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Control charts —

Part 7: **Multivariate control charts**

Cartes de contrôle —

Partie 7: Cartes de contrôle multivariées

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Foreword

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This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

A list of all parts in the ISO 7870 series can be found on the ISO website.

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Introduction

When a number of quality characteristics are to be controlled simultaneously, the usual practice has been to maintain a separate (univariate) chart for each characteristic. Unfortunately, this can give misleading results when the characteristics are highly correlated. Process monitoring of problems in which several related variables are of interest are collectively known as multivariate statistical process control (MSPC). The most useful tools of multivariate statistical process control are multivariate control charts. Multivariate control charts are applied for statistical process evaluation and control under the consideration of dependability between quality characteristics.

The function of a multivariate statistical process control system is to provide a statistical signal when assignable causes of variation are present. The systematic elimination of assignable causes of excessive variation, through continuous determined efforts, brings the process into a state of statistical control. Once the process is operating in statistical control, its performance is predictable and its capability to meet the specifications can then be assessed.

The main purpose of this document is to show how multivariate control charts can be used for process control in terms of SPC and how the state of process stability can be assessed in a multivariate way. ISO 22514-6 provides a calculation method for capability statistics for process parameters or product characteristics following a multivariate normal distribution or approximately multivariate normal.

Multivariate charts are based on multivariate characteristics where more than one characteristic is to be monitored in connection with others. In practice, a multivariate control chart is always applied with the support of software, such as Minitab, JMP, and Q-DAS[1)].

1) MINITAB is the trade name of a product supplied by Minitab Inc. JMP is the trade name of a product supplied by SAS Institute Inc. Q-DAS is the trade name of a product supplied by Q-DAS GmbH. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of these products.

Control charts —

Part 7:
Multivariate control charts

1 Scope

This document describes the construction and use of multivariate control charts in statistical process control (SPC) and establishes methods for using and understanding this generalized approach to control charts where the characteristics being measured are from variables data.

The use of principal component analysis (PCA) and partial least squares (PLS) in the field of multivariate statistical process control is not presented in this document

NOTE The document describes the current state of the art of multivariate control charts that are being applied in practice nowadays. It does not describe the current state of scientific research on the topic.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-2 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1
multivariate characteristics
multivariate quantity where the set of features consists of *d* quantities that are alone or combined with the quality of a product

Note 1 to entry: Following ISO 7870-2, these quantities are denoted as quality characteristics X_i where $i = 1, 2, ..., d$.

Note 2 to entry: The observation of multivariate characteristics can be expressed as the vector $\mathbf{x}=(x_1, x_2, ..., x_d)^T$. Thus, a multivariate quantity can be considered as a feature vector of a product. The value of the multivariate quantity is represented by a point in the *d*-dimensional feature space.

Note 3 to entry: All single quantities combined in the multivariate vector can be measured in the same product or object.

Note 4 to entry: If the multivariate quantity is described by means of statistics, the vector is considered as a random vector following a *d*-dimensional multivariate distribution.

3.2
confidence region
d-dimensional region for a multivariate characteristics of d-dimension and defined for a specified confidence level

Note 1 to entry: The region is limited by lines, surfaces or hyper-surfaces in the d-dimensional space.

Note 2 to entry: Form and size of the region are defined by one or more parameters.

4 Abbreviated terms and symbols

4.1 Abbreviated terms

SPC	statistical process control
MSPC	multivariate statistical process control
PCA	principal component analysis
PLS	partial least squares
UCL	upper control limit
LCL	lower control limit
ARL	average run length
EWMA	exponential weighted moving average
MEWMA	multivariate exponential weighted moving average

4.2 Symbols

$B_{1-\alpha,\nu_1,\nu_2}$	the 1 − α quantile of beta distribution with degree of freedom ν_1 and ν_2
d	number of dimensions for multivariate characteristics
D_j^2	the statistic plotted of a phase IIχ^2 control chart
$E(\lvert\mathbf{S}\rvert)$	mean of $\lvert\mathbf{S}\rvert$
$F_{1-\alpha,\nu_1,\nu_2}$	the 1 − α quantile of F distribution with degree of freedom ν_1 and ν_2
h	upper control limit of MEWMA control chart
L_{CL}	lower control limit
m	number of subgroups
n	size of each subgroup
$\mathrm{N_d}(\boldsymbol{\mu},\boldsymbol{\Sigma})$	d-dimensional normal distribution with $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$
s_{ab}	covariance between the a-th and b-th quality characteristics with n=1
s_{abj}	covariance between the a-th and b-th quality characteristics in the j-th subgroup with n>1
s_i^2	variance of the i-th quality characteristic with n=1

s_{ij}^2	variance of the i-th quality characteristic in the j-th subgroup with n>1
$\bar{s}_i^2$	average of s_{ij}^2 over all m subgroups for the i-th quality characteristic with n>1
$\bar{s}_{ab}$	average of s_{abj} over all m subgroups for the covariance between the a-th and b-th quality characteristics with n>1
$\mathbf{S}$	sample variance-covariance matrix with n=1
$\bar{\boldsymbol{S}}$	sample variance-covariance matrix with n>1
$\lvert\mathbf{S}\rvert$	determinant of the sample variance-covariance matrix **S**
T_j^2	the statistic plotted of a phase I T^2-chart.
T_f^2	the statistic plotted of a phase II T^2-chart
tr	trace operator
U_{CL}	upper control limit
$V(\lvert\mathbf{S}\rvert)$	variance of $\lvert\mathbf{S}\rvert$
x_{ij}	the j-th observation on the i-th quality characteristic with n=1
x_{ijk}	the k-th observation in the j-th subgroup on the i-th quality characteristic with n>1
$\bar{x}_{ij}$	mean of the i-th quality characteristic in the j-th subgroup with n>1
$\bar{\bar{x}}_i$	average of $\bar{x}_{ij}$ over all m subgroups for the i-th quality characteristic with n>1
$\mathbf{x}$	an observation vector
$\mathbf{x}_j$	vector of j-th observation with n=1
$\mathbf{x}_f$	vector of a future individual observation with n=1
$\bar{\mathbf{x}}$	sample mean vector with n=1
$\bar{\mathbf{x}}_j$	mean of the j-th rational subgroup with n>1
$\bar{\mathbf{x}}_f$	mean of a future rational subgroup with n>1
$\bar{\bar{\mathbf{x}}}$	sample mean vector with n>1
$\{\bar{\bar{x}}_i\}$	i-th element of the vector $\bar{\bar{\mathbf{x}}}$
Y_j^2	the statistic plotted of MEWMA control chart.
$\mathbf{Z}_j$	MEWMA statistic
$\chi_{1-\alpha,\nu}^2$	the $1-\alpha$ quantile of χ^2 distribution with degree of freedom ν
δ	shift size of the mean vector
$\boldsymbol{\lambda}$	MEWMA moving parameter vector
λ	EWMA moving parameter, $0 < \lambda \le 1$
$\boldsymbol{\mu}$	mean vector of multivariate characteristics

$\boldsymbol{\mu}_0$ pre-specified mean vector of multivariate characteristics

ρ_{y_1,y_2} correlation coefficient between y_1 and y_2

$\boldsymbol{\Sigma}$ variance-covariance matrix of multivariate characteristics

Σ_0 pre-specified variance-covariance matrix of multivariate characteristics

Σ_{Z_j} variance-covariance matrix of MEWMA statistic $\mathbf{Z}_j$

$(\cdot)^{-1}$ inverse operator

$(\cdot)^{\mathrm{T}}$ transpose operator

5 Purpose and classification of multivariate control charts

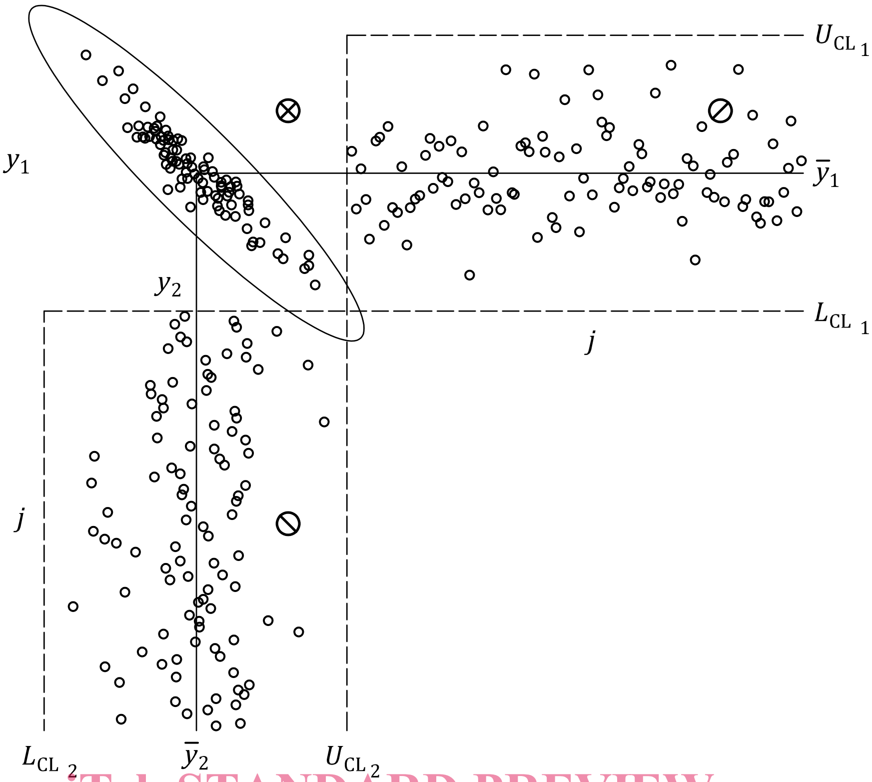
5.1 Purpose and applying conditions for multivariate control charts

There are many situations in which the simultaneous monitoring or control of two or more related quality characteristics is necessary. The difficulty with using independent univariate control charts is illustrated in Figure 1. Only two quality characteristics (y_1, y_2) are considered for ease of illustration.

Suppose that, when the process is in a state of statistical control where only common cause variation is present, both y_1 and y_2 follow a normal distribution but are correlated ($\rho_{y_1,y_2} = -0{,}94$) as illustrated in the joint plot of y_1 vs. y_2 in Figure 1. The ellipse represents a contour for the in-control process, with 0,997 3-quantile, corresponding to risk of a false alarm of 0,002 7 in the Shewhart chart, and the points represent a set of individual observations from this distribution. The same observations are also plotted in Figure 1 as individual Shewhart control charts on y_1 and y_2 vs. the observation number (time) with their corresponding upper and lower control limits (the 0,998 65-quantiles).

By looking at each of the individual Shewhart control charts, the process appears to be clearly in a state of statistical control, and none of the points give any indication of a problem. The true situation is only revealed in the bivariate y_1 vs. y_2 plot where it is seen that the lot of product indicated by the $\otimes$ is clearly outside the confidence region and is clearly different from the normal "in-control" population of the product.

If the quality characteristics are not independent, which usually would be the case if they relate to the same product, there is no easy way to measure the distortion in the joint control procedure. Process-monitoring problems in which several related variables are of interest are sometimes called multivariate quality control problems. This subject is particularly important, as automatic inspection procedures make it relatively easy to measure many parameters on each unit of product manufactured. For example, many chemical and process plants and semi-conductor manufacturers routinely maintain manufacturing databases with the process and quality data on hundreds of variables. Monitoring or analysing these data with univariate SPC procedures is often ineffective. Multivariate control charts are applied for statistical process evaluation and control under consideration of dependability between the product or process characteristics.



Key

$\bar{y}_1$	mean of y_1
$\bar{y}_2$	mean of y_2
j	observation number
U_{CL_1}	upper control limit of y_1
L_{CL_1}	lower control limit of y_1
U_{CL_2}	upper control limit of y_2
L_{CL_2}	lower control limit of y_2
⊘	point corresponding to y_1
⦸	point corresponding to y_2
⊗	point corresponding to (y_1, y_2)

Figure 1 — Quality control of two variables

Multivariate control charts work well when the number of process variables is not too large – ten or fewer. As the number of variables grows, however, traditional multivariate control charts lose efficiency with regard to shift detection. A popular approach in these situations is to reduce the dimensionality of the problem. This can be done with the use of projection methods such as principal component analysis (PCA) or partial least squares (PLS). These two methods are based on building a model from a historical data set, that is assumed to be in control. After the model has been built, a future observation is checked as to whether it fits well or not in the model.

In the SPC univariate case, the normal distribution is generally used to describe the behaviour of a continuous quality characteristic. The same approach can be used in the multivariate case. A multivariate normal distribution is applied as the basic assumption for a multivariate characteristics.

5.2 Classification of multivariate control charts

If the multivariate characteristics is considered to be a random vector with a multivariate normal distribution, this distribution is characterized by a mean vector $\boldsymbol{\mu}$ and a variance-covariance matrix $\boldsymbol{\Sigma}$ (see Annex C). Obviously from the viewpoint of the application of multivariate process control, multivariate control charts can be applied to monitor the mean shift and process dispersion separately. Thus, for the application, multivariate control charts can be classified as follows:

a) multivariate control charts for mean shift;

b) multivariate control charts for process dispersion.

For the mean shift, multivariate control charts with unweighted averages are analogous to the Shewhart $\bar{X}$ chart or chart for individuals. They use information only from the current sample and are relatively insensitive to small and moderate shifts in the mean vector. Multivariate control charts with weighted averages such as multivariate EWMA control chart can be used to overcome this problem. Just like EWMA charts are generally used for detecting small shifts in the process mean and they usually detect shifts of 0,5 sigma to 2 sigma much faster. Thus, multivariate control charts for mean shift can be classified as follows:

i) multivariate control charts with unweighted averages (see Clause 6), such as χ^2 and T^2 chart;

ii) multivariate control charts with weighted averages (see Clause 7), such as multivariate EWMA control chart.

Figure 2 is given to show how to select multivariate control charts.

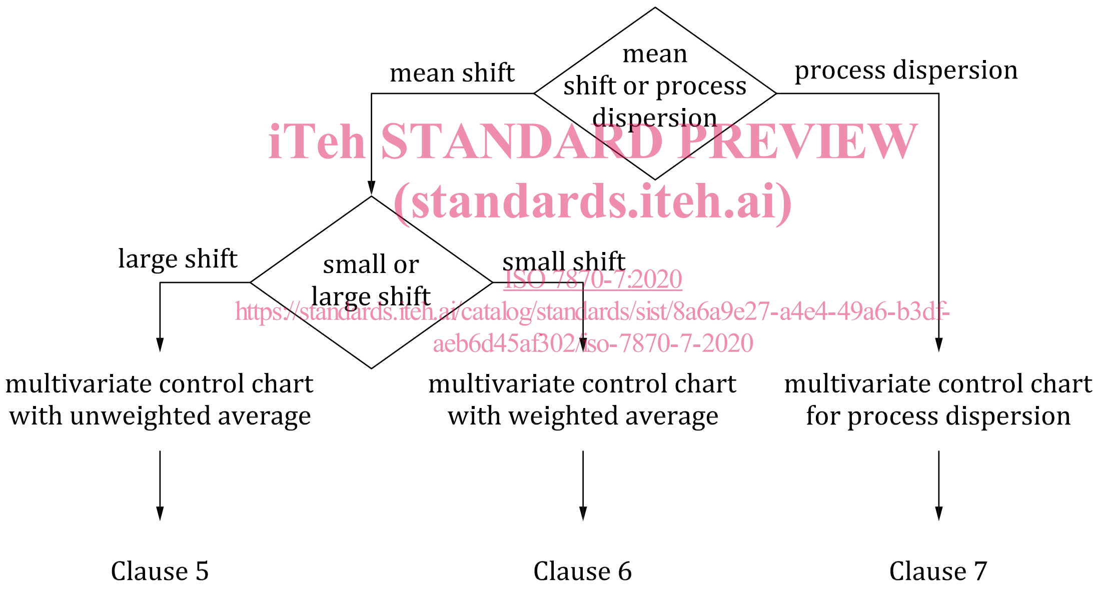


Figure 2 — Multivariate control chart selection flow chart

6 Multivariate control charts with unweighted averages for mean shift

6.1 General

For each of the multivariate control charts, there are two distinct situations:

a) when no pre-specified process parameter values are given, and

b) when pre-specified process parameter values are given.

The pre-specified or known process parameter values can be defined by target values or by requirements or by estimated values that have been determined by the data under the condition of a process in control.