
Quantities and units —

**Part 2:
Mathematics**

*Grandeurs et unités —
Partie 2: Mathématiques*

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 12, *Quantities and units*, in collaboration with Technical Committee IEC/TC 25, *Quantities and units*.

This second edition cancels and replaces the first edition (ISO 80000-2:2009), which has been technically revised.

The main changes compared to the previous edition are as follows:

- **Clause 4** revised to add clarification about writing of font types; revised rule for splitting equations over two or more lines;
- **Clause 18** revised to include clarification on scalars, vectors and tensors;
- missing symbols and expressions added in the second column "Symbol, expression" of the tables, and additional clarifications given in the fourth column "Remarks and examples" when necessary;
- Annex A deleted.

NOTE Although missing symbols and expressions have been added in this second edition of ISO 80000-1, the document remains non exhaustive.

A list of all parts in the ISO 80000 and IEC 80000 series can be found on the ISO and IEC websites.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

This corrected version of ISO 80000-2:2019 incorporates the following corrections:

- in 2-12.20, under "Remarks and examples", last line, f''' has been replaced with f'' ;
- in 2-13.1, under "Remarks and examples", the value 2,718 81 28 ... has been replaced with 2,718 281 828 ...;

- in 2-20.20, under "Remarks and examples", the first formula has been corrected to read $L_n^m(z) = (-1)^m \frac{d^m}{dz^m} L_{n+m}(z)$; i.e. addition of $+m$ in the subscript of L ;
- in 2-20.21, under "Remarks and examples", second line, the parenthesis has been corrected to read (for $n \in \mathbf{N}$, $|z| \leq 1$); i.e. addition of $|z| \leq 1$.

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Introduction

Arrangement of the tables

Each table of symbols and expressions (except [Table 13](#)) gives hints (in the third column) about the meaning or how the expression may be read for each item (numbered in the first column) of the symbol under consideration, usually in the context of a typical expression (second column). If more than one symbol or expression is given for the same item, they are on an equal footing. In some cases, e.g. for exponentiation, there is only a typical expression and no symbol. The purpose of the entries is identification of each concept and is not intended to be a complete mathematical definition. The fourth column “Remarks and examples” gives further information and is not normative.

[Table 13](#) has a different format. It gives the symbols of coordinates, as well as the position vectors and their differentials, for coordinate systems in three-dimensional spaces.

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Quantities and units —

Part 2: Mathematics

1 Scope

This document specifies mathematical symbols, explains their meanings, and gives verbal equivalents and applications.

This document is intended mainly for use in the natural sciences and technology, but also applies to other areas where mathematics is used.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 80000-1, *Quantities and units — Part 1: General*

3 Terms and definitions

Tables 1 to 16 give the symbols and expressions used in the different fields of mathematics.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

4 Variables, functions and operators

It is customary to use different sorts of letters for different sorts of entities, e.g. x, y, \dots for numbers or elements of some given set, f, g for functions, etc. This makes formulas more readable and helps in setting up an appropriate context.

Variables such as x, y , etc., and running numbers, such as i in $\sum_i x_i$ are printed in italic type. Parameters, such as a, b , etc., which may be considered as constant in a particular context, are printed in italic type. The same applies to functions in general, e.g. f, g .

An explicitly defined function not depending on the context is, however, printed in upright type, e.g. \sin, \exp, \ln, Γ . Mathematical constants, the values of which never change, are printed in upright type, e.g. $e = 2,718\ 281\ 828 \dots$; $\pi = 3,141\ 592 \dots$; $i^2 = -1$. Well-defined operators are also printed in upright type, e.g. **div**, δ in δx and each d in df/dx . Some transforms use special capital letters (see [Clause 19](#), Transforms).

Numbers expressed in the form of digits are always printed in upright type, e.g. 351 204; 1,32; 7/8.

Binary operators, for example $+$, $-$, $/$, shall be preceded and followed by thin spaces. This rule does not apply in case of unary operators, as in $-17,3$.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\omega t + \varphi)$. If the symbol for the function consists of two or more letters and the argument contains no operation symbol, such as $+$, $-$, \times , or $/$, the parentheses around the argument may be omitted. In these cases, there shall be a thin space between the symbol for the function and the argument, e.g. $\text{int } 2,4$; $\sin n\pi$; $\text{arcosh } 2A$; $\text{Ei } x$.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$; do not write $\cos x + y$, which could be mistaken for $\cos(x + y)$.

A comma, semicolon or other appropriate symbol can be used as a separator between numbers or expressions. The comma is generally preferred, except when numbers with a decimal comma are used.

If an expression or equation must be split into two or more lines, the following method shall be used:

- Place the line breaks immediately before one of the symbols $=$, $+$, $-$, \pm , or \mp , or, if necessary, immediately before one of the symbols \times , $:$, or $/$.

The symbol shall not be given twice around the line break; two minus signs could for example give rise to sign errors. If possible, the line break should not be inside of an expression in parentheses.

5 Mathematical logic

Table 1 — Symbols and expressions in mathematical logic

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-5.1	$p \wedge q$	conjunction of p and q , p and q	
2-5.2	$p \vee q$	disjunction of p and q , p or q	This “or” is inclusive, i.e. $p \vee q$ is true, if either p or q , or both are true.
2-5.3	$\neg p$	negation of p , not p	
2-5.4	$p \Rightarrow q$	p implies q , if p , then q	$q \Leftarrow p$ has the same meaning as $p \Rightarrow q$. \Rightarrow is the implication symbol. \rightarrow is also used as implication symbol.
2-5.5	$p \Leftrightarrow q$	p is equivalent to q	$(p \Rightarrow q) \wedge (q \Rightarrow p)$ has the same meaning as $p \Leftrightarrow q$. \Leftrightarrow is the equivalence symbol. \leftrightarrow is also used as equivalence symbol.
2-5.6	$\forall x \in A \ p(x)$	for every x belonging to A , the proposition $p(x)$ is true	If it is clear from the context which set A is considered, the notation $\forall x \ p(x)$ can be used. \forall is the universal quantifier. For $x \in A$, see 2-6.1.
2-5.7	$\exists x \in A \ p(x)$	there exists an x belonging to A for which $p(x)$ is true	If it is clear from the context which set A is considered, the notation $\exists x \ p(x)$ can be used. \exists is the existential quantifier. For $x \in A$, see 2-6.1. $\exists^1 x \ p(x)$ is used to indicate that there is exactly one element for which $p(x)$ is true. $\exists!$ is also used for \exists^1 .

6 Sets

Table 2 — Symbols and expressions for sets

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.1	$x \in A$	x belongs to A , x is an element of the set A	$A \ni x$ has the same meaning as $x \in A$.
2-6.2	$y \notin A$	y does not belong to A , y is not an element of the set A	$A \not\ni y$ has the same meaning as $y \notin A$. The negating stroke may also be vertical.
2-6.3	$\{x_1, x_2, \dots, x_n\}$	set with elements x_1, x_2, \dots, x_n	Also $\{x_i \mid i \in I\}$, where I denotes a set of subscripts.
2-6.4	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbf{R} \mid x \geq 5\}$ If it is clear from the context which set A is considered, the notation $\{x \mid p(x)\}$ can be used (for example $\{x \mid x \geq 5\}$, if it is clear that real numbers are considered). Instead of the vertical line often a colon is used as separator: $\{x \in A : p(x)\}$.
2-6.5	card A $ A $	number of elements in A , cardinality of A	The cardinality can be a transfinite number. The symbol $ $ is also used for absolute value of a real number (see 2-10.16), modulus of a complex number (see 2-15.4) and magnitude of a vector (see 2-18.4).
2-6.6	\emptyset $\{\}$	the empty set	
2-6.7	$B \subseteq A$	B is included in A , B is a subset of A	Every element of B belongs to A . \subset is also used, but see remark to 2-6.8. $A \supseteq B$ has the same meaning as $B \subseteq A$.
2-6.8	$B \subset A$	B is properly included in A , B is a proper subset of A	Every element of B belongs to A , but at least one element of A does not belong to B . If \subset is used for 2-6.7, then \subsetneq shall be used for 2-6.8. $A \supset B$ has the same meaning as $B \subset A$.
2-6.9	$A \cup B$	union of A and B	The set of elements which belong to at least one of the sets A and B . $A \cup B = \{x \mid x \in A \vee x \in B\}$
2-6.10	$A \cap B$	intersection of A and B	The set of elements which belong to both sets A and B . $A \cap B = \{x \mid x \in A \wedge x \in B\}$
2-6.11	$\bigcup_{i=1}^n A_i$	union of the sets A_1, A_2, \dots, A_n $\bigcup_{i=1}^n A_i = A_1 \cup \dots \cup A_n$	The set of elements belonging to at least one of the sets A_1, A_2, \dots, A_n $\bigcup_{i=1}^n$, $\bigcup_{i \in I}$ and $\bigcup_{i \in I}$ are also used, where I denotes a set of subscripts.

Table 2 (continued)

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.12	$\bigcap_{i=1}^n A_i$	intersection of the sets A_1, A_2, \dots, A_n $\bigcap_{i=1}^n A_i = A_1 \cap \dots \cap A_n$	The set of elements belonging to all sets A_1, A_2, \dots, A_n $\bigcap_{i=1}^n, \bigcap_{i \in I}$ and $\bigcap_{i \in I}$ are also used, where I denotes a set of subscripts.
2-6.13	$A \setminus B$	difference of A and B , A minus B	The set of elements which belong to A but not to B . $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ The notation $A - B$ should not be used. $C_A B$ is also used. $C_A B$ is mainly used when B is a subset of A , and the symbol A may be omitted if it is clear from the context which set A is considered.
2-6.14	(a, b)	ordered pair a, b , couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. If the comma can be mistaken as the decimal sign, then the semicolon (;) or a stroke () may be used as separator.
2-6.15	(a_1, a_2, \dots, a_n)	ordered n -tuple	See remark to 2-6.14.
2-6.16	$A \times B$	Cartesian product of the sets A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$
2-6.17	$\prod_{i=1}^n A_i$	Cartesian product of the sets A_1, A_2, \dots, A_n $\prod_{i=1}^n A_i = A_1 \times \dots \times A_n$	The set of ordered n -tuples (x_1, x_2, \dots, x_n) such that $x_1 \in A_1, x_2 \in A_2, \dots, x_n \in A_n$. $A \times A \times \dots \times A$ is denoted by A^n , where n is the number of factors in the product.
2-6.18	id_A	identity relation on set A , diagonal of $A \times A$	id_A is the set of all pairs (x, x) where $x \in A$. If the set A is clear from the context, the subscript A can be omitted.

7 Standard number sets and intervals

Table 3 — Symbols and expressions for standard number sets and intervals

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-7.1	N	the set of natural numbers, the set of positive integers and zero	N = {0, 1, 2, 3, ...} N* = {1, 2, 3, ...} Other restrictions can be indicated in an obvious way, as shown below. N _{>5} = { $n \in \mathbf{N} \mid n > 5$ } The symbols IN and N are also used.

Table 3 (continued)

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-7.2	Z	the set of integers	$\mathbf{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ $\mathbf{Z}^* = \{n \in \mathbf{Z} \mid n \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Z}_{>-3} = \{n \in \mathbf{Z} \mid n > -3\}$ The symbol \mathbb{Z} is also used.
2-7.3	Q	the set of rational numbers	$\mathbf{Q}^* = \{r \in \mathbf{Q} \mid r \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Q}_{<0} = \{r \in \mathbf{Q} \mid r < 0\}$ The symbols \mathbb{Q} and \mathbb{Q} are also used.
2-7.4	R	the set of real numbers	$\mathbf{R}^* = \{x \in \mathbf{R} \mid x \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{R}_{>0} = \{x \in \mathbf{R} \mid x > 0\}$ The symbols \mathbb{R} and \mathbb{R} are also used.
2-7.5	C	the set of complex numbers	$\mathbf{C}^* = \{z \in \mathbf{C} \mid z \neq 0\}$ The symbol \mathbb{C} is also used.
2-7.6	P	the set of prime numbers	$\mathbf{P} = \{2, 3, 5, 7, 11, 13, 17, \dots\}$ The symbol \mathbb{P} is also used.
2-7.7	$[a, b]$	closed interval from a included to b included	$[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$
2-7.8	$(a, b]$	left half-open interval from a excluded to b included	$(a, b] = \{x \in \mathbf{R} \mid a < x \leq b\}$ The notation $]a, b[$ is also used.
2-7.9	$[a, b)$	right half-open interval from a included to b excluded	$[a, b) = \{x \in \mathbf{R} \mid a \leq x < b\}$ The notation $]a, b[$ is also used.
2-7.10	(a, b)	open interval from a excluded to b excluded	$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$ The notation $]a, b[$ is also used.
2-7.11	$(-\infty, b]$	closed unbounded interval up to b included	$(-\infty, b] = \{x \in \mathbf{R} \mid x \leq b\}$ The notation $] -\infty, b[$ is also used.
2-7.12	$(-\infty, b)$	open unbounded interval up to b excluded	$(-\infty, b) = \{x \in \mathbf{R} \mid x < b\}$ The notation $] -\infty, b[$ is also used.
2-7.13	$[a, +\infty)$	closed unbounded interval onward from a included	$[a, +\infty) = \{x \in \mathbf{R} \mid a \leq x\}$ The notations $[a, \infty)$, $[a, +\infty[$ and $[a, \infty[$ are also used.
2-7.14	$(a, +\infty)$	open unbounded interval onward from a excluded	$(a, +\infty) = \{x \in \mathbf{R} \mid a < x\}$ The notations (a, ∞) , $]a, +\infty[$ and $]a, \infty[$ are also used.

8 Miscellaneous symbols

Table 4 — Miscellaneous symbols and expressions

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-8.1	$a = b$	a is equal to b a equals b	The symbol \equiv may be used to emphasize that a particular equality is an identity, i.e. holds universally. But see 2-8.18 for another meaning.
2-8.2	$a \neq b$	a is not equal to b	The negating stroke may also be vertical.
2-8.3	$a := b$	a is by definition equal to b	EXAMPLE $p := mv$, where p is momentum, m is mass and v is velocity. The symbols $\stackrel{\text{def}}{=}$ and $\stackrel{\text{def}}{\equiv}$ are also used.
2-8.4	$a \triangleq b$	a corresponds to b	EXAMPLES When $E = kT$, then $1 \text{ eV} \triangleq 11\,604,5 \text{ K}$. When 1 cm on a map corresponds to a length of 10 km, one may write $1 \text{ cm} \triangleq 10 \text{ km}$. The correspondence is not symmetric.
2-8.5	$a \approx b$	a is approximately equal to b	It depends on the user whether an approximation is sufficiently good. Equality is not excluded.
2-8.6	$a \simeq b$	a is asymptotically equal to b	EXAMPLE $\sin(x) \simeq \frac{1}{x-a}$ as $x \rightarrow a$ (For $x \rightarrow a$, see 2-8.16.)
2-8.7	$a \sim b$	a is proportional to b	The symbol \sim is also used for equivalence relations. The notation $a \propto b$ is also used.
2-8.8	$M \cong N$	M is congruent to N , M is isomorphic to N	M and N are point sets (geometrical figures). This symbol is also used for isomorphisms of mathematical structures.
2-8.9	$a < b$	a is less than b	
2-8.10	$b > a$	b is greater than a	
2-8.11	$a \leq b$	a is less than or equal to b	
2-8.12	$b \geq a$	b is greater than or equal to a	
2-8.13	$a \ll b$	a is much less than b	It depends on the situation whether a is sufficiently small as compared to b .
2-8.14	$b \gg a$	b is much greater than a	It depends on the situation whether b is sufficiently great as compared to a .
2-8.15	∞	infinity	This symbol does <i>not</i> denote a number but is often part of various expressions dealing with limits. The notations $+\infty$, $-\infty$ are also used.
2-8.16	$x \rightarrow a$	x tends to a	This symbol occurs as part of various expressions dealing with limits. a may be also ∞ , $+\infty$, or $-\infty$.

Table 4 (continued)

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-8.17	$m \mid n$	m divides n	For integers m and n : $\exists k \in \mathbf{Z} \quad m \cdot k = n$
2-8.18	$n \equiv k \pmod{m}$	n is congruent to k modulo m	For integers n, k and m : $m \mid (n - k)$ This concept of number theory must not be confused with identity of an equation, mentioned in 2-8.1, column 4.
2-8.19	$(a + b)$ $[a + b]$ $\{a + b\}$ $\langle a + b \rangle$	parentheses square brackets braces angle brackets	It is recommended to use only parentheses for grouping, since brackets and braces often have a specific meaning in particular fields. Parentheses can be nested without ambiguity.

9 Elementary geometry

Table 5 — Symbols and expressions in elementary geometry

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-9.1	$AB \parallel CD$	the straight line AB is parallel to the straight line CD	It is written $g \parallel h$ if g and h are the straight lines determined by the points A and B , and the points C and D , respectively.
2-9.2	$AB \perp CD$	the straight line AB is perpendicular to the straight line CD	It is written $g \perp h$ if g and h are the straight lines determined by the points A and B , and the points C and D , respectively. In a plane, the straight lines intersect.
2-9.3	$\sphericalangle ABC$	angle at vertex B in the triangle ABC	The angle is not oriented, it holds that $\sphericalangle ABC = \sphericalangle CBA$ and $0 \leq \sphericalangle ABC \leq \pi$ rad. For a more general definition including rotation angles see ISO 80000-3.
2-9.4	\overline{AB}	line segment from A to B	The line segment is the set of points between A and B on the straight line AB including the end points A and B .
2-9.5	\vec{AB}	vector from A to B	If $\vec{AB} = \vec{CD}$ then B , seen from A , is in the same direction and distance as D is, seen from C . It does not follow that $A = C$ and $B = D$.
2-9.6	$d(A, B)$	distance between points A and B	The distance is the length of the line segment \overline{AB} and also the magnitude of the vector \vec{AB} .