
Geographic information — Spatial schema

Information géographique — Schéma spatial

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing documents is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 211, *Geographic information/Geomatics*.

This second edition cancels and replaces the first edition (ISO 19107:2003), which has been technically revised. The main changes compared to the previous edition are as follows:

- It now forms a logical subset of this second edition. In other words, this document is 100 % backwardly compatible with its previous version, ISO 19107:2003, except in a few areas (in NURBS) where the previous version contained technical errors that are corrected in this revision.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document provides conceptual schemas for describing, representing and manipulating the spatial characteristics of geographic entities. Standardization in this area is the cornerstone for other geographic information design, specification and standardization.

"Vector" data consists of geometric primitives used to construct expressions of the spatial characteristics of geographic features. "Raster" data is based on the division of the extent covered into small units according to a tessellation of the space. This document deals only with vector data.

There is a hierarchy of complexity in the "geometry" of the underlying object used in various coordinate systems. These may use reference planes (map geometry – Euclidean), reference spheres (spherical geometry — using spherical trigonometry), reference ellipsoids (ellipsoidal geometry using Gaussian or Riemannian metrics) or more complex surfaces (usually using numeric approximations for calculation). The coordinates of a point locate it on, or in relation to, the reference geometry. With the exception of "map geometry," the usual Euclidean formulae for distance and area do not apply directly in the coordinate system.

Topology expressions provide qualitative descriptions of the spatial relations between geometry objects. Topology deals with the characteristics of geometric figures that remain invariant if the space is deformed elastically. Topological properties do not change when information is transformed from one coordinate system to another, usually including the coordinate function that map from R^2 or R^3 to the reference geometry. Topological properties in the domain of the coordinate system will be identical to those on the geographic surface; but the metric properties may change significantly (e.g. distance, area, direction).

Spatial operators are functions and procedures that use query, create, modify or delete spatial objects. This document defines the taxonomy of some of the more important operators, their definitions and implementations. The goals are to:

- Define spatial operators unambiguously, so that different implementations will yield comparable results within the limitations of accuracy and resolution.
- Use these definitions to define a set of standard operations that will form the basis of compliant systems and thus act as a test-bed for implementers and a benchmark set for validation of compliance.
- Define an operator algebra that will allow combinations of the base operators to be used predictably in the query and manipulation of geographic feature data.

Standardized conceptual schemas for spatial characteristics will increase the ability to share geographic information between applications. These schemas will be used by geographic information system and software developers and users of geographic information to provide consistently understandable spatial data structures and functions.

This document is technical because geometry is a technical topic. Euclid was speaking of a simpler form of geometry to the most powerful man in his world when he said:

There is no royal road to geometry (μή εἶναι βασιλικήν ἀτραπὸν ἐπὶ γεωμετρίας).

Euclid to Ptolemy I Soter (General with Alexander the Great, Pharaoh of Egypt) —

Attributed by Proclus (412–485 AD) in Commentary on the First Book of Euclid's Elements

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Geographic information — Spatial schema

1 Scope

This document specifies conceptual schemas for describing the spatial characteristics of geographic entities, and a set of spatial operations consistent with these schemas. It treats "vector" geometry and topology. It defines standard spatial operations for use in access, query, management, processing and data exchange of geographic information for spatial (geometric and topological) objects. Because of the nature of geographic information, these geometric coordinate spaces will normally have up to three spatial dimensions, one temporal dimension and any number of other spatially dependent parameters as needed by the applications. In general, the topological dimension of the spatial projections of the geometric objects will be at most three.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 19103, *Geographic information — Conceptual schema language*

ISO 19108, *Geographic information — Temporal schema*

ISO 19109, *Geographic information — Rules for application schema*

ISO 19111, *Geographic information — Spatial referencing by coordinates*

ISO/IEC 11404:2007, *Information technology — General-Purpose Datatypes (GPD)*

ISO/IEC 19505-2:2012, *Information technology — Object Management Group Unified Modeling Language (OMG UML) — Part 2: Superstructure*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC 11404, ISO 19103, ISO/IEC 19505-2 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

NOTE Common words from geometry, such as point, curve, line, surface solid, etc., take the common meanings unless they are used as classifier names (usually interfaces), in which case they are a digital representation of the geometric concept. Common mathematical terms that are not defined here take on their common meanings in mathematics (see [15], [10], ISO/IEC 11404 or a standard text on the topic, such as the "N. Bourbaki"¹⁾ series currently published by Springer Verlag, in French, English and German). Care should be taken since mathematical terms can be context sensitive, and can easily be confused with common words. For example, "open" set, "closed" curve, "closed" set, "rational" function, "boundary," "interior," "closure", "exterior", "function" and others from common language but have very specific meanings in mathematics and in this document. Where necessary to prevent confusion, existing definitions have been elaborated to make their intent in this document explicit. Mathematical terms include common vocabulary from geometry, topology, calculus, geodesy and differential geometry. Many of these terms can be sufficiently common that inclusion is not necessary. They are included here to prevent confusion especially for terms like the ones listed above that have another meaning in another context.

3.1 abstract root

<programming> common root classifier of a category which is a superclass of any other classifier in the category

Note 1 to entry: The class Any in some programming languages is the abstract root of all classes. Thus, it is the *de facto* union of all classes. In this document, Geometry is the (named and explicit) abstract root for all geometry objects. In the package Geometry and any of its subpackages (including those in its Requirements Classes), any interface will be a subtype of Geometry either directly or transitively.

3.2 arc

<geometry> *segment* (3.83) of a curve

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3.3 barycentric coordinates

<coordinate geometry> point in a **n-dimension** coordinate system using n+1 numbers, $[u_0, u_1, u_2, u_3, \dots, u_n] \ni [0 \leq u_i \leq 1 \wedge \sum u_i = 1, 0]$, in which the location of a point of an **n-simplex** (of any dimension) is specified by a weighted centre of mass of equal masses placed at its vertices using vector algebra of the \mathbb{R}^n used in the **coordinate reference system**

Note 1 to entry: Even though there are n+1 coordinates in a barycentric coordinate system, the topological dimension is n, since the restriction (sums to 1,0) loses 1 degree of freedom (once you have n ordinates, the

remaining one is determined such as $u_n = 1, 0 - \sum_{i=0}^{n-1} u_i$. The coordinates for the simplex are all non-negative, but

the system can be extended outside of the simples by using negative numbers. If the ordinates are all positive, then the point is inside (interior to) the **n-simplex**. If one of them is 1,0 and the other 0, this is a corner of the simplex. If one of them is zero and the others still each greater than or equal to zero, the point is on the n-1-simplex opposite the vertex zeroed out. If any are negative, the point is outside of the simplex. The coordinates are dependent on the underlying coordinate reference system of the source data.

1) N. Bourbaki is the pen-name for the "Association des collaborateurs de Nicolas Bourbaki" (Association of Collaborators of Nicolas Bourbaki) of mathematicians first published in 1935 and dedicated to "formalizing" mathematics. The group has an office at the "École Normale Supérieure" in Paris. See https://en.wikipedia.org/wiki/Nicolas_Bourbaki. Their books are held in high regard by the mathematical community.

3.4**bearing**

horizontal angle, tangent or direction at a point

Note 1 to entry: This definition (as opposed to the one in ISO 19162:2015) is required for this document because the concept is used in other definitions, such as first geodetic problem and second geodetic problem. The two definitions are nearly equivalent because the tangent of a curve on a surface is a tangent to the surface and does not specify a direction. Usual 2D measure of bearing can be an angle equivalently measured from North clockwise, or a unit tangent vector. If the coordinate system is spatially 3D, the horizontal bearing angle may also need to be a vertical altitude angle to be complete. If a reference curve (as used in ISO 19162) is parameterized by arc length, then the "derivative" is a unit vector. If another parameterization " t " is used, then the derivative should be normalized ($\vec{\tau} / \|\vec{\tau}\|; \dot{c}(t) = \vec{\tau}$). This is useful, since parameterization by arc length can be computationally difficult. The numeric representation of a vector depends on the coordinate system. The bearing is not dependent on a coordinate system, but it can be represented in any reasonable system. The bearing is not dependent on its various representations.

3.5**bicontinuous**

<mathematics> invertible, continuous and with a continuous inverse

3.6**boundary**

set that represents the limit of an entity

Note 1 to entry: Boundary is most commonly used in the context of geometry, where the set is a collection of points or a collection of objects that represent those points. In other arenas, the term is used metaphorically to describe the transition between an entity and the rest of its domain of discourse.

3.7**buffer**

geometric object containing all points and only those points whose distance from a specified geometric object is less than or equal to a given distance use in its construction

3.8**closure**

union of the interior and boundary of a topological object or geometric object

3.9**coboundary**

set of topological primitives of higher topological dimension associated with a particular topological object, such that this topological object is in each of their boundaries

Note 1 to entry: If a node is on the boundary of an edge, that edge is on the coboundary of that node. Any orientation parameter associated with one of these relations would also be associated with the other. The coboundary of a node can be called a "node star".

3.10**conformal, adj.**

angle-preserving

Note 1 to entry: Some projections are conformal. For example, a Mercator preserves the angle between curves, so that if two curves in a Mercator projected plane cross at a 90°, then the preimage curves on the ellipsoid also cross at 90°, such as lines of constant latitude and lines of constant longitude.

3.11**connected**

property of a topological space implying that only the entire space or the empty set are the only subsets which are both open and closed

Note 1 to entry: The formal definition of connected is that any pair of locally open sets whose union is the entire space must have a non-empty intersection.

a topological space T is connected if and only if

$$[\forall X, Y \subset T \ni X \cup Y = T] \Rightarrow [X \cap Y \neq \emptyset] \tag{1}$$

This formal definition is difficult to use. The term *path connected* (3.75), defined below is equivalent for the purposed of this document. The use of “finite precision” coordinates makes sets which are connected but not path connected impossible to represent. In all cases “connected” is used, but “path connected” is easier to test and to visualize.

3.12

connected node

<topology> node that starts or ends one or more edges

3.13

control point

<coordinate geometry> point used in the construction of a geometry that partially controls its shape but does not necessarily lie on the geometry

Note 1 to entry: A centre of an arc is a control point; poles in b-spline curves are control points.

3.14

convex

<geometry> containing all points on a "line" joining two interior points

Note 1 to entry: The definition of convex requires a definition of line. For coordinate systems, this is the usual linear interpolated arc, but in context, the "line" on a geometric reference surface will be a "geodesic arc". The default in this document is the linear interpolate.

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3.15

convex hull

<geometry> smallest convex set containing a given geometric object

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Note 1 to entry: "Smallest" is the set theoretic smallest, not an indication of a measurement. The definition can be rewritten as "the intersection of all convex sets that contain the geometric object". Another definition in a Euclidean space \mathbb{E}^n is the union of all lines with both end points in the given geometric object.

$$C = A.convexHull \Leftrightarrow$$

$$[C.convex = TRUE] \wedge [A \subset C] \wedge [[B.convex = true, A \subset B] \Rightarrow [A \subseteq C \subseteq B]] \tag{2}$$

3.16

coordinate

one of a sequence of numbers designating the position of a point

Note 1 to entry: In a coordinate reference system, the numbers are qualified by units. The number of offsets (generally called "ordinates") in a coordinate is not the dimension. If there is a constraint, the dimension can be smaller. See *coordinate dimension* (3.17).

3.17

coordinate dimension

<coordinate geometry> number of separate decisions needed to describe a position in a coordinate system

Note 1 to entry: The coordinate dimension represents the number of choices made, and constraints can restrict choices. A barycentric coordinate which has (n+1)-offsets, but the underlying space is dimension n. Homogeneous coordinates (wx, wy, wz, w) are actually 3 dimensional because the choice of "w" does not affect the position, i.e. (wx, wy, wz, w) = (x y, z, 1) → (x, y, z) which is not affected by w. The dimension will be at most the count of the numbers in the coordinate, but it can be less if the coordinates are constrained in some manner.

3.18 coordinate reference system

<differential geometry, geodesy> coordinate system that is related to an object by a datum

[SOURCE: ISO 19111:2007, 4.8, modified — Notes 1 and 2 to entry have been added]

Note 1 to entry: The original definition of coordinate reference system (CRS) uses a geometric object (a geodetic datum) which is referenced to the real world by a Datum. ISO 19111 can extend this to any "parameter" which essentially can be represented as the graph of the parameter relation. This graph is in the cross product of the Datum, the domain space of the parameter function, and the parameter space (which in turn may be multi dimensional). Thus, every coordinate system used in this document is logically a CRS. Since this was not the original intent of ISO 19111, this document will use CRS for coordinate systems associated to a geodetic datum, and use the more general term "coordinate system" for anything else that is either more or less. Here, CRS means spatial coordinates. In every case where the underlying datum (surface) is not flat, the coordinate system is not Euclidean, and the metric is not Pythagorean.

Note 2 to entry: In the event that later versions of ISO 19111 change the definition as copied above, this document should not be affected. In the case that dynamic datums are used, the measurements made by the geometry operations of the objects in this document would only be valid at the time used for the datum and the associated coordinates. If the datum varies, the measure may also vary, but the definitions of the measures remains the same, but the actual values may be a function of time.

3.19 curvature vector

<differential geometry> second derivative of a curve parameterized by arc length, at a point

Note 1 to entry: If $c(s) = (x(s), y(s), z(s))$ is a curve in 3D Cartesian space (\mathbb{E}^3), and s is the arc length along $c(s)$, then the unit tangent vector is $\dot{c}(s) = (\dot{x}(s), \dot{y}(s), \dot{z}(s))$, i.e. the derivative of the coordinate values of "c" with respect to "s". The curvature vector is $\ddot{c}(s) = (\ddot{x}(s), \ddot{y}(s), \ddot{z}(s))$. The curvature vector can be approximated by the inverse of the radius of a circle through any 3 nearby points on the curve (pointed from the curve to towards the centre of the circle)

3.20 cycle

<geometry, topology> bounded spatial object with an empty boundary

Note 1 to entry: Cycles are used to describe boundary components. A cycle usually has no boundary because it closes on itself, but it is bounded (i.e., it does not have infinite extent). A circle or a sphere, for example, has no boundary (i.e., its boundary is empty), but is bounded.

3.21 data point

<coordinate geometry> point that lies on the geometry

Note 1 to entry: The vertices in a line string are data points, the points used to construct a polynomial spline are data points. Data points can be used as control points, but are often derived after the geometry is constructed.

3.22 diameter

<metric> maximum distance between two points in the set of points

3.23 directed edge

<topology> directed topological object that represents an association between an edge and one of its orientations

Note 1 to entry: A directed edge that is in agreement with the orientation of the edge has a + orientation, otherwise, it has the opposite (-) orientation. Directed edge is used in topology to distinguish the right side (-) from the left side (+) of the same edge and the start node (-) and end node (+) of the same edge and in computational topology to represent these concepts.