
**Sequential sampling plans for
inspection by variables for percent
nonconforming (known standard
deviation)**

*Plans d'échantillonnage progressif pour le contrôle par mesures des
pourcentages de non-conformes (écart-type connu)*

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ISO copyright office
CP 401 • Ch. de Blandonnet 8
CH-1214 Vernier, Geneva
Phone: +41 22 749 01 11
Fax: +41 22 749 09 47
Email: copyright@iso.org
Website: www.iso.org

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This first edition of ISO 39511 cancels and replaces ISO 8423:2008, of which it constitutes a minor revision to change the reference number from 8423 to 39511.

Introduction

In contemporary production processes, quality is often expected to reach such high levels that the number of nonconforming items is reported in parts per million. Under such circumstances, popular acceptance sampling plans by attributes, such as those presented in ISO 2859-1, require prohibitively large sample sizes. When it is possible to apply acceptance sampling plans by variables, such as those presented in ISO 3951-1, the sample sizes are much smaller. However, especially in the case of acceptance of a product of extremely high quality, those sample sizes are still too large. Therefore, there is a need to apply standardized statistical procedures that require the smallest possible sample sizes; sequential sampling plans are the only statistical procedures that satisfy that need. It has been mathematically proved that among all possible sampling plans having similar statistical properties the sequential sampling plan has the smallest average sample size.

The principal advantage of sequential sampling plans is the reduction in the average sample size. The *average sample size* is the average of all the sample sizes that may occur under a sampling plan for a given lot or process quality level. The use of sequential sampling plans leads to a smaller average sample size than single sampling plans having the equivalent operating characteristic.

Other factors that should be taken into account are as follows.

a) Complexity

The rules of a sequential sampling plan are more easily misunderstood by inspectors than the simple rules for a single sampling plan.

b) Variability in the amount of inspection

As the actual number of items inspected for a particular lot is not known in advance, the use of sequential sampling plans brings about various organizational difficulties. For example, scheduling of inspection operations may be difficult.

c) Difficulty of drawing sample items

If drawing sample items is rather difficult, the reduction in the average sample size by sequential sampling plans may be cancelled out by the increased sampling cost.

d) Duration of test

If the test of a single item is of long duration and a number of items can be tested simultaneously, sequential sampling plans are much more time-consuming than the corresponding single sampling plan.

e) Variability of quality within the lot

If the lot consists of two or more sublots from different sources and if there is likely to be any substantial difference between the qualities of the sublots, drawing of a representative sample under a sequential sampling plan is far more difficult than under the corresponding single sampling plan.

The balance between the advantage of a smaller average sample size of the sequential sampling plan and the above disadvantages leads to the conclusion that sequential sampling plans are suitable only when inspection of individual items is costly in comparison with inspection overheads.

The choice between single and sequential sampling plans should be made before the inspection of a lot is started. During inspection of a lot, it is not permitted to switch from one type to another, because the operating characteristic of the plan may be drastically changed if the actual inspection results influence the choice of acceptability criteria.

Although a sequential sampling plan is on average much more economical than the corresponding single sampling plan, it may occur, during inspection of a particular lot, that acceptance or non-acceptance comes at a very late stage because the cumulative leeway (the statistic used for the determination of lot acceptability) remains between the acceptance value and the rejection value for a long time. With the

graphical method, this corresponds to the random progress of the step-wise linear curve remaining in the indecision zone.

In order to alleviate this disadvantage, the curtailment values are set before the inspection of a lot (or a process) is started, and inspection terminates if the cumulative sample size reaches the curtailment value, n_t , without determination of lot acceptability. The acceptance and non-acceptance of the lot (or the process) is then determined using the curtailment acceptance and rejection values.

For sequential sampling plans in common use, curtailment usually represents a deviation from their intended usage, leading to a distortion of their operating characteristics. In this International Standard, however, the operating characteristics of the sequential sampling plans have been determined with curtailment taken into account, so curtailment is an integral component of the provided plan.

Sequential sampling plans for inspection by variables are also provided in ISO 3951-5. However, the design principle of those plans is fundamentally different from that of this International Standard. The sampling plans in ISO 3951-5 are designed to supplement the ISO 3951-1 acceptance sampling system for inspection by variables, which is a counterpart of the popular ISO 2859-1 acceptance sampling system for inspection by attributes. Thus, they should be used for the inspection of a continuing series of lots, that is, a series long enough to permit the switching rules of the ISO 3951 system to take effect. The application of the switching rules is the only means of providing enhanced protection to the consumer (by means of tightened sampling inspection criteria or discontinuation of sampling inspection) when the sequential sampling plans from ISO 3951-5 are used. However, in certain circumstances, there is a strong need to have both producer's and consumer's risks under strict control. Such circumstances occur, for example, when sampling is performed for regulatory reasons, for the demonstration of quality of production processes or for hypothesis testing. In such cases, individual sampling plans selected from the ISO 3951-5 sampling scheme may be inappropriate. The sampling plans from this International Standard have been designed in order to meet these specific requirements.

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Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)

1 Scope

This International Standard specifies sequential sampling plans and procedures for inspection by variables of discrete items.

The plans are indexed in terms of producer's risk point and the consumer's risk point. Therefore, they are suitable not only for the purposes of acceptance sampling, but for the more general purpose of the testing of simple statistical hypotheses for proportions.

The purpose of this International Standard is to provide procedures for the sequential assessment of inspection results that may be used to induce the supplier to supply lots of a quality having a high probability of acceptance. At the same time, the consumer is protected by a prescribed upper limit to the probability of accepting a lot (or process) of poor quality.

This International Standard is primarily designed for use under the following conditions:

- a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process. In such a case, sampling of particular lots is equivalent to the sampling of the process. If there are different producers or production processes, this International Standard shall be applied to each one separately;
- b) where only a single quality characteristic x of these products is taken into consideration, which must be measurable on a continuous scale;
- c) where the measurement error is negligible (i.e. with a standard deviation no more than 10 % of the process standard deviation);
- d) where production is stable (under statistical control) and the quality characteristic x has a known standard deviation, and is distributed according to a normal distribution or a close approximation to the normal distribution;

CAUTION — The procedures in this International Standard are not suitable for application to lots that have been screened previously for nonconforming items.

- e) where a contract or standard defines an upper specification limit U , a lower specification limit L , or both; an item is qualified as conforming if and only if its measured quality characteristic, x , satisfies the appropriate one of the following inequalities:
 - 1) $x \leq U$ (i.e. the upper specification limit is not violated);
 - 2) $x \geq L$ (i.e. the lower specification limit is not violated);
 - 3) $x \leq U$ and $x \geq L$ (i.e. neither the upper nor the lower specification limit is violated.)

Inequalities 1) and 2) are called cases with a “single specification limit”, and 3) is the case with “double specification limits”.

In this International Standard, it is assumed that, where double specification limits apply, conformance to both specification limits is either equally important to the integrity of the product or is considered separately for both specification limits. In the first case, it is appropriate to control the combined percentage of product outside the two specification limits. This is referred to as combined control. In the second case, nonconformity beyond each of the limits is controlled separately, and this is referred to as separate control.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2:2006, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-1, *Sampling procedures for inspection by variables — Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 3534-2 and ISO 3951-1 and the following apply.

3.1 inspection by variables

inspection by measuring the magnitude(s) of the characteristic(s) of an item

[SOURCE: ISO 3534-2:2006, definition 4.1.4]

3.2 sampling inspection

inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2:2006, definition 4.1.6]
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3.3 acceptance sampling

sampling after which decisions are made to accept or not to accept a lot, or other grouping of products, materials or services, based on sample results

[SOURCE: ISO 3534-2:2006, definition 1.3.17]

3.4 acceptance sampling inspection

acceptance inspection where the acceptability is determined by means of sampling inspection

[SOURCE: ISO 3534-2:2006, definition 4.1.8]

3.5 acceptance sampling inspection by variables

acceptance sampling inspection in which the acceptability of a process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

[SOURCE: ISO 3534-2:2006, definition 4.2.11]

3.6 quality level

quality expressed as a rate of occurrence of nonconforming units

3.7 nonconformity

non-fulfilment of a requirement

[SOURCE: ISO 9000:2015, definition 3.6.9, and ISO 3534-2:2006, definition 3.1.11]

3.8**nonconforming unit**

unit with one or more nonconformities

[SOURCE: ISO 3534-2:2006, definition 1.2.15]

3.9**specification limit**

limiting value stated for a characteristic

[SOURCE: ISO 3534-2:2006, definition 3.1.3]

3.10**lower specification limit**

L

specification limit that defines the lower limiting value

[SOURCE: ISO 3534-2:2006, definition 3.1.5]

3.11**upper specification limit**

U

specification limit that defines the upper limiting value

[SOURCE: ISO 3534-2:2006, definition 3.1.4]

3.12**combined control**

requirement when both upper and lower limits are specified for the quality characteristic and specified risks apply to the combined percent nonconforming beyond the two limits

Note 1 to entry: The use of combined control implies that nonconformities beyond either specification limit are believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.13**separate control**

requirement when both upper and lower limits are specified for the quality characteristic and separate risks are given which apply to each limit

Note 1 to entry: The use of separate control implies that nonconformities beyond either specification limit are believed to be of different importance to the lack of integrity of the product.

3.14**maximum process standard deviation**

$\sigma_{\max}$

largest process standard deviation for a given sampling plan for which it is possible to satisfy the acceptance criteria for a combined double specification limit when the process variability is known

Note 1 to entry: Maximum process standard deviation $\sigma_{\max}$ was denoted by its acronym MPSD in older standards.

Note 2 to entry: This definition is different from the similar definition given in ISO 3534-2 in which the concept of AQL is used.

3.15**measurement**

set of operations having the object of determining a value of a quantity

[SOURCE: ISO 3534-2:2006, definition 3.2.1]

3.16

leeway

quantity derived from a measured value of an item

Note 1 to entry: In the case of a single lower specification limit and in the case of double specification limits, the leeway is obtained by subtracting the numerical value of the lower specification limit from the measured value. In the case of an upper specification limit, the leeway is obtained by subtracting the measured value from the numerical value of the upper specification limit.

3.17

cumulative leeway

value calculated by summing the leeways obtained from the start of the inspection up to, and including, that of the item last inspected

3.18

cumulative sample size

total number of inspected items, counting from the start of the inspection up to, and including, the item last inspected

3.19

acceptance value for sequential sampling

value derived from the specified parameters of the sampling plan and the cumulative sample size

Note 1 to entry: Whether the lot may yet be accepted is determined by comparing the cumulative leeway with the acceptance value.

3.20

rejection value for sequential sampling

value derived from the specified parameters of the sampling plan and the cumulative sample size

Note 1 to entry: Whether the lot may yet be considered unacceptable is determined by comparing the cumulative leeway with the rejection value.

3.21

consumer's risk quality

CRQ

Q_{CR}

(acceptance sampling) quality level of a lot or process which, in the acceptance sampling plan, corresponds to a specified consumer's risk

Note 1 to entry: The specified consumer's risk is usually 10 %.

[SOURCE: ISO 3534-2:2006, definition 4.6.9]

3.22

producer's risk quality

PRQ

Q_{PR}

(acceptance sampling) quality level of a lot or process which, in the acceptance sampling plan, corresponds to a specified producer's risk

[SOURCE: ISO 3534-2:2006, definition 4.6.10]

Note 1 to entry: The specified producer's risk is usually 5 %.

3.23

average sample size

ASSI

(acceptance sampling) average number of units in a sample inspected per lot in reaching decisions to accept or not to accept when using a given acceptance sampling plan

[SOURCE: ISO 3534-2:2006, definition 4.7.3]

3.24**sequential acceptance sampling inspection**

acceptance sampling inspection in which, after each item has been inspected, the decision to accept the lot, not accept the lot, or to inspect another item is taken based on the cumulative sampling evidence to date

[SOURCE: ISO 3534-2:2006, definition 4.2.7]

3.25**sequential sampling plan**

plan which states acceptance criteria in sequential acceptance sampling inspection

3.26**operating characteristic curve**

curve showing the relationship between probability of acceptance of product and the incoming quality level for a given acceptance sampling plan

[SOURCE: ISO 3534-2:2006, definition 4.5.1]

3.27**producer's risk point****PRP**

(acceptance sampling) point on the operating characteristic curve corresponding to a predetermined high probability of acceptance

[SOURCE: ISO 3534-2:2006, definition 4.6.7]

3.28**consumer's risk point****CRP**

(acceptance sampling) point on the operating characteristic curve corresponding to a predetermined low probability of acceptance

[SOURCE: ISO 3534-2:2006, definition 4.6.5]

4 Symbols

The symbols used are as follows.

A	acceptance value for sequential sampling
A_t	acceptance value corresponding to the curtailed value of the cumulative sample size
f	a factor given in Tables 5 and 6 , that relates the maximum process standard deviation to the difference between U and L
g	multiplier of the cumulative sample size that is used to determine the acceptance values and the rejection values (slope of the acceptance and rejection lines)
h_A	constant that is used to determine the acceptance values (intercept of the acceptance line)
h_R	constant that is used to determine the rejection values (intercept of the rejection line)
L	lower specification limit (as a suffix to a variable, denotes its value at L)
N	lot size (number of items in a lot)
n	sample size (number of items in a sample)
n_{cum}	cumulative sample size

n_t	curtailment value of the cumulative sample size
P_a	probability of acceptance
Q_{CR}	consumer's risk quality
Q_{PR}	producer's risk quality
R	rejection value for sequential sampling
U	upper specification limit (as a suffix to a variable, denotes its value at U)
x	measured value of the quality characteristic for the item of the sample
y	leeway, defined as $y = U - x$ for a single upper specification limit $y = x - L$ for a single lower specification limit $y = x - L$ for double specification limits
Y	cumulative leeway obtained by adding the leeways up to, and including, the item last inspected
α	producer's risk
β	consumer's risk
σ	standard deviation of a process that is under statistical control
NOTE	σ^2 , the square of the process standard deviation, is known as the process variance.
$\sigma_{\max}$	maximum process standard deviation

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5 Principles of sequential sampling plans for inspection by variables

Under a sequential sampling plan by variables, sample items are drawn at random and inspected one by one, and the cumulative leeway (which measures a “distance” between the process level and specification limits) is obtained. After the inspection of each item, the cumulative leeway is compared with the acceptability criteria in order to assess whether there is sufficient information to determine lot or process acceptability at that stage of the inspection.

If, at a given stage, the cumulative leeway is such that the risk of accepting a lot of unsatisfactory quality level is sufficiently low, the lot is considered acceptable and the inspection is terminated.

If, on the other hand, the cumulative leeway is such that the risk of non-acceptance of a lot of satisfactory quality level is sufficiently low, the lot is considered not acceptable and the inspection is terminated.

If the cumulative leeway does not allow either of the above decisions to be taken, then an additional item is sampled and inspected. The process is continued until sufficient sample information has been accumulated to warrant a decision that the lot is acceptable or not acceptable.

6 Selection of a sampling plan

6.1 Producer's risk point and consumer's risk point

The general method described in 6.1 and 6.2 is used when the requirements of the sequential sampling plan are specified in terms of two points on the operating characteristic curve of the plan. The point

corresponding to the higher probability of acceptance shall be designated the “producer’s risk point”; the other shall be designated the “consumer’s risk point”.

The first step when designing a sequential sampling plan is to choose these two points, if they have not already been dictated by circumstances. For this purpose, the following combination is often used:

- a producer’s risk of $\alpha \leq 0,05$ and the corresponding producer’s risk quality (Q_{PR});
- a consumer’s risk of $\beta \leq 0,10$ and the corresponding consumer’s risk quality (Q_{CR}).

This combination of requirements is used in this International Standard for the design of the sampling plans.

When the desired sequential sampling plan is required to have approximately the same operating characteristic curve as an existing single sampling plan, the producer’s risk point and the consumer’s risk point may be read off from a graph or a table of the operating characteristic of that plan. When no such plan exists, the producer’s and the consumer’s risk points have to be determined from direct consideration of the conditions under which the sampling plan operates.

6.2 Preferred values of Q_{PR} and Q_{CR}

Table 4 gives 21 preferred values of Q_{PR} (producer’s risk quality) ranging from 0,1 % to 10,0 %, and 17 preferred values of Q_{CR} (consumer’s risk quality) ranging from 0,8 % to 31,5 %. This International Standard is only applicable to a combination of the preferred values of Q_{PR} and Q_{CR} .

6.3 Pre-operation preparations

6.3.1 Obtaining the parameters h_A , h_R and g

The criteria for acceptance and non-acceptance of a lot are determined from the parameters h_A , h_R and g .

Table 4 gives the values of these parameters corresponding to each combination of preferred values of Q_{PR} and Q_{CR} together with the producer’s risk α approximately equal to 0,05 and the consumer’s risk β approximately equal to 0,1.

6.3.2 Obtaining the curtailment values

The curtailment value, n_t , of the cumulative sample size of the sequential sampling plan is given in Table 4 together with the other parameters.

7 Operation of a sequential sampling plan

7.1 Specification of the plan

Before operating a sequential sampling plan, the inspector shall record on the sampling document the specified values of the parameters, h_A , h_R , g and n_t .

7.2 Drawing a sample item

As a rule, the individual sample items shall be drawn at random from the lot and inspected one by one in the order in which they were drawn. If, for convenience, successive items are drawn at the same time, the order in which each sample item is inspected shall be at random.

7.3 Leeway and cumulative leeway

Following the inspection of each item, record the inspection result x against the current value, n_{cum} , of the cumulative sample size.