INTERNATIONAL STANDARD

ISO 7870-9

First edition
2020-06

Control charts —

Part 9: Control charts for stationary processes

Cartes de contrôle —

Partie 9: Cartes de contrôle de processus stationnaires

Reference number
ISO 7870-9:2020(E)

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in product and process management*.

A list of all parts in the ISO 7870 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user’s national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Statistical process control (SPC) techniques are widely used in industry for process monitoring and quality improvement. Various statistical control charts have been developed to monitor the process mean and variability. Traditional SPC methodology is based on a fundamental assumption that process data are statistically independent. Process data, however, are not always statistically independent from each other. In the industry for continuous productions such as the chemical industry, most process data on quality characteristics are self-correlated over time or autocorrelated. In general, autocorrelation can be caused by the measurement system, the dynamics of the process, or both. In many cases, the data can exhibit a drifting behaviour. In biology, random biological variation, for example the random burst in the secretion of some substance that influences the blood pressure, can have a sustained effect so that several consecutive measurements are all influenced by the same random phenomenon. In data collection, when the sampling interval is short, autocorrelation, especially the positive autocorrelation of the data, is a concern. Under such conditions, traditional SPC procedures are not effective and appropriate for monitoring, controlling and improving process quality.

Autocorrelated processes can be classified in two kinds of processes, based on whether they are stationary or nonstationary.

1) Stationary process – a direct extension of an independent and identically distributed (i.i.d.) sequence. An autocorrelated process is stationary if it is in a state of "statistical equilibrium". This implies that the basic behaviour of the process does not change in time. In particular, a stationary process has identical means and variances.

2) Nonstationary process.

Detailed information about stochastic process and time series can be found in Annex A.

To accommodate autocorrelated data, some SPC methodologies have been developed. Mainly, there are two approaches. The first approach is to use a process residual chart after fitting a time series model or other mathematical model to the data. Another more direct approach is to modify the existing charts, for example by adjusting the control limits based on process autocorrelation.

The aim of this document is to outline the major process control charts for monitoring both of the process mean and the process variance when the process is autocorrelated.

Control charts —

Part 9:
Control charts for stationary processes

1 Scope

This document describes the construction and applications of control charts for stationary processes.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions, and abbreviated terms and symbols

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-2 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1.1
autocovariance
internal covariance between members of series of observations ordered in time

3.1.2
control charts for autocorrelated processes
statistical process control charts applied to autocorrelated processes

3.2 Abbreviated terms and symbols

3.2.1 Abbreviated terms

ARL	average run length
i.i.d.	independent and identically distributed
SPC	statistical process control
ACF	autocorrelation function
AR(1)	first order autoregressive process
EWMA	exponentially weighted moving average
EWMAST	exponentially weighted moving average for a stationary process
EWMS	exponentially weighted mean squared deviation
CUSUM	cumulative sum

3.2.2 Symbols

T	index set for a stochastic process
μ	true process mean
σ	true process standard deviation
$N(\mu,\sigma^2)$	normal distribution with a mean of μ and variance of σ^2
γ	autocovariance
$\hat{\gamma}$	estimator of autocovariance
ρ	autocorrelation
$\hat{\rho}$	estimator of autocorrelation
ϕ	dependent parameter of an AR(1) process
λ	smoothing parameter for EWMA
r	smoothing parameter for EWMS
τ	time lag between two time points
S_t^2	EWMS at t
S_0^2	initial value of S_t^2
X_t	random variable X at t
a_t	random variable a at t in an AR(1) process
Δ	step mean change as a multiple of the process standard deviation
$\overline{x}$	arithmetic mean value of a sequence of x
s	standard deviation of a sequence of x
$\hat{X}_t$	prediction of X_t
R_t	residual at t
$\overline{R}$	arithmetic mean value of R_t
S_R	standard deviation of $\{R_t\}$
Z_t	EWMA statistic at t
Z_0	initial value of Z_t
L_Z	value of the control limit for Z_t (expresses in number of standard deviation of Z_t)

σ_Z	standard deviation of EWMA statistic
σ_a	standard deviation of the random variables a_t from white noise in an AR(1) process

4 Control charts for autocorrelated processes for monitoring process mean

4.1 General

Many statisticians and statistical process control practitioners have found that autocorrelation in process data has an impact on the performance of the traditional SPC charts. Similar to autocovariance (see 3.1.1), autocorrelation is internal correlation between members of a series of observations ordered in time. Autocorrelation can be caused by the measurement system, the dynamics of the process, or both. In Annex B, the impact of positive autocorrelation on the performance of various traditional control charts is demonstrated.

4.2 Residual charts

The residual charts have been used to monitor possible changes of the process mean. To construct a residual chart, time series or other mathematical modelling has to be applied to the process data.

The residual chart requires modelling the process data and to obtain the process residuals[1]. For a set of time series data, $\{x_t; t=1,2,...,N\}$, a time series or other mathematical model is established to fit the data. A residual at t is defined as:

$$R_t = x_t - \hat{x}_t$$

where $\hat{x}_t$ is the prediction of the time series at t based on a time series or other mathematical model.

Assuming that the model is true, the residuals are statistically uncorrelated to each other. Then, traditional SPC charts such as X charts, CUSUM charts and EWMA charts can be applied to the residuals. When an X chart is applied to the residuals, it is usually called an X residual chart. Once a change of the mean in the residual process is detected, it is concluded that the mean of the process itself has been out-of-control.

Similarly, the CUSUM residual chart and EWMA residual chart are proposed[2][3]. See Reference [4] for comparisons between residual charts and other control charts.

Advantage of the residual charts:

— a residual chart can be applied to any autocorrelated data, even if it is nonstationary. Usually, a model is established with time series or other model fitting software.

Disadvantages of the residual charts:

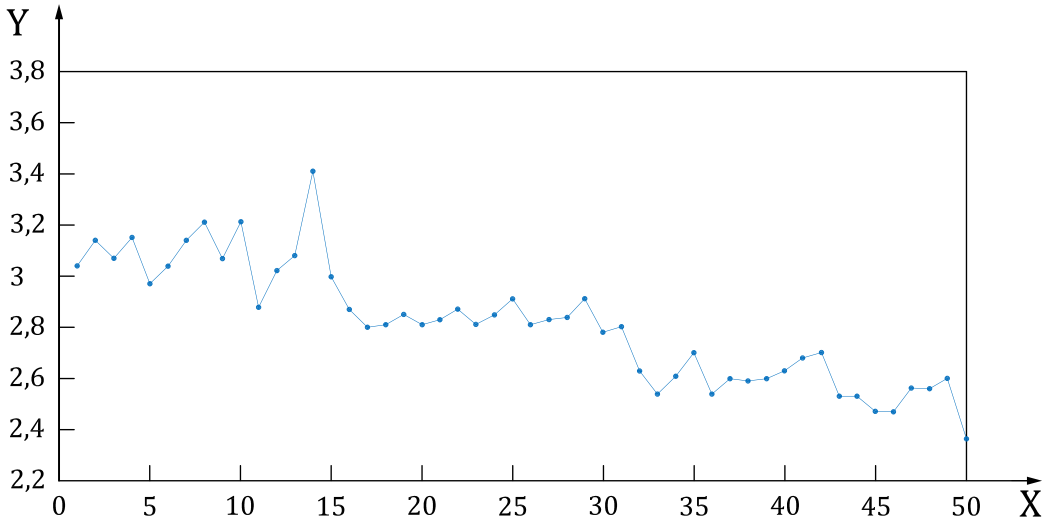
— the residual charts do not have the same properties as the traditional charts. The X residual chart for an AR(1) process (for an AR(1) process, see A.3.3) can have poor capability to detect a mean shift. Reference [5] shows that when the process is positively autocorrelated, the X residual chart does not perform well. Reference [6] shows that the detection capability of an X residual chart sometimes is small comparing to that of an X chart;

— the residual charts require time series or other modelling. The user of a residual chart shall check the validity of the model over time to reduce the mixed effect of modelling error and process change.

An example is illustrated in which the data, with a size of 50, are the daily measurements of the viscosity of a coolant in an aluminium cold rolling process[7]. Figure 1 shows the data with a decreasing trend. It is suspected that the measurements are not independent. Figure 2 shows the sample autocorrelation function (ACF) for lags from 0 to 12. For sample autocorrelation and ACF, see A.4.2 and A.5 in Annex A, and Reference [8]. As indicated in A.5, under the assumption for an i.i.d. normal sequence, approximately

95 % of the sample autocorrelations with a lag larger than one should fall between the bounds of $\pm 1{,}96/\sqrt{50}$. Based on that, the data are not independent. Reference [7] provides a model with the predicted viscosity at a period t given by:

$$\hat{x}_t = a + bx_{t-1} + cx_{t-2} + dx_{t-3} + ex_{t-4}, \; t = 1,\ldots,50$$

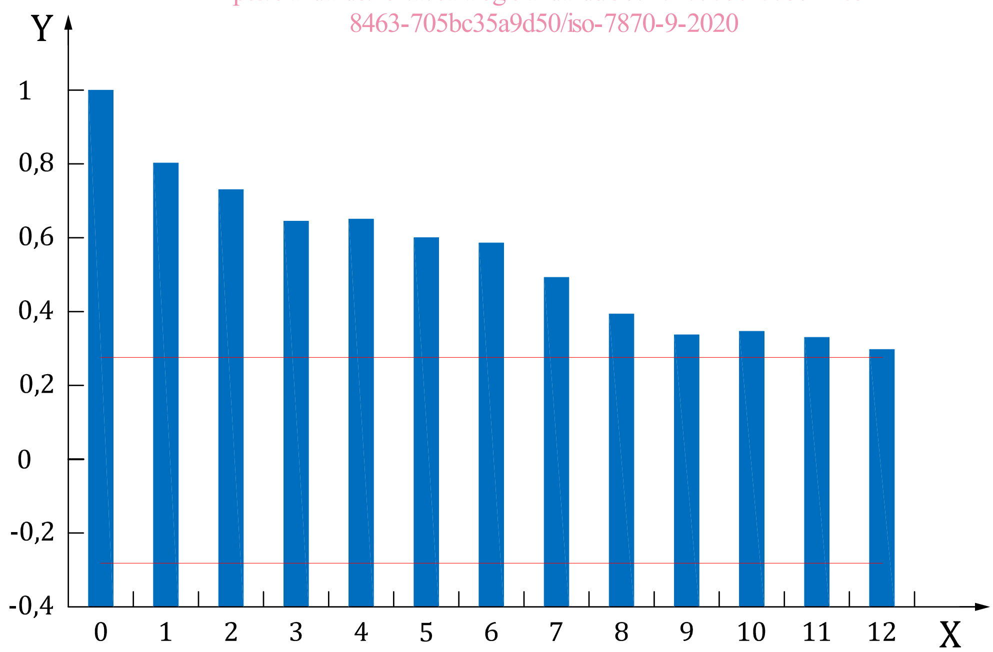


Key

X observation

Y viscosity

Figure 1 — Example



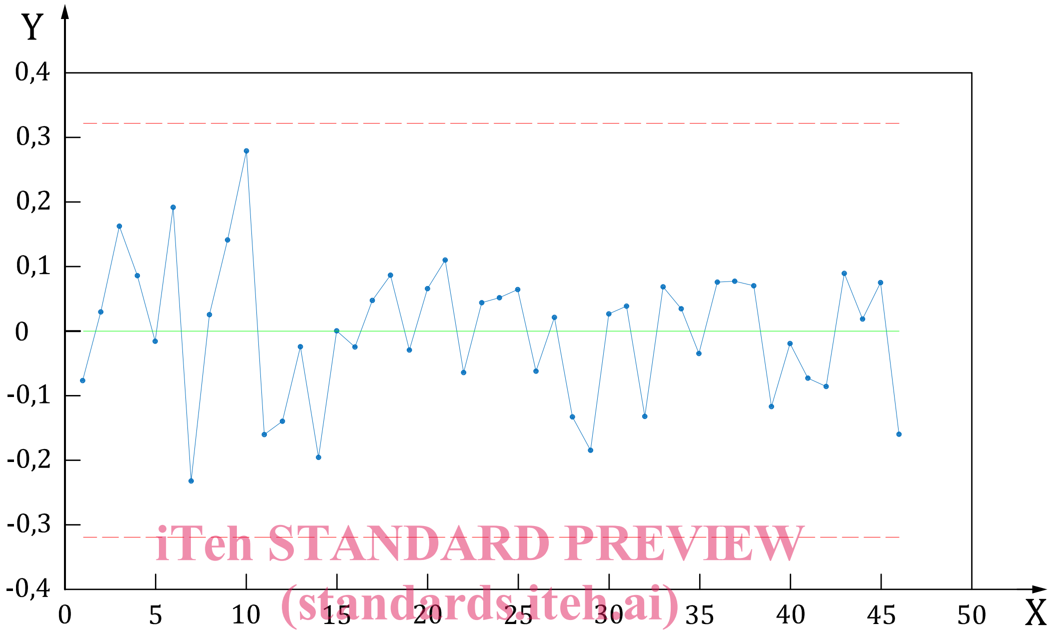
Key

X lag

Y autocorrelation

Figure 2 — Sample autocorrelations for the series of daily measurements of viscosity and an approximate 95 % confidence band

For the estimates of *a*, *b*, *c*, and *d* given in Reference [7], the residuals are calculated by $R_t = x_t - \hat{x}_t$, $t = 1,\ldots, 46$ which are shown in Figure 3. To test whether the residuals are independent from each other, the ACF with a confidence band is again applied and shown in Figure 4. Since the residuals are determined to be not autocorrelated, a *X* chart with 3*σ* control limits ($\bar{R} \pm 3S_R$, where $\bar{R}$ is the average of $\{R_t\}$ and S_R is the standard deviation of $\{R_t\}$) applies to the residuals, as shown in Figure 3. It is concluded that the mean of the residuals, as well as the process, is in control.



Key

X time

Y residual

Figure 3 — Residuals of the viscosity series and the *X* chart with 3*σ* control limits

