
Hydrometric uncertainty guidance (HUG)

Lignes directrices relatives à l'incertitude en hydrométrie

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 113, *Hydrometry*.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

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Introduction

The management of a natural environment requires knowledge, by measurement, of what is happening. Only then can effective action be taken and the effectiveness of the action assessed. Much depends on the quality of the knowledge itself.

The quality of measurable knowledge is stated in terms of measurement uncertainty. The internationally agreed method for assessing measurement quality is the guide to the estimation of uncertainty in measurement (GUM). Without this uniformity of measurement standards, equitable sharing of the environment is not possible and international obligations to care for the environment would be weakened.

The essential purpose of the GUM is for a statement of the quality of a measurement result to be presented with all measurements described in technical standards. Without this, no two measurements can be compared, or standards set. Whereas the GUM is a reference document serving the universal requirements of metrology, the Hydrometric uncertainty guidance (HUG) document is specific to hydrometry, i.e. to the measurement of the components of the hydrological cycle. It borrows from the GUM the methods that are the most applicable to hydrometry and applies them to techniques and equipment used in hydrometry.

In the past, error analysis has provided an indication of measurement quality, but such statements cannot properly convey the quality of the result because it presupposes knowledge of a true, error-free, value against which the measured result can be compared. The true value can never be known and uncertainty remains. For this reason, the GUM uses the concept of uncertainty and uses it for all stages and components of the measurement process. This ensures consistency.

The GUM defines standard uncertainty of a result as being equivalent to a standard deviation. This can be the standard deviation of a set of measured values or of probable values. This is broadly similar to the approach used in error analysis that preceded the uncertainty technique. However, the GUM provides additional methods of estimating uncertainty based on probability models. The two approaches are equivalent, but uncertainty requires only a knowledge or estimate of the dispersion of measurement about its mean value, and not the existence of a true value. It is assumed that a careful evaluation of the components of measurement uncertainty brings the mean value close to a probable true value, at least well within its margin of uncertainty.

In more general terms, uncertainty is a parameter that characterizes the dispersion of measurable values that can be attributed to their mean value.

By treating standard deviations and probability models as if they approximated to Gaussian (or normal) distributions, the GUM provides a formal methodology for combining components of uncertainty in measurement systems where several input variables combine to determine the result.

Within this formal framework, the GUM can be consistently applied to a range of applications and, thereby, be used to make meaningful comparisons of results.

The HUG seeks to promote an understanding of the nature of measurement uncertainty and its significance in estimating the 'quality' of a measurement or a determination in hydrometry.

Hydrometry is principally concerned with the determination of flow in rivers and man-made channels. This includes:

- environmental hydrometry, i.e. the determination of the flow of natural waters (largely concerned with hydrometric networks, water supply and flood protection);
- industrial hydrometry, i.e. the determination of flows within industrial plants and discharges into the natural environment (largely concerned with environment protection and also irrigation).

Both are the subject of international treaties and undertakings. For this reason, measured data is intended to conform to the GUM to assure that results can be compared.

Hydrometry is also concerned with the determination of rainfall, the movement/diffusion of groundwater and the transport by water flow of sediments and solids. This version of the HUG is concerned with flow determination only.

The results from hydrometry are used by other disciplines to regulate and manage the environment. If knowledge is required of biomass, sedimentary material, toxins, etc., the concentration of these components is determined and their uncertainty estimated. The uncertainty of mass-load can then be determined from the uncertainty of flow determination. The components of this calculation are made compatible through compliance with the GUM.

For practitioners of hydrometry and for engineers, the GUM is not a simple document to refer to. The document has been drafted to provide a legal framework for professional metrologists with a working knowledge of statistical methods and their mathematical representation. A helpful document, see Reference [2], is an abbreviated version of the GUM written to be more accessible to engineers and to specialists in fields other than metrology.

The HUG, although simplifying the concepts, in no way conflicts with the principles and methods of the GUM. Accordingly, the HUG interprets the GUM to apply its requirements to hydrometry in a practical way, and, hopefully, in a way accessible to engineers and those responsible for managing the environment.

In addition, the HUG introduces and develops the Monte Carlo Simulation, a complementary technique, which has benefits for hydrometry, inasmuch as complex measurement systems can be represented realistically.

The HUG summarizes basic hydrometric methods defined in various technical standards. The HUG develops uncertainty estimation formulae from the GUM for these basic methods. The basic hydrometric methods described in the HUG might not be identical to those recited in the published technical standards. In such cases, the methods described in these standards are to be taken as authoritative. However, clauses in technical standards that concern uncertainty should be adapted to be in accordance with the HUG.

NOTE 1 There is no unified definition of space coordinates within the hydrometric standards. The textbook conventional axes are adopted in this document when describing open channel flow: the x axis being horizontal and positive in the mean flow direction, the y axis being orthogonal to the x axis in the horizontal plane and the z axis being vertical positive.

NOTE 2 For a complete appreciation of the scope of definitions used in measurement uncertainty, the reader is referred to the GUM^[1] or to NIST Technical Note 1297^[2].

Hydrometric uncertainty guidance (HUG)

1 Scope

This document provides an understanding of the nature of measurement uncertainty and its significance in estimating the "quality" of a measurement or a determination in hydrometry.

This document is applicable to flow measurements in natural and man-made channels. Rainfall measurements are not covered.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 772, *Hydrometry — Vocabulary and symbols*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 772 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

4 Symbols

Symbols	Explanations	Units
α	coefficient representing the effects of non-uniform energy (velocity) in a channel	b
$\gamma_{xx}, \gamma_{xy}, \gamma_{xz}$	angles between boat axes and the x axis	rad
σ	standard deviation	a
$\Delta'x, \Delta'y$	dispersion of measurement from the mean value of the set of x, y measurements for a symmetric distribution: $\Delta'x = 0,5(x_{\max} - x_{\min})$, etc.	a
$\Delta'x^+, \Delta'x^-$	\pm dispersion about the mean value, \bar{x} , for an asymmetric distribution of measurements where $\Delta'x^+ = (x_{\max} - \bar{x})$ and $\Delta'x^- = (\bar{x} - x_{\min})$	a
Δ	small difference in a measured quantity $\Delta Q, \Delta h, \Delta T$, etc.	a
$\Delta y, \Delta z$	notional small distances in the y and z directions at a cross-section in the channel	m
Dc_2	in the dilution method, the downstream mixed change $(c_m - c_b)$ of concentration of the tracer	mg/l
$A, A(z), A(h)$	cross-section area (in the y, z plane) of the flow	m ²
a Dimensional order depends on its meaning in context.		
b Non-dimensional quantity.		

Symbols	Explanations	Units
B	channel width	m
b	contracted channel width or flume throat width	m
c_b	dilution method, the background concentration of tracer	mg/l
c_T	dilution method, the feed concentration of tracer	mg/l
c_m	dilution method, the downstream mixed concentration of the tracer	mg/l
C	discharge coefficient	b
C_v	velocity coefficient	b
d_i	deviation of a measurement (the i th measurement of a series) from the mean value of that series	a
E	datum elevation of a range measuring device	m
$f(h)$	general function of parameter h	a
F_x, F_y	multiplying factors to be applied to the summation of velocity-area elements to account for the approximation of a summation process to a true integration of continuously varying parameters	b
g	gravitation acceleration	m/s ²
h	head of water relative to a defined datum level in the channel	m
H	total head relative to a defined datum level in the channel	m
i, j	indices of a count $i = 1$ to n , or $j = 1$ to m of a series	a
J	false measurement detection factor	b
K	constant of a flow determination formula for a weir or flume	b
k_1, k_2	constants for the determination of flow by the dilution method	b
M	dilution method, the mass of tracer introduced into the stream	g
n	exponent of a flow determination formula for a weir or flume	b
n, m	number of measurement in a series	a
$p(x)$	probability function	b
Q	flow	m ³ /s
Q_p	estimated flow passing close to boundaries or any region where measurement cannot be determined by the primary means	m ³ /s
Q_T	dilution method, the flow of tracer into the stream	m ³ /s
S	sampling standard deviation of a set of measurements	a
t_e	factor to be applied to small numbers of samples to enable the standard deviation to be representative of large numbers of samples (see Annex A)	b
t_1, t_2	in the dilution method, the interval during which a change in concentration is detectable	s
T	absolute temperature, in Kelvin	°C
T_n	Grubbs' test parameter	°C
$U(x), u(y)$	uncertainty of measured variables x, y , etc.	a
$u_c(p), u_c(q)$	the combined uncertainty of determined results p, q , etc.	a
$u^*(x)$	the percentage uncertainty of a measurement of any quantity x	a
U_{95}	measurement uncertainty expanded to the 95 % level of confidence	a
$V_{\bar{x}}$	mean velocity through a $y \times z$ plane intersecting a channel cross-section of the channel	m/s
$V_x(y, z)$	velocity in the x direction at point y, z in the channel	m/s
\vec{V}	water velocity vector relative to channel	m/s
a Dimensional order depends on its meaning in context.		
b Non-dimensional quantity.		

Symbols	Explanations	Units
\vec{V}_b	boat velocity vector relative to the channel	m/s
\vec{V}'	water velocity vector relative to boat	m/s
V'_x, V'_y, V'_z	water velocity components relative to boat along boat coordinate axes	m/s
$V'_{bx}, V'_{by}, V'_{bz}$	components of boat velocity relative the boat axes	m/s
$\gamma_{xx}, \gamma_{xy}, \gamma_{xz}$	angles between boat axes and the channel x axis	rad
x, y, z	channel coordinates	m
x', y', z'	boat coordinates	m
x, y	measurable variables	a
a Dimensional order depends on its meaning in context.		
b Non-dimensional quantity.		

In this document, the term “uncertainty” refers to measurement uncertainty and the following formulae are used to signify

- a sum of n values of x $x_1 + x_2 + x_3 + \dots x_i + \dots x_n = \sum_{i=1}^n x_i$;
- a difference, $df(x)$ in the function, $f(x)$, due to a small change, Δx in the value x $df(x) = \frac{df}{dx} \Delta x$;
- a value of an integral, F , of a function, $f(x)$, between, $x = x_1$, and $x = x_n$ $F = \int_{x_1}^{x_n} f(x) dx$.

5 ISO/IEC Guide 98-3 (GUM) — Basic definitions and rules

5.1 General

This clause summarizes the methods described in the GUM for the expression of uncertainty in measurement. For a general introduction to measurement uncertainty, refer to [Annex A](#).

5.2 Uncertainty of sets of measurements

The GUM describes measurement uncertainty as a value that characterizes the dispersion of measurements that could reasonably be attributed to the result. The GUM goes on to define standard uncertainty as uncertainty expressed as a standard deviation, s .

From this definition, it follows that uncertainty only deals with the natural spreading in a series of measurement results. It should, therefore, be emphasized that uncertainty does not describe any constant (systematic) deviations of these measurements from the true value. The difference between random and systematic effects is further elaborated upon in [5.3](#).

So, for a set of n measurements, uncertainty is related to the difference between each measured value, x_i , from the average value, \bar{x} of the set. The standard deviation, and hence the uncertainty, $u(x)$ is:

$$u(x) = s = \sqrt{\frac{1}{n-1} \left[(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + (\bar{x} - x_3)^2 + \dots + (\bar{x} - x_n)^2 \right]}$$

where component $d_i = \bar{x} - x_i$ is the deviation of the i th measurement, x_i , from the mean value, \bar{x} .

Or, more concisely:

$$u(x) = s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n d_i^2} \quad (1)$$

where $d_i = \bar{x} - x_i$ is the deviation of the i th measurement from the mean value, \bar{x} shown as [Formula \(2\)](#):

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (2)$$

[Formula \(1\)](#) for the uncertainty of x only applies to steady, stationary stochastic processes, where the mean value and the standard deviation of the sampled process remain unchanged during the whole measurement process.

The uncertainty of the mean value, $u(\bar{x})$ decreases as the number of measurements, n , increases. The GUM relationship for this is shown as [Formula \(3\)](#):

$$u(\bar{x}) = \frac{1}{\sqrt{n}} u(x) \quad (3)$$

It should be noted that [Formula \(3\)](#) applies only for uncorrelated measurements, which means that there is no mutual relationship or connection among these measurements.

5.3 Random and systematic effects

[Formula \(3\)](#) applies only to the random variations of the measured quantity. This random effect is determined from the measured data and, as such, is evaluated after a set of measurements have been taken. Random effects can be determined from analysis of the historic data or by the instrumentation itself if it is designed to analyse the data in real time. Random effects diminish the uncertainty in the average value of a set of n uncorrelated measurements by the factor $\frac{1}{\sqrt{n}}$. Random conditions often

exist as natural turbulence. However, random variation can sometimes occur through human interpretation of a reading of an indicator, such as a staff gauge.

Constant deviations in the measurements that are inherent to the measurement equipment or to the method are called systematic effects. They should be clearly distinguished from the hereto described stochastic or random deviations as described by the term uncertainty. Systematic effects cannot be diminished by the use of [Formula \(3\)](#). During each measurement session, systematic effects can usually be taken as constant for the measurement device. Systematic components:

- a) should be assessed as part of an installation or commissioning procedure, and/or
- b) can be specified beforehand for the equipment by the manufacturer. Refer to [Clause 9](#)
- c) can sometimes be detected and quantified by a careful comparison of a few independent measuring techniques.

Refer to [A.6](#) for more information on random and systematic effects.

For the evaluation of the uncertainty of a continuous stochastic process, include the presence of any unsteady effects also as a random component. This is only allowed, however, if the physical quantity being measured is varying slowly and in a relatively small manner during the measurement process. Such a procedure will of course widen the dispersion of measured values and hence add to the assessment of the random component. Such variation becomes then part of the randomness of the measurement. If during the measurement process the rate and amount of change are such that it significantly exceeds the natural dispersion of measurements, then the result shall be discarded.

5.4 Uncertainty models — Probability distributions

In hydrometry, measurements are often made manually or using automated instruments. They have a margin within which measured values can vary randomly in steady conditions. If they also have a steady offset inherent to the measurement process, this is termed a systematic component. It is commonly expressed as a probability distribution. Probability distributions have standard deviations about the mean value which are equivalent to the standard deviation of discrete measurements as defined above. The standard uncertainty equivalent to [Formulae \(1\)](#) and [\(2\)](#) are shown as [Formulae \(4\)](#) and [\(5\)](#):

$$\bar{x} = \int_{-\Delta x}^{\Delta x} x \cdot p(x) dx \quad (4)$$

and

$$u(x) = \sqrt{\int_{-\Delta x}^{\Delta x} d(x)^2 \cdot p(x) dx} \quad (5)$$

where

- $p(x)$ is a probability density function;
- d is the deviation from the mean;
- Δx is the bin of discrete measurements.

Refer to [Annex A](#) for details.

5.5 Combining uncertainties — Law of propagation of uncertainties

The GUM also defines a rule for combining uncertainties from several sources. It is called "the law of the propagation of uncertainties". For a relationship, f , between a result, y , and variables, x_1, x_2, \dots, x_n , defined as, $y = f(x_1, x_2, \dots, x_n)$, the combined uncertainty, $u_c(y)$, of y is

$$u_c(y)^2 = \sum_{i=1}^{i=n} \left[\frac{\partial f}{\partial x_i} u(x_i) \right]^2$$

or

$$u_c(y)^2 = \left(\frac{\partial f}{\partial x_1} u(x_1) \right)^2 + \left(\frac{\partial f}{\partial x_2} u(x_2) \right)^2 + \dots + \left(\frac{\partial f}{\partial x_n} u(x_n) \right)^2 \quad (6)$$

where x_1, x_2, \dots, x_n are independent variables. This linear approximation with the first derivatives is only allowed, however, when the deviations in the variables x_i are relatively small compared with their mean values.

[Formula \(6\)](#) applies only where the variables x_1, x_2, \dots, x_n are uncorrelated, i.e. if variable x_i changes value, no other x variable is affected by that change. If two or more variables x do influence each other (i.e. they are correlated), then an additional component of uncertainty exists. [Formula \(6\)](#) then becomes

$$u_c(y)^2 = \sum_{i=1}^{i=n} \left(\frac{\partial f}{\partial x_i} u(x_i) \right)^2 + 2 \sum_{i=1}^{i=n-1} \sum_{j=i+1}^{j=n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i x_j) \quad (7)$$

Almost all hydrometric uncertainty estimations require the use of the simpler form, i.e. [Formula \(6\)](#).

The components can be random or systematic. The partial derivatives $\frac{\partial f}{\partial x_n}$ are referred to as “sensitivity coefficients”, and $u(x_i, x_j) = \text{cov}(x_i, x_j)$.

5.6 Expressing results

[Formula \(6\)](#) expresses the final result in terms of standard uncertainty. For the Gaussian probability, used as a model distribution for general analysis, one standard deviation covers only 68 % of the range of possible results. This means that for a result expressed as

$$\text{Flow rate} = 10,8 \text{ m}^3/\text{s} \pm 0,6 \text{ m}^3/\text{s}$$

or

$$Q = \bar{Q} \pm u(Q)$$

only 68 % of the measurement will lie between 10,2 m³/s and 11,4 m³/s. Almost one third of the measurement can be expected to lie outside this band. Such a statement is of little value in hydrometry. A more meaningful statement is required that will cover a larger portion of possible results.

[A.9](#) defines expanded uncertainty. By expanding the margin of uncertainty, a greater portion of the expected range of measurements is covered. For the Gaussian probability distribution, it can be shown that by doubling the uncertainty margin, 95 % of expected measurements are covered. [Subclause A.9](#) defines expanded uncertainty.

The same result expressed in the form

$$\text{Flow rate} = 10,8 \text{ m}^3/\text{s} \pm 1,2 \text{ m}^3/\text{s} \text{ at the 95 \% confidence level}$$

or

$$Q = \bar{Q} \pm U_{95}(Q)$$

means that 95 % of the measurements are expected to lie between 9,6 m³/s and 12,0 m³/s. This is a more practical expression of the result.

In hydrometry, all measurements shall be expressed at the 95 % confidence level with a statement of the form:

$$\text{Quantity} = \text{Value} \pm \text{uncertainty at the 95 \% confidence level}$$

or

$$\text{Quantity} = \text{Value} \pm \text{percentage uncertainty at the 95 \% confidence level}$$

Refer to [A.9](#) for more detail.

6 Open channel flow — Velocity area methods

6.1 General

[Figure 1](#) shows the coordinate system used in this document with orthogonal axes x , y , z . The mean velocity is calculated in the x direction. The xy -plane is horizontal. The z axis is vertical. Note that a velocity \vec{v} vector representing the mean velocity does not have to align with the x axis. The flow in the channel can be determined from velocities passing obliquely through an intersecting yz plane.