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Part 31:
Robust profile filters: Gaussian regression filters

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Partie 31: Filtres de profil robustes: Filtres de régression gaussiens

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

The committee responsible for this document is ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This first edition of ISO 16610-31 cancels and replaces ISO/TS 16610-31, which has been technically revised.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences the chain link C of all chains of standards.

For more detailed information of the relation of this document to the GPS matrix model, see Annex C.

The ISO/GPS matrix model given in ISO 14638 gives an overview of the ISO/GPS system of which this document is a part. The fundamental rules of ISO/GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to specifications made in accordance with this document, unless otherwise indicated.

This document develops the concept of the discrete robust Gaussian regression filter. The robust process reduces the influence of the deep valleys and high peaks. The subject of this document is the robust Gaussian regression filter of degree p = 2, which has very good robust behaviour and form approximation for functional stratified engineering surfaces.

Geometrical product specifications (GPS) — Filtration —

Part 31:
Robust profile filters: Gaussian regression filters

1 Scope

This document specifies the characteristics of the discrete robust Gaussian regression filter for the evaluation of surface profiles with spike discontinuities such as deep valleys and high peaks.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1:2015, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO/IEC Guide 99, ISO 16610-1, ISO 16610-20, ISO 16610-30 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— IEC Electropedia: available at http://www.electropedia.org/

— ISO Online browsing platform: available at http://www.iso.org/obp

3.1
robust filter
filter that is insensitive to output data against specific phenomena in the input data

3.2
regression filter
M-estimator based on the local polynomial modelling of the profile

3.3
robust Gaussian regression filter
regression filter (3.2) based on the Gaussian weighting function and a *biweight influence function* (3.4)

3.4
biweight influence function
asymmetric function which is scale-invariant, expressed by

$$\psi(x) = \begin{cases} x \times \left(1 - \left(\dfrac{x}{c}\right)^2\right)^2 & \text{for} \quad |x| \le c \\ 0 & \text{for} \quad |x| > c \end{cases}$$

where c is the scale parameter.

4 Robust Gaussian regression filter

4.1 Weighting function

The weighting function of the robust Gaussian regression filter depends on the profile values (distance to the reference line) and the location of the weighting function along the profile.

4.2 Filter equation

4.2.1 General

The robust Gaussian regression filter is derived from the general discrete regression filter (see Annex A) by setting the degree to $p = 2$, using the biweight influence function and the Gaussian weighting function according to ISO 16610-21. In the case of $p = 2$, the robust Gaussian regression filter follows form components up to the second degree.

4.2.2 Filter equation for the robust Gaussian regression filter for open profiles

For open profiles, the filter equation for the robust Gaussian regression filter is given by Formula (1):

$$w_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \left(\boldsymbol{X}_k^T \times \boldsymbol{S}_k \times \boldsymbol{X}_k \right)^{-1} \times \boldsymbol{X}_k^T \times \boldsymbol{S}_k \times \boldsymbol{z} \tag{1}$$

The regression function is spanned by the matrix

$$\boldsymbol{X}_k = \begin{bmatrix} 1 & x_{1,k} & x_{1,k}^2 \\ \vdots & \vdots & \vdots \\ 1 & x_{n,k} & x_{n,k}^2 \end{bmatrix} \tag{2}$$

where

$$x_{l,k} = \left(l - k \right) \times \Delta x, \qquad l = 1, \ldots, n \tag{3}$$

The space variant weighting function, $\boldsymbol{S}_k$, is given by Formula (4):

$$\boldsymbol{S}_k = \begin{bmatrix} s_{1,k} \times \delta_1 & 0 & \cdots & 0 \\ 0 & s_{2,k} \times \delta_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & s_{n,k} \times \delta_n \end{bmatrix} \tag{4}$$

with the Gaussian function

$$s_{l,k} = \frac{1}{\gamma \times \lambda_\mathrm{c}} \times \exp\left(-\pi \left(\frac{x_{l,k}}{\gamma \times \lambda_\mathrm{c}} \right)^2 \right), \qquad l = 1, .., n \tag{5}$$

and the parameter

$$\gamma = \sqrt{\frac{-1 - W\left(-\frac{1}{2 \times \exp(1)} \right)}{\pi}} \approx 0,730\,9 \tag{6}$$

The additional weights

$$\delta_l = \begin{cases} \left(1 - \left(\frac{z_l - w_l}{c} \right)^2 \right)^2 & \text{for } |z_l - w_l| \le c \\ 0 & \text{for } |z_l - w_l| > c \end{cases}, \qquad l = 1, .., n \tag{7}$$

are derived from the biweight influence function with the parameter

$$c = \frac{3}{\sqrt{2} \times \mathrm{erf}^{-1}(0,5)} \times \mathrm{median}\left| \boldsymbol{z} - \boldsymbol{w} \right| \approx 4,447\,8 \times \mathrm{median}\left| \boldsymbol{z} - \boldsymbol{w} \right| \tag{8}$$

The definition for c is equivalent to three times Rq of the surface roughness for Gaussian distributed profiles and is the default case.

where

$W(X)$	is the “Lambert W” function;
$\mathrm{erf}^{-1}(x)$	is the inverse error function;
n	is the number of values in the profile;
k	is the index of the profile ordinate $k = 1, \ldots, n$;
$\boldsymbol{z}$	is the vector of dimension n of the profile values before filtering;
$\boldsymbol{w}$	is the vector of dimension n of the profile values of the filter reference line;
w_k	is the value of the filter mean line at position k;
λ_c	is the cut-off wavelength of the profile filter;
Δx	is the sampling interval.

NOTE 1 Vector $\boldsymbol{w}$ gives the profile values of the long-wave component (reference line). The short-wave component, $\boldsymbol{r}$, can be obtained by the difference vector, $\boldsymbol{r} = \boldsymbol{z} - \boldsymbol{w}$.

NOTE 2 For surfaces with big pores or peaks at the profile boundaries, the robustness can be increased by setting $P = 0$. In this case, the nominal form is eliminated by using a pre-filtering technique. The filter equation for $P = 0$ results in

$$w_k = \left(\boldsymbol{X}_k^T \times \boldsymbol{S}_k \times \boldsymbol{X}_k\right)^{-1} \times \boldsymbol{X}_k^T \times \boldsymbol{S}_k \times \boldsymbol{z} = \left(\sum_{l=1}^{n} s_{l,k} \times \delta_l\right)^{-1} \times \sum_{l=1}^{n} \left(s_{l,k} \times \delta_l \times z_l\right)$$

where

$$\boldsymbol{X}_k = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \text{ and } \gamma = \sqrt{\frac{\ln 2}{\pi}}$$

4.2.3 Filter equation for robust Gaussian regression filter for closed profiles

For closed profiles, the filter equation for the robust Gaussian regression filter is given by Formula (9):

$$\tilde{w}_k = \begin{pmatrix}1 & 0 & 0\end{pmatrix} \times \left(\tilde{\boldsymbol{X}}_k^T \times \tilde{\boldsymbol{S}}_k \times \tilde{\boldsymbol{X}}_k\right)^{-1} \times \tilde{\boldsymbol{X}}_k^T \times \tilde{\boldsymbol{S}}_k \times \tilde{\boldsymbol{z}} \tag{9}$$

The regression function is spanned by the matrix

$$\tilde{\boldsymbol{X}}_k = \begin{bmatrix} 1 & \tilde{x}_{1,k} & \tilde{x}_{1,k}^2 \\ \vdots & \vdots & \vdots \\ 1 & \tilde{x}_{n,k} & \tilde{x}_{n,k}^2 \end{bmatrix} \tag{10}$$

with

$$\tilde{x}_{l,k} = \left(\left(l - k + \frac{n}{2}\right) \bmod n - \frac{n}{2}\right) \times \Delta x, \qquad l = 1, \ldots, n \tag{11}$$

The space variant weighting function, $\tilde{\boldsymbol{S}}_k$, is given by

$$\tilde{\boldsymbol{S}}_k = \begin{bmatrix} \tilde{s}_{1,k} \times \tilde{\delta}_1 & 0 & \cdots & 0 \\ 0 & \tilde{s}_{2,k} \times \tilde{\delta}_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{s}_{n,k} \times \tilde{\delta}_n \end{bmatrix} \tag{12}$$

with the Gaussian function

$$\tilde{s}_{l,k} = \frac{1}{\gamma \times \lambda_c} \times \exp\left(-\pi\left(\frac{\tilde{x}_{l,k}}{\gamma \times \lambda_c}\right)^2\right), \qquad l = 1, \ldots, n \tag{13}$$

and the parameter

$$\gamma = \sqrt{\frac{-1 - W\left(-\dfrac{1}{2 \times \exp(1)}\right)}{\pi}} \approx 0{,}730\,9 \tag{14}$$

The additional weights

$$\tilde{\delta}_l = \begin{cases} \left(1 - \left(\dfrac{\tilde{z}_l - \tilde{w}_l}{\tilde{c}}\right)^2\right)^2 & \text{for} \quad \left|\tilde{z}_l - \tilde{w}_l\right| \le \tilde{c} \\ 0 & \text{for} \quad \left|\tilde{z}_l - \tilde{w}_l\right| > \tilde{c} \end{cases}, \qquad l = 1, \ldots, n \tag{15}$$

are derived from the biweight influence function with the parameter

$$\tilde{c} = \frac{3}{\sqrt{2} \times \operatorname{erf}^{-1}(0{,}5)} \times \operatorname{median}\left|\tilde{\boldsymbol{z}} - \tilde{\boldsymbol{w}}\right| \approx 4{,}447\,8 \times \operatorname{median}\left|\tilde{\boldsymbol{z}} - \tilde{\boldsymbol{w}}\right| \tag{16}$$

The definition for c is equivalent to three times Rq of the surface roughness for Gaussian distributed profiles and is the default case.