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**Railway applications — Calculation  
of braking performance (stopping,  
slowing and stationary braking) —**

**Part 2:  
General algorithms utilizing step by  
step calculation**

iTeh STANDARD PREVIEW

*Applications ferroviaires — Calcul des performances de freinage  
(freinage d'arrêt, de ralentissement et d'immobilisation) —*

*Partie 2: Algorithmes généraux utilisant le calcul pas à pas*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see [www.iso.org/iso/foreword.html](http://www.iso.org/iso/foreword.html).

This document was prepared by Technical Committee ISO/TC 269, *Railway applications*, Subcommittee SC 2, *Rolling stock*.

A list of all parts in the ISO 20138 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at [www.iso.org/members.html](http://www.iso.org/members.html).

## Introduction

This document describes methodologies for calculation of braking performance such as stopping distance, deceleration, power and energy for railway rolling stock.

The objective of this document is to enable the railway industry and operators to work with common calculation methods.

The ISO 20138 series consists of two parts (ISO 20138-1 and this document) which complement each other.

This document describes the step by step calculation methods for railway applications applicable to all countries. In addition, the algorithms provide a means of comparing the results of other braking performance calculation methods.

The methodology of step by step calculation is based on numerical time integration.

The step by step calculation method cannot be used for stationary braking. This document considers an example for stationary braking of a multiple unit in accordance with ISO 20138-1.

When calculating stopping and slowing distances using the step by step calculation method, it is intended that both ISO 20138-1 and this document be considered.

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# Railway applications — Calculation of braking performance (stopping, slowing and stationary braking) —

## Part 2: General algorithms utilizing step by step calculation

### 1 Scope

This document specifies the methodologies for calculation of braking performance for railway rolling stock.

This document describes the general algorithms/formulae using instantaneous value inputs to perform calculations of brake equipment and braking performance, in terms of stopping/slowing distances, braking power and energy for all types of rolling stock, either as vehicles or units.

The calculations can be performed at any stage of the assessment process (design, manufacture, testing, verification, investigation, etc.) of railway rolling stock. This document does not set out specific acceptance criteria (pass/fail).

This document is not intended to be used as a design guide for the selection of brake systems and does not specify performance requirements. This document does not provide a method to calculate the extension of stopping distances when the level of demanded adhesion exceeds the available adhesion (wheel slide activity).

This document contains examples of the calculation of brake forces for different brake equipment types and examples of the calculation of stopping distance for vehicles or units.

### 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 20138-1:2018, *Railway applications — Calculation of braking performance (stopping, slowing and stationary braking) — Part 1: General algorithms utilizing mean value calculation*

### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 20138-1 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

#### 3.1 slowing distance

$s$   
distance run between the initial brake demand and achieving the final speed  $v_{fin}$

Note 1 to entry: When the final speed  $v_{fin} = 0$  m/s, slowing distance is also known as stopping distance.

**3.2  
slowing time**

$t$   
elapsed time from the initial brake demand until the final speed  $v_{fin}$  is reached

Note 1 to entry: When the final speed  $v_{fin} = 0$  m/s, slowing time is also known as stopping time.

**4 Symbols**

For the purposes of this document, the general symbols given in [Table 1](#) and ISO 20138-1:2018, Table 1 apply.

**Table 1 — Symbols**

Symbol	Definition	Unit
$a$	Instantaneous deceleration of the vehicle/unit	m/s <sup>2</sup>
$a_{f(t) = 100 \%}$	Deceleration during each chosen time step	m/s <sup>2</sup>
$a_j$	Constant deceleration during iteration step $j$	m/s <sup>2</sup>
$D_{max}$	Wheel diameter max.	m
$D_{min}$	Wheel diameter min.	m
$F_{B,ax,st}$	Stationary brake force acting on that wheelset	N
$F_{pad,i}$	Force acting on single disc surface ( $i$ is an index used for sorting)	N
$F_{r,n}$	Instantaneous retarding force of brake equipment type $n$	N
$F_{r,n,j}$	Instantaneous retarding force for brake equipment type $n$ during iteration step $j$	N
$F_{r,nom}$	Nominal retarding force	N
$f(t)$	Factor dependent on time	—
$f(t) = 100 \%$	Index for 100 % applied braking force without consideration of any time characteristics	—
$f(v)$	Factor dependent on speed	—
$f(x)$	Factor (common characteristic) dependent on another variable $x$	—
$i_{tra}$	Transmission ratio	—
$j$	Iteration step number	—
$P_n$	Instantaneous braking power of brake equipment type $n$	W
$S_j$	Distance travelled from brake command at time $t_0$ to time $t_j$	m
$S_{n,j}$	Distance travelled during iteration step $j$ whilst the brake equipment type $n$ is applied	m
$S_{ref(\Delta t)}$	Stopping/slowng distance, calculated with time step $\Delta t$	m
$S_{comp(2 \cdot \Delta t)}$	Stopping/slowng distance, calculated with doubled time step ( $2 \cdot \Delta t$ )	m
$S_{f(t) = 100 \%}$	Braking distance without consideration of any time characteristics from initial speed $v_0$ to final speed $v_{fin}$	m
$t$	Slowing time/stopping time	s
$t_j$	Elapsed time from brake command to iteration step $j$	s
$\Delta t$	Time step	s
$v$	Current speed	m/s
$v_j$	Speed at time $t_j$	m/s
$v_{1,ECB}$	Deactivating speed of eddy current brake	m/s
$\varepsilon$	Speed deviation from $v_{fin}$	m/s
$\mu$	Coefficient of friction (brake pad or block)	—
$\xi$	Relative distance deviation	%



Table 1 (continued)

Symbol	Definition	Unit
$\tau_{ax}$	Value of the instantaneous adhesion required between wheel and rail for the braked wheelset	—
$\tau_{req}$	Required wheel/rail adhesion	—

## 5 General explanation of step-by-step calculation

### 5.1 Method

The step-by-step method is used when it is not appropriate or desirable to represent the non-constant retarding and braking forces by mean values. Further details of when the use of mean value calculations is appropriate are given in ISO 20138-1.

Time steps are defined in such a way that the braking forces can be considered as constant throughout each step's duration. The duration of each step can depend on changes in the braking force and is not necessarily fixed (i.e. algorithms can be based either on constant or adaptive time step integration schemes). Each time step is characterised by an initial state and a final state, e.g. an initial and a final speed.

For each time step, the distance travelled during that time step as well as the final speed are calculated and the deceleration at the end of that time step is calculated according to Newton's laws. The outputs of the calculations for each time step are used as inputs to the calculations for each subsequent time step.

The calculation shall be done in accordance with the workflow as shown in [Annex A](#).

### 5.2 Retarding force models

Mathematical models for common brake systems (e.g. magnetic track brakes, electrodynamic brakes, etc.) are described in [Annex B](#). The mathematical models for disc brakes, tread brakes and external deceleration forces (e.g. wind forces, running resistance) are described in ISO 20138-1.

The impact of time, speed, load, temperature, etc. on the nominal retarding force can also be expressed in terms of dimensionless factors (functions), e.g. time dependency  $f(t)$ , speed dependency  $f(v)$ , etc. Thus, any deceleration force characteristics due to brake system applications or acting external forces (e.g. wind forces) can be modelled.

These dimensionless factors can take effect at the same time and are thus superposed by multiplication as set out in [Formula \(1\)](#).

$$F_r = F_{r,nom} \cdot f(t) \cdot f(v) \cdot \dots \cdot f(x) \quad (1)$$

where

- $F_r$  is the instantaneous retarding force acting at the rail generated by the brake equipment, expressed in N;
- $F_{r,nom}$  is the nominal retarding force, expressed in N;
- $f(t)$  is the factor dependent on time;
- $f(v)$  is the factor dependent on speed;
- $f(x)$  is the factor (common characteristic) dependent on another variable  $x$ .

NOTE For nominal retarding forces  $F_{r,nom}$ , the factors  $f(t)$ ,  $f(v)$  and  $f(x)$  are equal to 1.

### 5.3 Algorithm

#### 5.3.1 General description

Instantaneous values are the input data for step-by-step (iterative) calculation. The workflow of [Figure A.1](#) shall be used for performing stopping and slowing calculations.

The numerical integration is time-based.

Every calculation begins with the initial brake demand and the initial vehicle/unit speed.

The initial time step begins at time  $t_0 = 0$  s simultaneously with the start of the braking demand. The braking forces which are acting in the initial time step are calculated.

The result of the first iteration step refers to  $j = 1$ , i.e.  $v_1 = v_0 - a_0 \Delta t$ , whereas initial values (e.g. initial speed) refer to index  $j = 0$ .

The vehicle/unit speed at the end of the time step and the distance travelled during this time step are calculated.

The value of the selected parameter (e.g. speed, distance) at the end of the time step is compared with its target value.

If the target value has not been reached, the calculations are repeated for the next time step.

The time step calculation continues until the target value is reached.

#### 5.3.2 Time integration

The time integration should continue until the calculated value of the selected parameter (e.g. speed) is considered equal to the target value of that parameter, i.e. when the condition given in [Formula \(2\)](#) is achieved (where speed is used as an example selected parameter):

$$|v_j - v_{fin}| < \varepsilon \quad (2)$$

where

- $v_j$  is the speed at time  $t_j$ , expressed in m/s;
- $v_{fin}$  is the final speed, expressed in m/s;
- $\varepsilon$  is the speed deviation from  $v_{fin}$ , expressed in m/s.

A speed deviation not greater than  $10^{-3}$  m/s is considered as suitable for high speed train calculations. For lower speeds or slowing calculations, other values may be used.

Based on the calculation of retarding forces and external forces, the constant deceleration  $a_j$  during iteration step  $j$  can be calculated as set out in [Formula \(3\)](#):

$$a_j = \frac{(\sum F_{r,n} + \sum F_{\text{ext}})_j}{m_{\text{dyn}}} \quad (3)$$

where

- $j$  is the iteration step number;
- $a_j$  is the constant deceleration during iteration step  $j$ , expressed in  $\text{m/s}^2$ ;
- $F_{r,n}$  is the instantaneous retarding force of brake equipment type  $n$ , expressed in N;
- $F_{\text{ext}}$  is the external force, expressed in N;
- $m_{\text{dyn}}$  is the dynamic mass, expressed in kg.

If the target value of the selected parameter has not been achieved, the next time step integration is conducted, utilising the outputs of the preceding step, as shown in [Formulae \(4\) to \(8\)](#):

$$\text{Speed at start of step } t_{j+1} \quad v_{j+1} = v_j - a_j \cdot \Delta t \quad (4)$$

$$\text{Distance at start of step } t_{j+1} \quad s_{j+1} = s_j + v_j \cdot \Delta t - \frac{1}{2} \cdot a_j \cdot \Delta t^2 \quad (5)$$

$$\text{Deceleration during step } t_{j+1} \quad a_{j+1} = \frac{(\sum F_{r,n} + \sum F_{\text{ext}})_{j+1}}{m_{\text{dyn}}} \quad (6)$$

$$\text{Next time step} \quad t_{j+1} = t_j + \Delta t \quad (7)$$

$$\text{Next time increment} \quad j \rightarrow j+1 \quad (8)$$

where

- $a_j$  is the constant deceleration during iteration step  $j$ , expressed in  $\text{m/s}^2$ ;
- $F_{r,n}$  is the instantaneous retarding force of brake equipment type  $n$ , expressed in N;
- $F_{\text{ext}}$  is the external force, expressed in N (for decelerating force **positive** value, for accelerating force **negative** value);
- $j$  is the iteration step number;
- $m_{\text{dyn}}$  is the dynamic mass, expressed in kg;
- $s_j$  is the distance travelled from brake command at time  $t_0$  to time  $t_j$ , expressed in m;
- $t_j$  is the elapsed time from brake command to iteration step  $j$ , expressed in s;
- $\Delta t$  is the time step, expressed in s.

The final time step sometimes needs to be adjusted, if necessary, to meet the target value of the selected parameter (see [5.3.1](#)).

Other more detailed algorithms may be used if considered necessary.

**5.3.3 Determination of time step/relative distance deviation  $\xi$**

The relative distance deviation  $\xi$  has to be calculated if the applied integration procedure imposes constant time steps. If an adaptive time integration is used, the requirements in this clause are not applicable.

The time step  $\Delta t$  shall be chosen in such a way that the relative distance deviation is not greater than the minimum precision required. The relative distance deviation  $\xi$  is obtained by two separate integrations. The original calculation with time step  $\Delta t$  determines the reference stopping/slowing distance  $s_{ref(\Delta t)}$  and the second integration with doubled time step  $2 \cdot \Delta t$  determines a new stopping/slowing distance  $s_{comp(2 \cdot \Delta t)}$  for comparison. The relative distance deviation  $\xi$  is calculated in accordance with [Formula \(9\)](#) and shall not be greater than the minimum precision required.

The value of the relative distance deviation  $\xi$  shall not exceed a predefined limit value and can be calculated as set out in [Formula \(9\)](#):

$$\xi = \left| \frac{s_{comp(2 \cdot \Delta t)} - s_{ref(\Delta t)}}{s_{ref(\Delta t)}} \right| \cdot 100 \tag{9}$$

where

- $\xi$  is the relative distance deviation, expressed in %;
- $s_{ref(\Delta t)}$  is the stopping/slowing distance, calculated with time step  $\Delta t$ , expressed in m;
- $s_{comp(2 \cdot \Delta t)}$  is the stopping/slowing distance, calculated with doubled time step  $(2 \cdot \Delta t)$ , expressed in m.

Usually, a relative distance deviation of  $\xi \leq 0,1\%$  is considered as acceptable. For low speeds and slowing calculations, greater values of deviation ratio may be used.

NOTE The definition of validation requirements of any numerical integration procedure is outside the scope of this document.

**5.3.4 Equivalent system response time  $t_e$**

The calculation of equivalent system response time allows the assumption that braking consists first of a "free running time" with braking force equal to zero, followed by a braking time with fully applied braking force. ISO 20138-1 describes the equivalent response time when considering the free running time.

The equivalent system response time  $t_e$  based on stopping and braking distance shall be calculated with two separate time integrations:

- a) the stopping/slowing distance calculated taking into account the time characteristics of each acting brake equipment type starting at time  $t_0 = 0$  s simultaneously with the start of the braking demand until achieving the final speed  $v_{fin}$ ;
- b) the stopping/slowing distance calculated assuming each acting brake equipment type fully applies (100 %) at time  $t_0 = 0$  s simultaneously with the start of the braking demand until achieving the final speed  $v_{fin}$ .

The equivalent system response time can be calculated as set out in [Formula \(10\)](#).

$$t_e = \frac{s - s_{f(t)=100\%}}{v_0} \tag{10}$$

where

- $v_0$  is initial speed, in m/s;
- $s$  is the stopping/slowing distance with all time characteristics taken into account, expressed in m;
- $s_{f(t)=100\%}$  is the braking distance without consideration of any time characteristics from initial speed  $v_0$  to final speed  $v_{fin}$ .

## 5.4 Supplementary dynamic calculations

### 5.4.1 Energy dissipated by each brake equipment type

ISO 20138-1 describes the calculation of energy dissipated during braking based on mean retarding forces.

The total energy dissipated by each brake equipment type during iteration steps  $j = 0$  to  $j = J$  can be calculated based on instantaneous values as set out in [Formula \(11\)](#).

$$W_{B,n} = \sum_{j=0}^J (F_{r,n,j} \cdot s_{n,j}) \quad (11)$$

where

- $W_{B,n}$  is the energy dissipated by brake equipment type  $n$ , expressed in J;
- $F_{r,n,j}$  is the instantaneous retarding force for brake equipment type  $n$  during iteration step  $j$ , expressed in N;
- $s_{n,j}$  is the distance travelled during iteration step  $j$  whilst the brake equipment type  $n$  is applied, expressed in m.

### 5.4.2 Value of the instantaneous adhesion required between wheel and rail for the braked wheelset ( $\tau_{ax}$ )

The value of the instantaneous adhesion required between wheel and rail for the braked wheelset can be calculated as set out in [Formula \(12\)](#).

$$\tau_{ax} = \frac{\sum_{n=1}^N F_{r,n} - m_{rot,ax} \cdot a}{m_{st,ax} \cdot g} \cdot \sqrt{i^2 + 1} \quad (12)$$

where

- $\tau_{ax}$  is the value of the instantaneous adhesion required between wheel and rail for the braked wheelset;
- $N$  is the number of brake equipment types;
- $\sum_{n=1}^N F_{r,n}$  is the sum of all adhesion dependent retarding forces from all brake equipment types per wheelset, expressed in N;
- $F_{r,n}$  is the instantaneous retarding force of brake equipment type  $n$ , expressed in N;
- $a$  is the instantaneous deceleration of the vehicle/unit, expressed in m/s<sup>2</sup>;
- $g$  is the standard acceleration due to gravity, expressed in m/s<sup>2</sup>;