figure 3](#).

Subscript 1 refers to the pinion and subscript 2 to the wheel.

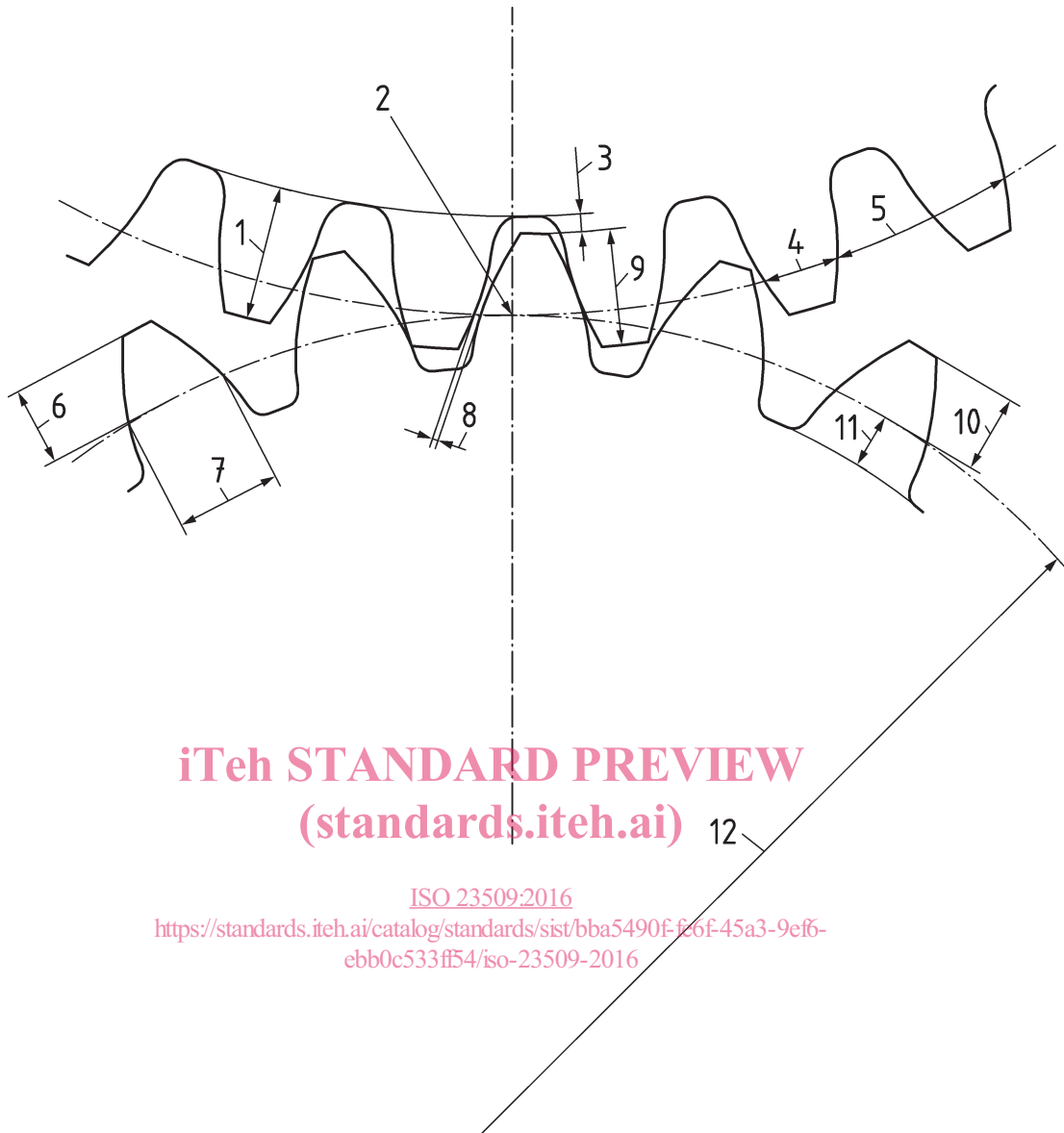


Key

- | | | |
|--|--|---|
| 1 back angle | 10 front angle | 19 outer pitch diameter, d_{e1}, d_{e2} |
| 2 back cone angle | 11 mean cone distance, R_m | 20 root angle, δ_{f1}, δ_{f2} |
| 3 back cone distance | 12 mean point | 21 shaft angle, Σ |
| 4 clearance, c | 13 mounting distance | 22 equivalent pitch radius |
| 5 crown point | 14 outer cone distance, R_e | 23 mean pitch diameter, d_{m1}, d_{m2} |
| 6 crown to back | 15 outside diameter, d_{ae1}, d_{ae2} | 24 pinion |
| 7 dedendum angle, θ_{f1}, θ_{f2} | 16 pitch angle, δ_1, δ_2 | 25 wheel |
| 8 face angle δ_{a1}, δ_{a2} | 17 pitch cone apex | |
| 9 face width, b | 18 crown to crossing point, t_{x01}, t_{x02} | |

NOTE See [Figure 2](#) for mean transverse section, A-A.

Figure 1 — Bevel gear nomenclature — Axial plane



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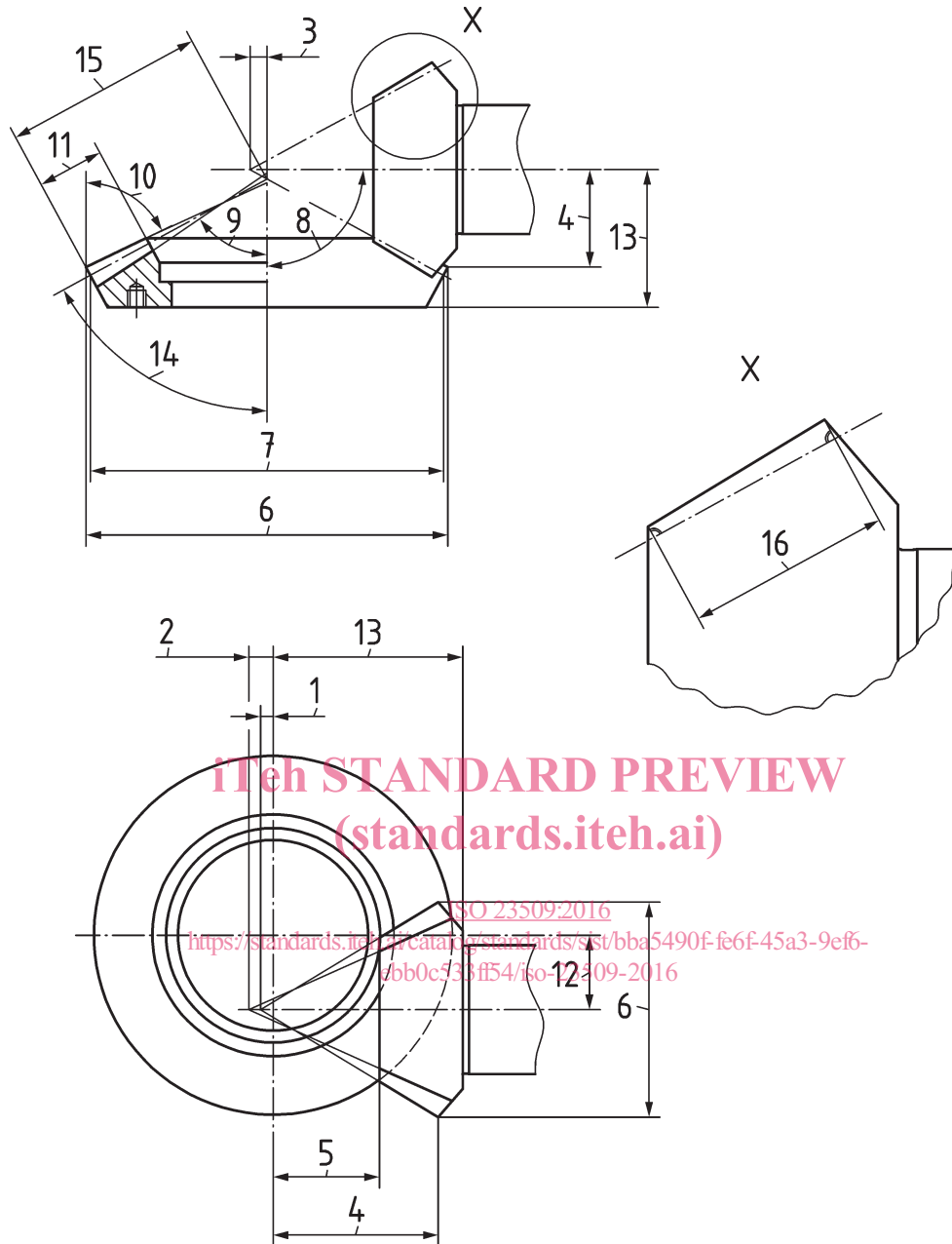
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Key

1	whole depth, h_m	5	circular pitch	9	working depth, h_{mw}
2	pitch point	6	chordal addendum	10	addendum, h_{am}
3	clearance, c	7	chordal thickness	11	dedendum, h_{fm}
4	circular thickness	8	backlash	12	equivalent pitch radius

NOTE See A-A in [Figure 1](#).

Figure 2 — Bevel gear nomenclature — Mean transverse section



Key

- | | | | | | |
|---|---|----|---|----|----------------------------|
| 1 | face apex beyond crossing point, t_{zF1} | 7 | outer pitch diameter, d_{e1}, d_{e2} | 13 | mounting distance |
| 2 | root apex beyond crossing point, t_{zR1} | 8 | shaft angle, Σ | 14 | pitch angle, δ_2 |
| 3 | pitch apex beyond crossing point, t_{z1} | 9 | root angle, δ_{f1}, δ_{f2} | 15 | outer cone distance, R_e |
| 4 | crown to crossing point, t_{x01}, t_{x02} | 10 | face angle of blank, δ_{a1}, δ_{a2} | 16 | pinion face width, b_1 |
| 5 | front crown to crossing point, t_{xi1} | 11 | wheel face width, b_2 | | |
| 6 | outside diameter, d_{ae1}, d_{ae2} | 12 | hypoid offset, a | | |

NOTE Apex beyond crossing point values are positive when crossing point lies inside the respective cone.

Figure 3 — Hypoid nomenclature

3.1 Terms and definitions

3.1.1

mean chordal addendum

h_{amc1}, h_{amc2}

height from the top of the gear tooth to the chord subtending the circular thickness arc at the mean cone distance in a plane normal to the tooth face

3.1.2

mean addendum

h_{am1}, h_{am2}

height by which the gear tooth projects above the pitch cone at the mean cone distance

3.1.3

outer normal backlash allowance

j_{en}

amount by which the tooth thicknesses are reduced to provide the necessary backlash in assembly

Note 1 to entry: It is specified at the outer cone distance.

3.1.4

coast side

<by normal convention> convex pinion flank in mesh with the concave wheel flank

3.1.5

cutter radius

r_{c0}

nominal radius of the face type cutter or cup-shaped grinding wheel that is used to cut or grind the spiral bevel teeth

3.1.6

sum of dedendum angles

$\Sigma\theta_f$

sum of the pinion and wheel dedendum angles

3.1.7

sum of constant slot width dedendum angles

$\Sigma\theta_{fC}$

sum of dedendum angles for constant slot width

3.1.8

sum of modified slot width dedendum angles

$\Sigma\theta_{fM}$

sum of dedendum angles for modified slot width taper

3.1.9

sum of standard depth dedendum angles

$\Sigma\theta_{fS}$

sum of dedendum angles for standard depth taper

3.1.10

sum of uniform depth dedendum angles

$\Sigma\theta_{fU}$

sum of dedendum angles for uniform depth

3.1.11

mean dedendum

h_{fm1}, h_{fm2}

depth of the tooth space below the pitch cone at the mean cone distance

3.1.12
mean whole depth

h_m
tooth depth at mean cone distance

3.1.13
mean working depth

h_{mw}
depth of engagement of two gears at mean cone distance

3.1.14
direction of rotation

direction determined by an observer viewing the gear from the back looking towards the pitch apex

3.1.15
drive side

by normal convention, concave pinion flank in mesh with the convex wheel flank

3.1.16
face width

b
length of the teeth measured along a pitch cone element

3.1.17
mean addendum factor

c_{ham}
apportions the mean working depth between wheel and pinion mean addendums

Note 1 to entry: The gear mean addendum is equal to c_{ham} times the mean working depth.

3.1.18
mean radius of curvature

$\rho_{m\beta}$
radius of curvature of the tooth surface in the lengthwise direction at the mean cone distance

3.1.19
number of blade groups

z_0
number of blade groups contained in the circumference of the cutting tool

3.1.20
number of teeth

z_1, z_2
number of teeth contained in the whole circumference of the pitch cone

3.1.21
number of crown gear teeth

z_p
number of teeth in the whole circumference of the crown gear

Note 1 to entry: The number may not be an integer.

3.1.22
mean normal chordal tooth thickness

s_{mnc1}, s_{mnc2}
chordal thickness of the gear tooth at the mean cone distance in a plane normal to the tooth trace

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3.1.23 mean normal circular tooth thickness

s_{mn1}, s_{mn2}

length of arc on the pitch cone between the two sides of the gear tooth at the mean cone distance in the plane normal to the tooth trace

3.1.24 tooth trace

curve of the tooth on the pitch surface

3.1.25 mean point

point where the calculation of basic geometry is executed

Note 1 to entry: Mean point does not necessarily coincide with middle point of face width.

Note 2 to entry: In all the methods listed in this document, the term “mean point” refers to “calculation point”. See [A.3](#) for calculation points.

3.2 Symbols

Table 1 — Symbols used in this document

Symbol	Description	Unit
a	hypoid offset	mm
b_1, b_2	face width	mm
b_{e1}, b_{e2}	face width from calculation point to outside	mm
b_{i1}, b_{i2}	face width from calculation point to inside	mm
c	clearance	mm
c_{be2}	face width factor	—
c_{ham}	mean addendum factor of wheel	—
d_{ae1}, d_{ae2}	outside diameter	mm
d_{e1}, d_{e2}	outer pitch diameter	mm
d_{m1}, d_{m2}	mean pitch diameter	mm
F_{ax}	axial force	N
F_{mt1}, F_{mt2}	tangential force at mean diameter	N
F_{rad}	radial force	N
$f_{\alpha lim}$	influence factor of limit pressure angle	—
h_{ae1}, h_{ae2}	outer addendum	mm
h_{am1}, h_{am2}	mean addendum	mm
h_{amc1}, h_{amc2}	mean chordal addendum	mm
h_{e1}, h_{e2}	outer whole depth	mm
h_{fe1}, h_{fe2}	outer dedendum	mm
h_{fi1}, h_{fi2}	inner dedendum	mm
h_{fm1}, h_{fm2}	mean dedendum	mm
h_m	mean whole depth	mm
h_{mw}	mean working depth	mm
h_{t1}	pinion whole depth	mm
j_{en}	outer normal backlash	mm
j_{et}	outer transverse backlash	mm
j_{mn}	mean normal backlash	mm

Table 1 (continued)

Symbol	Description	Unit
j_{mt}	mean transverse backlash	mm
k_c	clearance factor	—
k_d	depth factor	—
k_{hap}	basic crown gear addendum factor (related to m_{mn})	—
k_{hfp}	basic crown gear dedendum factor (related to m_{mn})	—
k_t	circular thickness factor	—
m_{et}	outer transverse module	mm
m_{mn}	mean normal module	mm
n_1	pinion speed	min ⁻¹
P	power	kW
R_{e1}, R_{e2}	outer cone distance	mm
R_{i1}, R_{i2}	inner cone distance	mm
R_{m1}, R_{m2}	mean cone distance	mm
r_{c0}	cutter radius	mm
s_{mn1}, s_{mn2}	mean normal circular tooth thickness	mm
s_{mnc1}, s_{mnc2}	mean normal chordal tooth thickness	mm
T_1	pinion torque	Nm
t_{xi1}, t_{xi2}	front crown to crossing point	mm
t_{xo1}, t_{xo2}	pitch cone apex to crown (crown to crossing point, hypoid)	mm
t_{z1}, t_{z2}	pitch apex beyond crossing point	mm
t_{zF1}, t_{zF2}	face apex beyond crossing point	mm
t_{zi1}, t_{zi2}	crossing point to inside point along axis	mm
t_{zm1}, t_{zm2}	crossing point to mean point along axis	mm
t_{zR1}, t_{zR2}	root apex beyond crossing point	mm
u	gear ratio	—
u_a	equivalent ratio	—
W_{m2}	wheel mean slot width	mm
x_{hm1}	profile shift coefficient	—
x_{sm1}, x_{sm2}	thickness modification coefficient (backlash included)	—
x_{smn}	thickness modification coefficient (theoretical)	—
Z_0	number of blade groups	—
Z_1, Z_2	number of teeth	—
Z_p	number of crown gear teeth	—
α_{dC}	nominal design pressure angle on coast side	°
α_{dD}	nominal design pressure angle on drive side	°
α_{eC}	effective pressure angle on coast side	°
α_{eD}	effective pressure angle on drive side	°
α_{nD}	generated pressure angle on drive side	°
α_{nC}	generated pressure angle on coast side	°
α_{lim}	limit pressure angle	°
β_{e1}, β_{e2}	outer spiral angle	°
β_{i1}, β_{i2}	inner spiral angle	°
β_{m1}, β_{m2}	mean spiral angle	°
Δb_{x1}	pinion face width increment	mm

Table 1 (continued)

Symbol	Description	Unit
Δg_{xi}	increment along pinion axis from calculation point to inside	mm
Δg_{xe}	increment along pinion axis from calculation point to outside	mm
$\Delta \Sigma$	shaft angle departure from 90°	°
δ_{a1}, δ_{a2}	face angle	°
δ_{f1}, δ_{f2}	root angle	°
δ_1, δ_2	pitch angle	°
ε_{β}	face contact ratio	-
η	wheel offset angle in axial plane	°
θ_{a1}, θ_{a2}	addendum angle	°
θ_{f1}, θ_{f2}	dedendum angle	°
ν	lead angle of cutter	°
ρ_b	epicycloid base circle radius	mm
ρ_{lim}	limit curvature radius	mm
ρ_{P0}	crown gear to cutter centre distance	mm
Σ	shaft angle	°
$\Sigma \theta_f$	sum of dedendum angles	°
$\Sigma \theta_{fC}$	sum of dedendum angles for constant slot width taper	°
$\Sigma \theta_{fS}$	sum of dedendum angles for standard taper	°
$\Sigma \theta_{fM}$	sum of dedendum angles for modified slot width taper	°
$\Sigma \theta_{fU}$	sum of dedendum angles for uniform depth taper	°
ζ_o	pinion offset angle in face plane	°
ζ_m	pinion offset angle in axial plane	°
ζ_{mp}	offset angle in pitch plane, pinion and wheel	°
ζ_R	pinion offset angle in root plane	°

4 Design considerations

4.1 General

Loading, speed, accuracy requirements, space limitations and special operating conditions influence the design. For details, see ISO 10300 (all parts), [Annex B](#) and handbooks of gear manufacturing companies.

“Precision finish”, as used in this document, refers to a machine finishing operation which includes grinding, skiving and hard cut finishing. However, the common form of finishing known as “lapping” is specifically excluded as a form of precision finishing.

Users should determine the cutting methods available from their gear manufacturer prior to proceeding. Cutting systems used by bevel gear manufacturers are heavily dependent upon the type of machine tool that will be used.