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**Statistical methods in process  
management — Capability and  
performance —**

Part 9:

**Process capability statistics for  
characteristics defined by geometrical  
specifications**

*Méthodes statistiques dans la gestion de processus — Aptitude et  
performance —*

*Partie 9: Méthodes statistiques pour l'aptitude des processus dont les  
caractéristiques sont définies par des spécifications géométriques*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

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This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in products and process management*.

This document is a second draft for approval and only editorial changes will be made before publication.

A list of all parts in the ISO 22514 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at [www.iso.org/members.html](http://www.iso.org/members.html).

## Introduction

Many organizations will need to evaluate the capability and performance of their key processes when the specifications are defined by requirements other than linear size. The methods described in this document are intended to assist the organization in this respect.

During the last couple of years, it has been more common in the design and development departments in companies to not only use linear tolerances alone, but also including modifiers as well as geometrical tolerances with or without use of the maximum material requirements.

This situation has been supported by new measurement methods used in production, where it is common to use measurement equipment, where the results are given in form of point clouds instead of one single value.

It is a challenge in such cases to calculate capability and performance, but organizations and customers still require the capability indices in acceptance of produced or delivered batches of parts.

This document describes how to calculate capability or performance where functional requirements on parts are given.

As an example, the “maximum material requirement”, MMR, covers “assemble ability” and the “least material requirement”, LMR, covers, for example, “minimum wall thickness” of a part. Each requirement (MMR and LMR) combines two independent requirements into one collective requirement, which simulates the intended function of the workpiece. In some cases of both MMR and LMR, the “reciprocity requirement”, RPR, can be added.

In [Annex D](#), a case study of process analysis, where the characteristic to be improved is perpendicularity, is introduced.

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# Statistical methods in process management — Capability and performance —

## Part 9: Process capability statistics for characteristics defined by geometrical specifications

### 1 Scope

This document describes process capability and performance measures when the specifications are given by geometrical product specifications e.g. maximum material requirements or linear size with a modifier.

The purpose of this document of the international series of standards on capability calculation is to assist the organizations to calculate the PCIs (process capability index) when geometrical product specifications are used on drawings.

### 2 Normative references

There are no normative references in this document.

### 3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

#### 3.1 Terms

##### 3.1.1

##### **feature of size**

feature of linear size

geometrical feature, having one or more intrinsic characteristics, only one of which can be considered as variable parameter, that additionally is a member of a “one parameter family”, and obeys the monotonic containment property for that parameter

EXAMPLE 1 A single cylindrical hole or shaft is a feature of linear size. Its linear size is its diameter.

EXAMPLE 2 Two opposite parallel plane surfaces are a feature of linear size. Its linear size is the distance between the two parallel planes.

[SOURCE: ISO 17450-1:2011, 3.3.1.5.1., modified: “may” replaced by “can”, deleted Note 1 to Note 4, deleted reference to [Figure 5](#) (ISO 17450-1:2011), deleted EXAMPLE 2, added new EXAMPLE 2]

### 3.1.2

#### **local size**

local linear size

local size characteristic

local linear size characteristic

size characteristic having by definition a non-unique result of evaluation along and around the feature of size

Note 1 to entry: For a given feature, an infinity of local sizes exists.

[SOURCE: ISO 14405-1:2016, 3.6, modified: “and/or” replaced by “and”, Note 2 to entry to Note 4 to entry deleted.]

### 3.1.3

#### **two-point size**

<local size>

distance between two opposite points on an extracted integral linear feature of size

Note 1 to entry: A two-point size taken on a cylinder can be called a “two-point diameter”. In ISO 17450-3, this is defined as a local diameter of an extracted cylinder.

Note 2 to entry: A two-point size taken on two opposite planes can be called “two-point distance”. In ISO 17450-3, this is defined as a local size of two parallel extracted surfaces.

[SOURCE: ISO 14405-1:2016, 3.6.1, modified: deleted Note 1 to entry to Note 3 to entry, added two new notes]

### 3.1.4

#### **envelope requirement**

combination of the two-point size applied for the least material limit of the size and either the minimum circumscribed size or the maximum inscribed size applied for the maximum material limit of the size

Note 1 to entry: The “envelope requirement” was previously referred to as the “Taylor principle”.

Note 2 to entry: According to ISO 8015, the surface of a single feature of size (e.g. cylindrical surface or a feature based on two parallel plane surfaces) cannot violate the envelope of a geometrical ideal form at a maximum material limit of size

[SOURCE: ISO 14405-1:2016, 3.8, modified: Note 2 to entry added]

### 3.1.5

#### **maximum material virtual size**

##### **MMVS**

size generated by the collective effect of the maximum material size, MMS, of a feature of size and the geometrical tolerance (form, orientation or location) given for the derived feature of the same feature of size

Note 1 to entry: Maximum material virtual size, MMVS, is a parameter for size used as a numerical value connected to maximum material virtual condition, MMVC.

Note 2 to entry: For external features, MMVS is the sum of MMS and the geometrical tolerance, whereas for internal features, it is the difference between MMS and the geometrical tolerance.

Note 3 to entry: The MMVS for external features of size,  $l_{MMVS,e}$ , is given by the following formula:

$$l_{MMVS,e} = l_{MMS} + \delta$$

and the MMVS for internal features of size,  $l_{MMVS,i}$ , is given by the following one:

$$l_{MMVS,i} = l_{MMS} - \delta$$

where



$l_{MMS}$  is the maximum material size;

$\delta$  is the geometrical tolerance.

### 3.1.6

#### least material virtual size

##### LMVS

size generated by the collective effect of the least material size, LMS, of a feature of size and the geometrical tolerance (form, orientation or location) given for the derived feature of the same feature of size

Note 1 to entry: Least material virtual size, LMVS, is a parameter for size used as a numerical value connected to least material virtual condition, LMVC.

Note 2 to entry: For external features, LMVS is the difference between LMS and the geometrical tolerance, whereas for internal features, it is the sum of LMS and the geometrical tolerance.

Note 3 to entry: The LMVS for external features of size,  $l_{LMVS,e}$ , is given by the following formula:

$$l_{LMVS,e} = l_{LMS} - \delta$$

and the LMVS for internal features of size,  $l_{LMVS,i}$ , is given by the following one:

$$l_{LMVS,i} = l_{LMS} + \delta$$

where

$l_{LMS}$  is the least material size;

$\delta$  is the geometrical tolerance.

### 3.1.7

#### maximum material requirement

##### MMR

requirement for a feature of linear size, defining a geometrical feature of the same type and of perfect form, with a given value for the intrinsic characteristic (dimension) equal to the maximum material virtual size, which limits the non-ideal feature on the outside of the material

Note 1 to entry: Maximum material requirement, MMR, is used to control the assembly ability of a workpiece.

[SOURCE: ISO 2692:2021, 3.12, modified Note 2 to entry deleted.]

### 3.1.8

#### least material requirement

##### LMR

requirement for a feature of linear size, defining a geometrical feature of the same type and of perfect form, with a given value for the intrinsic characteristic (dimension) equal to the last material virtual size, which limits the non-ideal feature on the inside of the material

Note 1 to entry: Least material requirements, LMR is used, for example, to control the minimum wall thickness between two symmetrical or coaxially located similar features of size.

[SOURCE: ISO 2692:2021, 3.13, modified Note 2 to entry deleted.]

**3.1.9**  
**reciprocity requirement**  
**RPR**

additional requirement for a feature of linear size indicated in addition to the maximum material requirement, MMR, or the least material requirement, LMR to indicate that the size tolerance is increased by the difference between the geometrical tolerance and the actual geometrical deviation

[SOURCE: ISO 2692:2021, 3.14]

**3.2 Abbreviated terms**

ASME	American Society of Mechanical Engineers
LMC	least material conditions
LMS	least material size
LMR	least material requirement
LMVC	least material virtual condition
LMVS	least material virtual size
MMC	maximum material condition
MMR	maximum material requirement
MMS	maximum material size
MMVS	maximum material virtual size
PCI	process capability index
RPR	reciprocity requirement

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**3.3 Symbols**

In addition to the symbols listed below, some symbols are defined where they are used within the text.

$C_p$	process capability index
$C_{pk}$	minimum process capability index
$C_{pkL}$	lower process capability index
$C_{pkU}$	upper process capability index
$D$	Diameter
$\Delta$	geometrical tolerance
$\delta_A$	measured geometrical tolerance
$l_{LMS}$	least material size
$l_{LMVS,e}$	LMVS for external features of size
$l_{LMVS,i}$	LMVS for internal features of size
$l_{MMS}$	maximum material size

$l_{\text{MMVS}}$	maximum material virtual size
$l_{\text{MMVS,e}}$	MMVS for external features of size
$l_{\text{MMVS,i}}$	MMVS for internal features of size
$L_{\text{SL}}$	lower specification limit
$N$	total sample size
$n$	subgroup sample size
$\mu$	location of the process; population mean value
$P_p P_{\text{po}}$	process performance index
$P_{\text{pk}} P_{\text{pok}}$	minimum process performance index
$P_{\text{pk}_L}$	lower process performance index
$P_{\text{pk}_U}$	upper process performance index
$\theta$	Scale parameter of the Rayleigh distribution
$s$	standard deviation, sample statistic
$\bar{s}$	average sample standard deviation
$\sigma$	standard deviation, population
$U_{\text{SL}}$	upper specification limit
$\bar{x}$	average from sample
$X_{99,865\%}$	upper 99,865 % quantile
$X_{0,135\%}$	lower 0,135 % quantile

## 4 Statistical measures used in the calculation of process capability or performance

### 4.1 General

The statistical analysis described in this document is designed to determine capability or performance indices when the characteristic of interest is a feature of linear size, and this size has a geometrical modifier added to the specification or a geometrical tolerance with or without maximum material condition.

### 4.2 Independency principle

#### 4.2.1 General

A GPS specification for a feature or relation between features can be fulfilled independent of other specifications except when it is stated by special indication e.g.  $\textcircled{M}$  modifiers according to ISO 2692, CZ according to ISO 1101 or  $\textcircled{E}$  modifiers according to ISO 14405-1 as part of the specification. Each requirement ( $\textcircled{E}$ , MMR and LMR) combines two independent requirements into one collective requirement, which more accurately simulates the intended function of the workpiece. In some cases of both MMR and LMR, the “reciprocity requirement”, RPR, can be added.

If those special indications are used as requirements, they need to be considered as a collective requirement and the capability indices can be calculated as one common value.

### 4.2.2 Maximum Material ISO versus ASME

In this standard the ISO definitions as defined in ISO 8015 are used. Geometrical product specifications in ASME are defined in Y 14,5 that often differs from the definitions in ISO. Tolerancing in ISO geometrical features are individual and independent of each other. In ASME tolerancing of the mating behaviour of the part in the assembly group used.

### 4.2.3 Measurement procedure

The measurement procedure is especially important when measuring properties with modifiers or geometric tolerances. The tolerance applies to the entire surface of the workpiece in three dimensions with an infinite number of points, therefore a sufficient number of measuring points defined in the procedure can be measured on every workpiece. You also have to consider the distribution of these points. More information can be found in [Annex C](#).

### 4.3 Location

It is a precondition, that the size of the characteristic of interest can have only one value assigned and a characterisations of process location can be the mean,  $\mu$ , or the median,  $X_{50\%}$ . If the variation of the characteristic can be described by a symmetric distribution the mean is the most natural selection, with non-symmetric distributions the median is the preferred selection.

### 4.4 Dispersion

It is important to differentiate between a standard deviation that measures only variation based on e.g. 50 samples and the standard deviation which measures variation from more than 100 samples. Methods for calculating standard deviations representing these two cases are given in [Annex A](#). Very often, when data are gathered over a long period of time, the standard deviation is larger due to the effects of fluctuations in the process. It is important that the use and calculation of the standard deviation in the formulae only make sense if the data are normally distributed.

In case of a characteristics with modifiers added or characteristics defined with geometrical tolerances the actual distribution in most cases cannot be described by a normal distribution therefore, the capability calculation formula based on reference limits can be used instead. The formulae for the distribution models can be found in [Annex B](#).

### 4.5 Reference limits

The lower and upper reference limits are respectively defined as the 0,135 % and the 99,865 % quantiles of the distribution that describes the output of the process characteristic. They are described as  $X_{0,135\%}$  and  $X_{99,865\%}$ .

### 4.6 Reference interval

The reference interval is the interval between the upper and the lower reference limits. The reference interval includes 99,73 % of the individual values in the population from a process.

## 5 Geometrical product specifications

### 5.1 General

Produced workpieces exhibit deviations from the ideal geometric form shown on a drawing. The real value of the dimension of a feature of size is dependent on the form deviations and on the specific type of size applied.