INTERNATIONAL STANDARD

ISO 80000-11

Second edition
2019-10

Quantities and units —

Part 11: **Characteristic numbers**

Grandeurs et unités —

Partie 11: Nombres caractéristiques

Reference number
ISO 80000-11:2019(E)

Contents

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see: www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 12, *Quantities and units*, in collaboration with Technical Committee IEC/TC 25, *Quantities and units*.

This second edition cancels and replaces the first edition (ISO 80000-11:2008), which has been technically revised.

The main changes compared to the previous edition are as follows:

— the table giving the quantities and units has been simplified;

— all items have been revised in terms of the layout of the definitions, and a worded definition has been added to each item;

— the number of items has been increased from 25 to 108 (concerns all Clauses);

— item 11-9.2 (Landau-Ginzburg number) has been transferred in this document from ISO 80000-12:2009 (revised as ISO 80000-12:2019).

A list of all parts in the ISO 80000 and IEC 80000 series can be found on the ISO and IEC websites.

Any feedback or questions on this document should be directed to the user’s national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Characteristic numbers are physical quantities of unit one, although commonly and erroneously called “dimensionless” quantities. They are used in the studies of natural and technical processes, and (can) present information about the behaviour of the process, or reveal similarities between different processes.

Characteristic numbers often are described as ratios of forces in equilibrium; in some cases, however, they are ratios of energy or work, although noted as forces in the literature; sometimes they are the ratio of characteristic times.

Characteristic numbers can be defined by the same equation but carry different names if they are concerned with different kinds of processes.

Characteristic numbers can be expressed as products or fractions of other characteristic numbers if these are valid for the same kind of process. So, the clauses in this document are arranged according to some groups of processes.

As the amount of characteristic numbers is tremendous, and their use in technology and science is not uniform, only a small amount of them is given in this document, where their inclusion depends on their common use. Besides, a restriction is made on the kind of processes, which are given by the Clause headings. Nevertheless, several characteristic numbers are found in different representations of the same physical information, e.g. multiplied by a numerical factor, as the square, the square root, or the inverse of another representation. Only one of these have been included, the other ones are declared as deprecated or are mentioned in the remarks column.

Quantities and units —

Part 11: Characteristic numbers

1 Scope

This document gives names, symbols and definitions for characteristic numbers used in the description of transport and transfer phenomena.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

Names, symbols and definitions for characteristic numbers are given in Clauses 4 to 9.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

4 Momentum transfer

Table 1 gives the names, symbols and definitions of characteristic numbers used to characterize processes in which momentum transfer plays a predominant role. The transfer of momentum (ISO 80000-4) basically occurs during a collision of 2 bodies, and is governed by the law of momentum conservation. Energy dissipation can occur. In a more generalized meaning momentum transfer occurs during the interaction of 2 subsystems moving with velocity v relative to each other. Typically, one of the subsystems is solid and possibly rigid, with a characteristic length, which can be a length, width, radius, etc. of a solid object, often the effective length is given by the ratio of a body's volume to the area of its surface.

The other subsystem is a fluid, in general liquid or gaseous, with the following properties amongst others:

— mass density ρ (ISO 80000-4);

— dynamic viscosity η (ISO 80000-4);

— kinematic viscosity $\nu = \eta / \rho$ (ISO 80000-4), or

— pressure drop Δp (ISO 80000-4).

The field of science is mainly fluid dynamics (mechanics). Characteristic numbers of this kind allow the comparison of objects of different sizes. They also can give some estimation about the change of laminar flow to turbulent flow.

Table 1 — Characteristic numbers for momentum transfer

No.	Name	Symbol	Definition	Remarks
11-4.1	Reynolds number	*Re*	quotient of inertial forces and viscous forces in a fluid flow, expressed by $Re = \frac{\rho v l}{\eta} = \frac{v l}{\nu}$; where ρ is mass density (ISO 80000-4), v is speed (ISO 80000-3), l is characteristic length (ISO 80000-3), η is dynamic viscosity (ISO 80000-4), and ν is kinematic viscosity (ISO 80000-4)	The value of the Reynolds number gives an estimate on the flow state: laminar flow or turbulent flow. In rotating movement, the speed $v = \omega l$, where l is the distance from the rotation axis and ω *is* the angular velocity.
11-4.2	Euler number	*Eu*	relationship between pressure drop in a flow and the kinetic energy per volume for flow of fluids in a pipe, expressed by $Eu = \frac{\Delta p}{\rho v^2}$; where Δp is drop of pressure (ISO 80000-4), ρ is mass density (ISO 80000-4), and v is speed (ISO 80000-3)	The Euler number is used to characterize losses in the flow. A modification of the Euler number is considering the dimensions of the containment (pipe): $Eu' = \frac{d}{l} Eu$; where d is inner diameter (ISO 80000-3) of the pipe, and l is length (ISO 80000-3).
11-4.3	Froude number	*Fr*	quotient of a body's inertial forces and its gravitational forces for flow of fluids, expressed by $Fr = \frac{v}{\sqrt{l g}}$; where v is speed (ISO 80000-3) of flow, l is characteristic length (ISO 80000-3), and g is acceleration of free fall (ISO 80000-3)	The Froude number can be modified by buoyancy. Sometimes the square and sometimes the inverse of the Froude number as defined here is wrongly used.

Table 1 *(continued)*

No.	Name	Symbol	Definition	Remarks
11-4.4	Grashof number	*Gr*	quotient of buoyancy forces due to thermal expansion which results in a change of mass density and viscous forces for free convection due to temperature differences, expressed by $Gr = l^3 g \alpha_V \Delta T / \nu^2$; where l is characteristic length (ISO 80000-3), g is acceleration of free fall (ISO 80000-3), α_V is thermal cubic expansion coefficient (ISO 80000-5), ΔT is difference of thermodynamic temperature T (ISO 80000-5) between surface of the body and the fluid far away from the body, and ν is kinematic viscosity (ISO 80000-4)	Heating can occur near hot vertical walls, in pipes, or by a bluff body. The characteristic length can be the vertical height of a hot plate, the diameter of a pipe, or the effective length of a body. See also Rayleigh number (item 11-5.3).
11-4.5	Weber number	*We*	relation between inertial forces and capillary forces due to surface tension at the interface between two different fluids, expressed by $We = \rho v^2 l / \gamma$; where ρ is mass density (ISO 80000-4), v is speed (ISO 80000-3), l is characteristic length (ISO 80000-3), and γ is surface tension (ISO 80000-4)	The fluids can be gases or liquids. The different fluids often are drops moving in a gas or bubbles in a liquid. The characteristic length is commonly the diameter of bubbles or drops. The square root of the Weber number is called Rayleigh number. Sometimes the square root of the Weber number as defined here is called the Weber number. That definition is deprecated. Interfaces only exist between two fluids which are not miscible.
11-4.6	Mach number	*Ma*	quotient of the speed of flow and the speed of sound, expressed by $Ma = v / c$; where v is speed (ISO 80000-3) of the body, and c is speed of sound (ISO 80000-8) in the fluid	The Mach number represents the relationship of inertial forces compared to compression forces. For an ideal gas $c = \sqrt{\gamma \frac{p}{\rho}} = \sqrt{\gamma \frac{RT}{M}} = \sqrt{\gamma \frac{kT}{m}}$; where γ is ratio of the specific heat capacity (ISO 80000-5).

Table 1 *(continued)*

No.	Name	Symbol	Definition	Remarks
11-4.7	Knudsen number	*Kn*	quotient of free path length of a particle and a characteristic length, expressed by $Kn = \lambda / l$; where λ is mean free path (ISO 80000-9), and l is characteristic length (ISO 80000-3)	The Knudsen number is a measure to estimate whether the gas in flow behaves like a continuum. The characteristic length, *l*, can be a characteristic size of the gas flow region like a pipe diameter.
11-4.8	Strouhal number; Thomson number	*Sr*, *Sh*	relation between a characteristic frequency and a characteristic speed for unsteady flow with periodic behaviour, expressed by $Sr = f l / v$; where f is frequency (ISO 80000-3) of vortex shedding, l is characteristic length (ISO 80000-3), and v is speed (ISO 80000-3) of flow	The characteristic length, *l*, can be the diameter of an obstacle in the flow which can cause vortex shedding, or the length of it.
11-4.9	drag coefficient	c_D	relation between the effective drag force and inertial forces for a body moving in a fluid, expressed by $c_D = \frac{2F_D}{\rho v^2 A}$; where F_D is drag force (ISO 80000-4) on the body, ρ is mass density (ISO 80000-4) of the fluid, v is speed (ISO 80000-3) of the body, and A is cross-sectional area (ISO 80000-3)	The drag coefficient is strongly dependant on the shape of the body.
11-4.10	Bagnold number	*Bg*	quotient of drag force and gravitational force for a body moving in a fluid, expressed by $Bg = \frac{c_D \rho v^2}{l g \rho_b}$; where c_D is drag coefficient (item 11-4.9) of the body, ρ is mass density (ISO 80000-4) of the fluid, v is speed (ISO 80000-3) of the body, l is characteristic length (ISO 80000-3), g is acceleration of free fall (ISO 80000-3), and ρ_b is mass density (ISO 80000-4) of the body	The characteristic length, *l*, is the body's volume divided by its cross-sectional area.

Table 1 *(continued)*

No.	Name	Symbol	Definition	Remarks
11-4.11	Bagnold number <solid particles>	Ba_2	quotient of drag force and viscous force in a fluid transferring solid particles, expressed by $Ba_2 = \frac{\rho_s d^2 \dot{\gamma}}{\eta} \sqrt{1/\left(f_s^{1/2} - 1\right)}$; where ρ_s is mass density (ISO 80000-4) of particles, d is diameter (ISO 80000-3) of particles, $\dot{\gamma} = v/d$ is shear rate time-derivative of shear strain (ISO 80000-4), η is dynamic viscosity (ISO 80000-4) of fluid, and f_s is volumic fraction of solid particles	
11-4.12	lift coefficient	c_l, c_A	quotient of the lift force available from a wing at a given angle and the inertial force for a wing shaped body moving in a fluid, expressed by $c_l = \frac{2F_l}{\rho v^2 S} = \frac{F_l}{qS}$; where F_l is lift force (ISO 80000-4) on the wing, ρ is mass density (ISO 80000-4) of the fluid, v is speed (ISO 80000-3) of the body, $S = A \cos \alpha$ is effective area (ISO 80000-3) when α is the angle of attack and A is area of the wing, and $q = \rho v^2/2$ is dynamic pressure.	The lift coefficient is dependant on the shape of the wing.
11-4.13	thrust coefficient	c_t	quotient of the effective thrust force available from a propeller and the inertial force in a fluid, expressed by $c_t = F_T/\left(\rho n^2 d^4\right)$; where F_T is thrust force (ISO 80000-4) of the propeller, ρ is mass density (ISO 80000-4) of the fluid, n is rotational frequency (ISO 80000-3), and d is tip diameter (ISO 80000-3) of the propeller	The thrust coefficient is dependant on the shape of the propeller.

Table 1 *(continued)*

No.	Name	Symbol	Definition	Remarks
11-4.14	Dean number	*Dn*	relation between centrifugal force and inertial force, for flows of fluids in curved pipes, expressed by $Dn = \frac{2vr}{\nu}\sqrt{\frac{r}{R}}$; where v is (axial) speed (ISO 80000-3), r is radius (ISO 80000-3) of the pipe, ν is kinematic viscosity (ISO 80000-4) of the fluid, and R is radius of curvature (ISO 80000-3) of the path of the pipe	
11-4.15	Bejan number	*Be*	quotient of mechanical work and frictional energy loss in fluid dynamics in a pipe, expressed by $Be = \frac{\Delta p l^2}{\eta \nu} = \frac{\rho \Delta p\, l^2}{\eta^2}$; where Δp is drop of pressure (ISO 80000-4) along the pipe, l is characteristic length (ISO 80000-3), η is dynamic viscosity (ISO 80000-4), ν is kinematic viscosity (ISO 80000-4), and ρ is mass density (ISO 80000-4).	A similar number exists for heat transfer (item 11-5.9). The kinematic viscosity is also called momentum diffusivity.
11-4.16	Lagrange number	*Lg*	quotient of mechanical work and frictional energy loss in fluid dynamics in a pipe, expressed by $Lg = \frac{l \Delta p}{\eta v}$; where l is length (ISO 80000-3) of the pipe, Δp is drop of pressure (ISO 80000-4) along the pipe, η is dynamic viscosity (ISO 80000-4), and v is speed (ISO 80000-3)	The Lagrange number is also given by $La = Re \cdot Eu$; where Re is the Reynolds number (item 11-4.1), and Eu is the Euler number (item 11-4.2).

Table 1 *(continued)*

No.	Name	Symbol	Definition	Remarks
11-4.17	Bingham number; plasticity number	*Bm*, *Bn*	quotient of yield stress and viscous stress in a viscous material for flow of viscoplastic material in channels, expressed by $Bm = \frac{\tau d}{\eta v}$; where τ is shear stress (ISO 80000-4), d is characteristic diameter (ISO 80000-3), e.g. effective channel width, η is dynamic viscosity (ISO 80000-4), and v is speed (ISO 80000-3)	
11-4.18	Hedström number	*He*, *Hd*	quotient of yield stress and viscous stress of a viscous material at flow limit for visco-plastic material in a channel, expressed by $He = \frac{\tau_0 d^2 \rho}{\eta^2}$; where τ_0 is shear stress (ISO 80000-4) at flow limit, d is characteristic diameter (ISO 80000-3), e.g. effective channel width, ρ is mass density (ISO 80000-4), and η is dynamic viscosity (ISO 80000-4)	
11-4.19	Bodenstein number	*Bd*	mathematical expression of the transfer of matter by convection in reactors with respect to diffusion, $Bd = vl/D$; where v is speed (ISO 80000-3), l is length (ISO 80000-3) of the reactor, and D is diffusion coefficient (ISO 80000-9)	The Bodenstein number is also given by $Bd = Pe^* = Re \cdot Sc$; where Pe^* is the Péclet number for mass transfer (item 11-6.2), Re is the Reynolds number (item 11-4.1), and $Sc = \eta/(\rho D) = \nu/D$ is Schmidt number (item 11-7.2).

Table 1 *(continued)*

No.	Name	Symbol	Definition	Remarks
11-4.20	Rossby number; Kiebel number	*Ro*	quotient of inertial forces and Coriolis forces in the context of transfer of matter in geophysics, expressed by $Ro = v/(2l\omega_E \sin\varphi)$; where v is speed (ISO 80000-3) of motion, l is characteristic length (ISO 80000-3), the scale of the phenomenon, ω_E is angular velocity (ISO 80000-3) of the Earth's rotation, and φ is angle (ISO 80000-3) of latitude	The Rossby number represents the effect of Earth's rotation on flow in pipes, rivers, ocean currents, tornadoes, etc. The quantity $\omega_E \sin\varphi$ is called Coriolis frequency.
11-4.21	Ekman number	*Ek*	quotient of viscous forces and Coriolis forces in the context of transfer of matter for the flow of a rotating fluid, expressed by $Ek = \nu/(2l^2\omega_E \sin\varphi)$; where ν is kinematic viscosity (ISO 80000-4), l is characteristic length (ISO 80000-3), the scale of the phenomenon, ω_E is angular frequency (ISO 80000-3) of the Earth's rotation, and φ is angle of latitude	In plasma physics, the square root of this number is used. The Ekman number is also given by $Ek = Ro/Re$; where Ro is the Rossby number (item 11-4.20), and Re is the Reynolds number (item 11-4.1).
11-4.22	elasticity number	*El*	relation between relaxation time and diffusion time in viscoelastic flows, expressed by $El = t_r \nu / r^2$; where t_r is relaxation time (ISO 80000-12), ν is kinematic viscosity (ISO 80000-4), and r is radius (ISO 80000-3) of pipe	See also Deborah number (item 11-7.8).

Table 1 *(continued)*

No.	Name	Symbol	Definition	Remarks
11-4.23	Darcy friction factor; Moody friction factor	f_D	representation of pressure loss in a pipe due to friction within a laminar or turbulent flow of a fluid in a pipe, expressed by $f_D = \frac{2\Delta p}{\rho v^2}\frac{d}{l}$; where Δp is drop of pressure (ISO 80000-4) due to friction, ρ is mass density (ISO 80000-4) of the fluid, v is (average) speed (ISO 80000-3) of the fluid in the pipe, d is diameter (ISO 80000-3) of the pipe, and l is length (ISO 80000-3) of the pipe	
11-4.24	Fanning number	f_n, f	relation between shear stress and dynamic pressure in the flow of a fluid in a containment, expressed by $f_n = \frac{2\tau}{\rho v^2}$; where τ is shear stress (ISO 80000-4) at the wall, ρ is mass density (ISO 80000-4) of the fluid, and v is speed (ISO 80000-3) of the fluid in the pipe	The Fanning number describes the flow of fluids in a pipe with friction at the walls represented by its shear stress. Symbol f may be used where no conflicts are possible.
11-4.25	Goertler number; Goertler parameter	Go	characterization of the stability of laminar boundary layer flows in transfer of matter in a boundary layer on curved surfaces, expressed by $Go = \frac{v l_b}{\nu}\sqrt{\left(\frac{l_b}{r_c}\right)}$; where v is speed (ISO 80000-3), l_b is boundary layer thickness (ISO 80000-3), ν is kinematic viscosity (ISO 80000-4), and r_c is radius of curvature (ISO 80000-3)	The Goertler number represents the ratio of centrifugal effects to viscous effects.