
Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application —

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Applications to unfolding methods

ISO 11929-3:2019
<https://standards.iteh.ai/catalog/standards/sist/289589c7-779e-4dc7-b69a1572637021c/iso-11929-3-2019>
Détermination des limites caractéristiques (seuil de décision, limite de détection et extrémités de l'intervalle élargi) pour mesurages de rayonnements ionisants — Principes fondamentaux et applications —

Partie 3: Applications aux méthodes de déploiement



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Published in Switzerland

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by This document was prepared by ISO/TC 85, *Nuclear energy, nuclear technologies, and radiological protection*, Subcommittee SC 2, *Radiological protection*.

This second edition of ISO 11929-3 together with ISO 11929-1, ISO 11929-2, cancels and replaces ISO 11929:2010 which have been technically revised, specifically with reference to the type of statistical treatment of the data and extended with respect to the methodology of uncertainty assessment from the ISO/IEC Guide 98-3:2009, to the ISO/IEC Guide 98-3-1:2008.

A list of all the parts in the ISO 11929 series can be found on the ISO website.

Introduction

Measurement uncertainties and characteristic values, such as the decision threshold, the detection limit and limits of the coverage interval for measurements as well as the best estimate and its associated standard measurement uncertainty, are of importance in metrology in general and for radiological protection in particular. The quantification of the uncertainty associated with a measurement result provides a basis for the trust an individual can have in a measurement result. Conformity with regulatory limits, constraints or reference values can only be demonstrated by taking into account and quantifying all sources of uncertainty. Characteristic limits provide, at the end, the basis for deciding under uncertainty.

This standard provides characteristic values of a non-negative measurand of ionizing radiation. It is also applicable for a wide range of measuring methods extending beyond measurements of ionizing radiation.

The limits to be provided according to the ISO 11929 series for specified probabilities of wrong decisions allow detection possibilities to be assessed for a measurand and for the physical effect quantified by this measurand as follows:

- the “decision threshold” allows a decision to be made on whether or not the physical effect quantified by the measurand is present;
- the “detection limit” indicates the smallest true quantity value of the measurand that can still be detected with the applied measurement procedure; this gives a decision on whether or not the measurement procedure satisfies the requirements and is therefore suitable for the intended measurement purpose;
- the “limits of the coverage interval” enclose, in the case of the physical effect recognized as present, a coverage interval containing the true quantity value of the measurand with a specified probability.

Hereinafter, the limits mentioned are jointly called the “characteristic limits”.

NOTE According to ISO/IEC Guide 99:2007 updated by JCGM 200:2012 the term “coverage interval” is used here instead of “confidence interval” in order to distinguish the wording of Bayesian terminology from that of conventional statistics.

All the characteristic values are based on Bayesian statistics and on the ISO/IEC 98-3 Guide to the Expression of Uncertainty in Measurement as well as on the ISO/IEC Guide 98-3-1 and ISO/IEC 98-3-2. As explained in detail in ISO 11929-2, the characteristic values are mathematically defined by means of moments and quantiles of probability distributions of the possible measurand values.

Since measurement uncertainty plays an important part in ISO 11929, the evaluation of measurements and the treatment of measurement uncertainties are carried out by means of the general procedures according to the ISO/IEC Guide 98-3 and to the ISO/IEC Guide 98-3-1; see also References [9] to [13]. This enables the strict separation of the evaluation of the measurements, on the one hand, and the provision and calculation of the characteristic values, on the other hand. The ISO 11929 series makes use of a theory of uncertainty in measurement [14] to [16] based on Bayesian statistics (e.g. [17] to [22]) in order to allow to take into account also those uncertainties that cannot be derived from repeated or counting measurements. The latter uncertainties cannot be handled by frequentist statistics.

Because of developments in metrology concerning measurement uncertainty laid down in the ISO/IEC Guide 98-3, ISO 11929:2010 was drawn up on the basis of the ISO/IEC Guide 98-3, but using Bayesian statistics and the Bayesian theory of measurement uncertainty. This theory provides a Bayesian foundation for the ISO/IEC Guide 98-3. Moreover, ISO 11929:2010 was based on the definitions of the characteristic values [9], the standard proposal [10], and the introducing article [11]. It unified and replaced all earlier parts of ISO 11929 and was applicable not only to a large variety of particular measurements of ionizing radiation but also, in analogy, to other measurement procedures.

Since the ISO/IEC Guide 98-3-1 has been published, dealing comprehensively with a more general treatment of measurement uncertainty using the Monte Carlo method in complex measurement

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evaluations. This provided an incentive for writing a corresponding Monte Carlo supplement^[12] to ISO 11929:2010 and to revise ISO 11929:2010. The revised ISO 11929 is also essentially founded on Bayesian statistics and can serve as a bridge between ISO 11929:2010 and the ISO/IEC Guide 98-3-1. Moreover, more general definitions of the characteristic values (ISO 11929-2) and the Monte Carlo computation of the characteristic values make it possible to go a step beyond the present state of standardization laid down in ISO 11929:2010 since probability distributions rather than uncertainties can be propagated. It is thus more comprehensive and extending the range of applications.

The revised ISO 11929, moreover, is more explicit on the calculation of the characteristic values. It corrects also a problem in ISO 11929:2010 regarding uncertain quantities and influences, which do not behave randomly in measurements repeated several times. Reference ^[13] gives a survey on the basis of the revision. Furthermore, this document gives detailed advice how to calculate characteristic values in the case of multivariate measurements using unfolding methods. For such measurements, the ISO/IEC Guide 98-3-2 provides the basis of the uncertainty evaluation.

Formulas are provided for the calculation of the characteristic values of an ionizing radiation measurand via the “standard measurement uncertainty” of the measurand (hereinafter the “standard uncertainty”) derived according to the ISO/IEC Guide 98-3 as well as via probability density functions (PDFs) of the measurand derived in accordance with ISO/IEC Guide 98-3-1. The standard uncertainties or probability density functions take into account the uncertainties of the actual measurement as well as those of sample treatment, calibration of the measuring system and other influences. The latter uncertainties are assumed to be known from previous investigations.

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Determination of the characteristic limits (decision threshold, detection limit and limits of the coverage interval) for measurements of ionizing radiation — Fundamentals and application —

Part 3: Applications to unfolding methods

1 Scope

The ISO 11929 series specifies a procedure, in the field of ionizing radiation metrology, for the calculation of the “decision threshold”, the “detection limit” and the “limits of the coverage interval” for a non-negative ionizing radiation measurand when counting measurements with preselection of time or counts are carried out. The measurand results from a gross count rate and a background count rate as well as from further quantities on the basis of a model of the evaluation. In particular, the measurand can be the net count rate as the difference of the gross count rate and the background count rate, or the net activity of a sample. It can also be influenced by calibration of the measuring system, by sample treatment and by other factors.

ISO 11929 has been divided into four parts covering elementary applications in ISO 11929-1, advanced applications on the basis of the ISO/IEC Guide 98-3-1 in ISO 11929-2, applications to unfolding methods in this document, and guidance to the application in ISO 11929-4.

ISO 11929-1 covers basic applications of counting measurements frequently used in the field of ionizing radiation metrology. It is restricted to applications for which the uncertainties can be evaluated on the basis of the ISO/IEC Guide 98-3 (JCGM 2008). In Annex A of ISO 11929-1:2019, the special case of repeated counting measurements with random influences is covered, while measurements with linear analogous ratemeters, are covered in Annex B of ISO 11929-1:2019.

ISO 11929-2 extends the former ISO 11929:2010 to the evaluation of measurement uncertainties according to the ISO/IEC Guide 98-3-1. ISO 11929-2 also presents some explanatory notes regarding general aspects of counting measurements and on Bayesian statistics in measurements.

This document deals with the evaluation of measurements using unfolding methods and counting spectrometric multi-channel measurements if evaluated by unfolding methods, in particular, for alpha- and gamma-spectrometric measurements. Further, it provides some advice on how to deal with correlations and covariances.

ISO 11929-4 gives guidance to the application of the ISO 11929 series, summarizes shortly the general procedure and then presents a wide range of numerical examples.

ISO 11929 Standard also applies analogously to other measurements of any kind especially if a similar model of the evaluation is involved. Further practical examples can be found, for example, in ISO 18589[7], ISO 9696[2], ISO 9697[3], ISO 9698[4], ISO 10703[5], ISO 7503[1], ISO 28218[8], and ISO 11665[6].

NOTE A code system, named UncertRadio, is available for calculations according to ISO 11929- 1 to ISO 11929-3. UncertRadio[35][36] can be downloaded for free from <https://www.thuenen.de/en/fi/fields-of-activity/marine-environment/coordination-centre-of-radioactivity/uncertradio/>. The download contains a setup installation file which copies all files and folders into a folder specified by the user. After installation one has to add information to the PATH of Windows as indicated by a pop-up window during installation. English language can be chosen and extensive “help” information is available.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 80000-1, *Quantities and units — Part 1: General*

ISO 80000-10, *Quantities and units — Part 10: Atomic and nuclear physics*

ISO/IEC Guide 98-3, *Uncertainty of measurement — Part 1: Guide to the expression of uncertainty in measurement, JCGM 100:2008*

ISO/IEC Guide 98-3-1, *Evaluation of measurement data — Supplement 1 to the “Guide to the expression of uncertainty in measurement” — a Propagation of distributions using a Monte Carlo method, JCGM 101:2008*

ISO/IEC Guide 98-3-2, *Evaluation of measurement data — Supplement 2 to the “Guide to the expression of uncertainty in measurement” — Models with any number of output quantities, JCGM 102:2011*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 80000-1, ISO 80000-10, ISO/IEC Guide 98-3, ISO/IEC Guide 98-3-1, ISO/IEC 98-3-2, ISO/IEC Guide 99 and ISO 3534-1 and the following apply.

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

3.1 quantity value value of a quantity value

number and reference together expressing magnitude of a quantity

[SOURCE: JCGM 200:2012, 1.19]

3.2 measurement

process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity

[SOURCE: JCGM 200:2012, 2.1]

3.3 measurand

quantity intended to be measured

[SOURCE: JCGM 200:2012, 2.3]

3.4 coverage interval

interval containing the set of true quantity values of a measurand with a stated probability, based on the information available

[SOURCE: JCGM 200:2012, 2.36]

Note 1 to entry: A coverage interval does not need to be centred on the chosen measured quantity value (see JCGM 101:2008).

Note 2 to entry: A coverage interval should not be termed “confidence interval” to avoid confusion with the statistical concept.

3.5 measurement method method of measurement

generic description of a logical organization of operations used in a measurement

[SOURCE: JCGM 200:2012, 2.4]

3.6 measurement procedure

detailed description of a measurement according to one or more measurement principles and to a given measurement method, based on a measurement model and including any calculation to obtain a measurement result

[SOURCE: JCGM 200:2012, 2.6]

3.7 measurement result result of measurement

set of quantity values being attributed to a measurand together with any other available relevant information

[SOURCE: JCGM 200:2012, 2.9]

3.8 measured quantity value value of a measured quantity measured value

quantity value representing a measurement result

[SOURCE: JCGM 200:2012, 2.10]

3.9 true quantity value true value of a quantity true value

quantity value consistent with the definition of a quantity

[SOURCE: JCGM 200:2012, 2.11]

Note 1 to entry: In the Error Approach to describing measurement, a true quantity value is considered unique and, in practice, unknowable. The Uncertainty Approach is to recognize that, owing to the inherently incomplete amount of detail in the definition of a quantity, there is not a single true quantity value but rather a set of true quantity values consistent with the definition. However, this set of values is, in principle and in practice, unknowable. Other approaches dispense altogether with the concept of true quantity value and rely on the concept of metrological compatibility of measurement results for assessing their validity.

Note 2 to entry: When the definitional uncertainty associated with the measurand is considered to be negligible compared to the other components of the measurement uncertainty, the measurand may be considered to have an “essentially unique” true quantity value. This is the approach taken by the ISO/IEC Guide 98-3 and associated documents, where the word “true” is considered to be redundant.

3.10
measurement uncertainty
uncertainty of measurement
uncertainty

non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used

[SOURCE: JCGM 200:2012, 2.26]

Note 1 to entry: Measurement uncertainty includes components arising from systematic effects, such as components associated with corrections and the assigned quantity values of measurement standards, as well as the definitional uncertainty. Sometimes estimated systematic effects are not corrected for but, instead, associated measurement uncertainty components are incorporated.

Note 2 to entry: The parameter may be, for example, a standard deviation called standard measurement uncertainty (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability.

Note 3 to entry: Measurement uncertainty comprises, in general, many components. Some of these may be evaluated by Type A evaluation of measurement uncertainty from the statistical distribution of the quantity values from series of measurements and can be characterized by standard deviations. The other components, which may be evaluated by Type B evaluation of measurement uncertainty, can also be characterized by standard deviations, evaluated from probability density functions based on experience or other information.

Note 4 to entry: In general, for a given set of information, it is understood that the measurement uncertainty is associated with a stated quantity value attributed to the measurand. A modification of this value results in a modification of the associated uncertainty.

3.11
model of evaluation

set of mathematical relationships between all measured and other quantities involved in the evaluation of measurements

Note 1 to entry: The model of evaluation does not need to be an explicit function; it can also be an algorithm realized by a computer code.

3.12
decision threshold

value of the estimator of the measurand, which when exceeded by the result of an actual measurement using a given measurement procedure of a measurand quantifying a physical effect, is used to decide that the physical effect is present

Note 1 to entry: The decision threshold is defined such that in cases where the measurement result, y , exceeds the decision threshold, y^* , the probability that the true value of the measurand is zero is less or equal to a chosen probability for a wrong decision, α .

Note 2 to entry: If the result, y , is below the decision threshold, y^* , it is decided to conclude that the result cannot be attributed to the physical effect; nevertheless it cannot be concluded that it is absent.

3.13
detection limit

smallest true value of the measurand which ensures a specified probability of being detectable by the measurement procedure

Note 1 to entry: With the *decision threshold* (3.12), the detection limit is the smallest true value of the measurand for which the probability of wrongly deciding that the true value of the measurand is zero is equal to a specified value, β , when, in fact, the true value of the measurand is not zero. The probability of being detectable is consequently $(1 - \beta)$.

Note 2 to entry: The terms detection limit and decision threshold are used in an ambiguous way in different standards (e.g. standards related to chemical analysis or quality assurance). If these terms are referred to one has to state according to which standard they are used.

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3.14**limits of the coverage interval**

values which define a coverage interval

Note 1 to entry: A coverage interval is sometimes known as a credible interval or a Bayesian interval. Its limits are calculated in the ISO 11929 series to contain the true value of the measurand with a specified probability $(1 - \gamma)$.

Note 2 to entry: The definition of a coverage interval is ambiguous without further stipulations. In this standard two alternatives, namely the probabilistically symmetric and the shortest coverage interval are used.

3.15**best estimate of the true quantity value of the measurand**

expectation value of the probability distribution of the true quantity value of the measurand, given the experimental result and all prior information on the measurand

Note 1 to entry: The best estimate is the one, among all possible estimates of the measurand on the basis of given information, which is associated with the minimum uncertainty.

3.16**guideline value**

value which corresponds to scientific, legal or other requirements with regard to the detection capability and which is intended to be assessed by the measurement procedure by comparison with the detection limit

Note 1 to entry: The guideline value can be given, for example, as an activity, a specific activity or an activity concentration, a surface activity or a dose rate.

Note 2 to entry: The comparison of the detection limit with a guideline value allows a decision on whether or not the measurement procedure satisfies the requirements set forth by the guideline value and is therefore suitable for the intended measurement purpose. The measurement procedure satisfies the requirement if the detection limit is smaller than the guideline value.

Note 3 to entry: The guideline value shall not be confused with other values stipulated as conformity requests or as regulatory limits.

3.17**background effect**

measurement effect caused by radiation other than that caused by the object of the measurement itself

Note 1 to entry: The background effect can be due to natural radiation sources or radioactive materials in or around the measuring instrumentation and also to the sample itself (for instance, from other lines in a spectrum).

3.18**background effect in spectrometric measurement**

number of events of no interest in the region of a specific line in the spectrum

3.19**net effect**

contribution of the possible radiation of a measurement object (for instance, of a radiation source or radiation field) to the measurement effect

3.20**gross effect**

measurement effect caused by the background effect and the net effect

3.21**shielding factor**

factor describing the reduction of the background count rate by the effect of shielding caused by the measurement object

3.22

relaxation time constant

duration in which the output signal of a linear-scale ratemeter decreases to 1/e times the starting value after stopping the sequence of the input pulses

4 Quantities and symbols

The symbols for auxiliary quantities and the symbols only used in the annexes are not listed. Physical quantities are denoted by upper-case letters but shall be carefully distinguished from their values, denoted by the corresponding lower-case letters.

<i>A</i>	response matrix of the spectrometer
A_{ik}	elements of the response matrix <i>A</i>
a_0, a_1	parameters in an algebraic expression of the standard uncertainty of a net counting rate
<i>b</i>	width of a gamma peak, in channels
c_j	position parameter of a peak <i>j</i> , in gamma-ray or alpha-ray spectrometry
diag	indicator for a diagonal matrix
<i>D</i>	matrix converting measured activities to decay corrected activity concentrations
<i>d</i>	set of statistically independent quantities
f_B	function representing the analogue of the total peak area method design factor $[1 + b/(2L)]$ for the peak fitting case (gamma-ray spectrometry)
$f_{att,i}$	self-attenuation correction factor for gamma-line <i>i</i>
$f_{TCS,i}$	true-coincidence-summing correction factor for gamma-line <i>i</i>
f_d	decay correction factor including the decay during the measurement
G_k	function of the input quantities X_i ($i = 1, \dots, m$)
<i>G</i>	column matrix of the G_k
<i>h</i>	full width at half-maximum of a peak, in channels
$h(\cdot)$	function as part of an implicit model
$H(\vartheta_i)$	functional relationship representing the spectral density at ϑ_i of a multi-channel spectrum
<i>i</i>	number of a channel in a multi-channel spectrum obtained by a spectrometric nuclear radiation measurement ($i = 1, \dots, m$)
<i>J</i>	matrix of partial derivatives of y^+ with respect to parameters <i>y</i>
<i>L</i>	width of a background region (in channels) adjacent to a gamma peak
L_k	<i>k</i> -th element of a system of functions describing spectral densities, which constitute by superposition the total fitting function
<i>m</i>	number of input quantities; or number of channels in the spectrum; number of lines per nuclide used for activity calculation; or a parameter index

N_i	Poisson-distributed random variable of events counted in channel, i , during the measuring time, t ($i = 1, \dots, m$)
n_i	number of events counted in a channel, i , during the measuring time, t ($i = 1, \dots, m$), estimate of N_i
n	number of output quantities in unfolding
n_g	gross counts in a peak region
n_0	average background counts per channel (spectrum)
p_i	estimate of an input quantity which is not subject to fitting (parameter); contained in the response matrix A
p	column matrix of the p_i
p_c	values of non-linear parameters held fixed at their calibrated estimates
$p_{\alpha,i}$	alpha emission probability of gamma-line i
$p_{\gamma,i}$	gamma emission probability of gamma-line i
q	column matrix of input quantities considered as parameters; mainly contained in the matrix D
Q	matrix of partial derivatives of y with respect to parameters p
Q'	matrix of partial derivatives of y^+ with respect to parameters q
R_{ni}	net counting rate of the peak i of interest
$R_{ni,0}$	net counting rate of a background spectrum peak at the position of the peak i of interest
R_{gi}	gross counting rate of the peak i of interest
R_{Ti}	counting rate of the trapezoidal background continuum of the peak i of interest
t	duration of measurement
X_i	random variable of the rate of events counted in channel i during the measuring time, t , input quantity of the evaluation, $X_i = N_i/t$ ($i = 1, \dots, m$)
X	column matrix of the X_i
x_i	rate of events counted in channel, i , during the measuring time, t , $x_i = n_i/t$ ($i = 1, \dots, m$), estimate of X_i
x	column matrix of the x_i
x_{net}	column matrix of net counting rates
$u(x_i, x_j)$	covariance associated with x_i and x_j
$u(y_k)$	standard uncertainty associated with y_k
U_x	uncertainty matrix of X
U_y	uncertainty matrix of Y
w	column matrix of input estimates; $w = (x_1, \dots, x_m, p_1, p_2, \dots)^T$ (transposed row matrix)